Dariusz WIĘCKOWSKI

Warsaw University of Technology, Faculty of Automotive and Construction Machinery Engineering
ORCID: 0000-0002-6367-4489

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Using logistic mapping to determine the size of the set of autonomous vehicles in the monitored area

***Abstract**. The work is related to the issue of collecting and processing data on the number and type of vehicles in the monitored area. Attention was paid to mobility not only on roads, but also in space, without traffic jams and stress and without a driver. It was noted that, regarding drivers, many publications refer to research related to their physical fitness, mental fitness, knowledge, skills and attitudes. Very often, a characteristic parameter is selected, i.e. reaction time (braking, avoiding obstacles). The paper presents a method for supporting the management of autonomous vehicle traffic organization. Issues such as maintaining an appropriate distance between vehicles to avoid collisions were taken into account, making it possible to achieve driving velocities appropriate to given traffic conditions. The attention was paid to maintaining a smooth ride by limiting acceleration and braking, which results in energy reduction, and at the same time it is possible to optimize the energy recovery process in the case of electric drives. For this purpose, a control and management model was proposed. Logistic mapping was used also for this purpose. The logistic mapping allows us to explain specific processes with planned flow parameters. A function describing the model was proposed. A model for analysing the growth of the vehicle fleet on a year-to-year basis over a 12-year period is presented. The model proposed in this work eliminates subjective issues because it refers to objects moving without the participation of the operator (driver). A stability criterion was proposed. A logistic mapping stability process was carried out.*

***Streszczenie**. Praca dotyczy zagadnienia gromadzenia i przetwarzania danych o liczbie i rodzaju pojazdów na monitorowanym obszarze. Zwrócono uwagę na mobilność nie tylko na drogach, ale także w przestrzeni, bez korków i stresu oraz bez kierowcy. Zauważono, że w odniesieniu do kierowców wiele publikacji odwołuje się do badań związanych z ich sprawnością fizyczną, sprawnością psychiczną, wiedzą, umiejętnościami i postawami. Bardzo często wybierany jest charakterystyczny parametr, tj. czas reakcji (hamowanie, omijanie przeszkód). W artykule przedstawiono metodę wspomagania zarządzania organizacją ruchu pojazdów autonomicznych. Uwzględniono takie zagadnienia, jak utrzymanie odpowiedniej odległości między pojazdami w celu uniknięcia kolizji, co pozwala na osiągnięcie prędkości jazdy odpowiednich do danych warunków ruchu. Zwrócono uwagę na utrzymanie płynności jazdy poprzez ograniczenie przyspieszania i hamowania, co skutkuje redukcją energii, a jednocześnie możliwa jest optymalizacja procesu odzyskiwania energii w przypadku napędów elektrycznych. W tym celu zaproponowano model sterowania i zarządzania. Wykorzystano w tym celu mapowanie logistyczne. Mapowanie logistyczne pozwala na wyjaśnienie określonych procesów z zaplanowanymi parametrami przepływu. Zaproponowano funkcję opisującą model. Przedstawiono model do analizy wzrostu floty pojazdów w ujęciu rok do roku w okresie 12 lat. Model zaproponowany w tej pracy eliminuje kwestie subiektywne, ponieważ odnosi się do obiektów poruszających się bez udziału operatora (kierowcy). Zaproponowano kryterium stabilności. Przeprowadzono proces mapowania stabilności logistycznej. (**Wykorzystanie mapowania logistycznego do określenia wielkości zestawu pojazdów autonomicznych na monitorowanym obszarze**)*

Keywords: set increase analysis, number of autonomous vehicles in a specific area, logistic mapping
Słowa kluczowe: analiza wzrostu zestawu, liczba pojazdów autonomicznych na określonym obszarze, mapowanie logistyczne

Introduction

The idea of an autonomous car is not new, but it was only in the 21st century that work on it accelerated. Automatization of vehicles is a continuous, evolutionary process and more and more of its elements are included in currently produced vehicles [2]. A look at the 21st century means not only moving vehicles, but also flying autonomous objects used to transport people and loads. The goal is the mobility not only on roads, but also in space, without traffic jams and stress and without a driver. The effect is to increase transport efficiency. Systems that would control or manage the motion of such vehicles must primarily ensure the collection and processing of data on the number and type of vehicles (objects) located in the monitored area, their velocity and traffic intensity. As well as information about the road infrastructure and surroundings [13]. Regarding drivers, many publications refer to research related to physical fitness, mental fitness, knowledge, skills and attitudes of the driver. A very often chosen characteristic parameter is reaction time, both in the braking process and in the process of avoiding obstacles. An important issue is driver modeling taking into account real time. The safety of the driver's behavior, his driving style and the related probability of an accident or collision risk [10, 16]. The maneuver control process, use of appropriate sensors, preparation of the model and control algorithm. At the same time, the issue of recognizing the sensitivity of the vehicle maneuver control system to interference and errors in measurement signals. Use – modeling and numerical research [5, 8]. Proposed solutions for various autonomous vehicle traffic control and planning systems. Planning and assessment of car driving smoothness in various operating conditions. Driver identification using driver behavior signals while driving [19, 23-25]. As a part of intelligent transport network systems, striving to minimize traffic delays and minimize the costs of not only operation but also the entire infrastructure [1, 21]. All this must be supplemented with legal regulations needed to introduce autonomous vehicle traffic management systems, for example [14]. Generally, vehicle movement in autonomous mode requires solving many interdisciplinary issues. The most important of them are as follows [4]:

- defining a method for identifying the vehicle's position (with specified accuracy) and correlating this position with the current map
- defining the object recognition method (sensors, software, algorithms);
- determination of the way of the decisions' implementation during maneuvers
- defining the procedure for admitting vehicles to traffic
- solving legislative issues

In the process of managing the traffic of autonomous vehicles, existing tools can and even should be used, for example issues of classical mechanics through the use of logistic mapping [22]. Human is most often the weakest link in the entire road traffic process [17].

In this work, the author proposes a model that eliminates these problems because it refers to objects moving without the participation of the operator (driver).

Recently (the 20s of this century), we have been observing the phenomenon of the synergy between artificial intelligence techniques and transport systems. For engineers and scientists in many fields, the greatest IT, automotive and transport challenge have become the pursuit of transport autonomy, not only by supporting the driver, but by completely eliminating him. One of the characteristic features of the autonomy of autonomous transport is the implementation of the journey: "point to point" or "door to door" without intermediate stops [7]. Two autonomous passenger transport systems can be distinguished [3, 7]:

1) PRT – Personal Rapid Transit. It consists of 3-4 passenger vehicles running on light ground or aboveground infrastructure. Here we are dealing with a "point to point" or "door to door" system.

2) APM – Automated People Mover. In this case, we are dealing with vehicles (usually rail) moving along a precisely defined route, with intermediate stops.

At the same time, it should be noted that the fleet management system (FMS) covers issues from many areas, such as: freight and passenger transport logistics, road, air, ground and rail transport, material handling system, on-demand mobility services, operation planning, influence on design vehicles, vehicle supervision and diagnostics, velocity management, energy supply, etc. [6 ~~19~~]. The purpose of FMS systems is to: minimize the vehicle and infrastructure costs, improve the efficiency and productivity. The size of the fleet of any transport system depends on [7]: the size of the area in which the system is implemented, the expected number of people in a given area (as potential customers of the system), the level of the service quality, order the policy and vehicle control. Planning vehicle traffic, especially in large agglomerations, is a big challenge for: urban planners, architects, engineers and scientists of various disciplines, and the users themselves. Therefore, models are used to better predict the effects of new solutions.

Dynamic models are commonly used when considering issues related to control-ling motion of autonomous vehicles. Algorithms for control in various execution phases of various maneuvers are being developed. The model presented in the form of differential equations does not fully allow for drawing significant conclusions regarding the dynamics of the motion of the guided vehicle. It is advisable to transform it from the form of differential equations to the transmittance function, which allows for a broader analysis of traffic dynamics - examples of studies [9, 11, 12].

The issues related to the collection and processing of data on the number and type of vehicles in the monitored area are becoming important. They become particularly important in the context of the concept of traveling without a driver - autonomous vehicles. The lack of drivers in the autonomous vehicles eliminates the human factor from the driver-vehicle-environment system. As a result, there are no subjective influences in the driving process.

The work presents a method of supporting the management of the organization of such vehicle traffic, proposing a control and management model. Logistic mapping was used for this purpose. This is a function that, in a general sense, determines the value of the variable k for a sequence of equally spaced time values t, $k_{t+1} = qk_t(1 - kt)$. And although it is not a mechanical system, its evolution has many features in common with the evolution of nonlinear mechanical systems. Logistic mapping allows us to explain peculiar processes at planned flow parameters. The logistic mapping is useful not only for forecasting population size. Mitchell Feingenbaum developed the theory of universality, according to which mathematical models of nonlinear dynamic processes, whether biological, economic or physical, proceed in the same way [9]. The logistic mapping is worth knowing, because it allows to explain peculiar processes at the planned flow parameters. In the process of developing a community (set), logistics is a regulator that maintains the growth of the set within certain limits [22].

Discrete time and mappings

In almost all problems of mechanics, we are interested in the evolution of the system for continuously changing time. However, there are systems for which time is a discrete variable. Even when a variable is specified as a continuous function of time, it may turn out that we only need its value for certain discrete times. For example, when analysing the number of vehicles in a certain area (e.g. in an urban agglomeration), we may not be interested in daily fluctuations in the number of set, we would rather want to know the number of set once a year.

It follows that there are situations where the primary goal is to predict the state of a system for any given time, so that it may sometimes be useful to record its state only at discrete moments separated by an interval of one cycle. In such a situation, we can be satisfied with this smaller amount of information. But so that any information about the behaviour of the systems, with discrete time values that can be obtained, should make it easier to predict the state of a system of in the next cycles. We can imagine and assume that the size of some set, for example the number of autonomous vehicles in some agglomeration n_{t+1} in year $t + 1$, is uniquely determined by the size of the set n_t in the previous year t. For such a system there will be a function f transforming n_t into the corresponding number n_{t+1}:

$$n_{t+1} = f(n_t) \tag{1}$$

The equation in this form can be called the growth equation for a given set, in this case autonomous vehicles. Of course, this is a very simplified model. In the real world, the size of the set depends not only on n_t, but also on many other factors, such as the number of vehicles in working order or out of order. Nevertheless, one can adopt a model for which n_{t+1} will depend uniquely only on nt.

The simplest example of a growth model in the form (1) is the equation for the case when n_{t+1} is proportional to n_t.

$$n_{t+1} = f(n_t) = qn_t \tag{2}$$

That is, the function f(n) determining the size of the set in the next year, based on the population size in the current year, has the form.

$$f(n) = qn \tag{3}$$

where: q – positive constant, as a parameter of the growth rate for a given population.

For example, if in a given year one autonomous vehicle is withdrawn from use, and at the beginning of the next year two additional vehicles appear, the size of the set of these vehicles will satisfy equation (2) with the parameter $q = 2$.

The solution to equation (2) for n_t is expressed by introducing n_0 as the initial value. The process of behaviour n_t on a broad time scale is presented below. The solution to equation (2) can be presented as follows:

$$n_1 = f(n_0) = qn_0 \tag{4}$$

and

$$n_2 = f(n_1) = f(f(n_0)) = q^2 n_0 \tag{5}$$

from which it follows that for t-times

$$n_t = f(n_{t-1}) = f(f(\ldots f(n_0)\ldots)) = q^t n_0 \tag{6}$$

From the above it follows that:
- for q > 1 the size of the set nt grows exponentially, tending to infinity for t → ∞;
- for q = l the size of the set remains constant;
- for q < 1 the size of the set decreases exponentially to zero.

Note-comment. In mathematics, the terms "function" and "mapping" are treated almost as synonyms. Therefore, it can be assumed that equation (1) defines n_{t+1}, as a function of nt.

It can also be assumed that (1) is a mapping in which f transforms the number n_t into the corresponding number n_{t+1}:

$$n_t \xrightarrow{f} n_{t+1} = f(n_t) \quad (7)$$

or

$$n_0 \xrightarrow{f} n_1 \xrightarrow{f} n_2 \xrightarrow{f} n_3\ n_t \xrightarrow{f} \ldots \quad (8)$$

This is an iterated, one-dimensional mapping.

Logistic mapping

For further considerations, an exponential mapping for $q > 1$ was adopted as a real model for the initial increase in the number of vehicles. However, it should be noted that in real conditions, the set of vehicles cannot grow exponentially indefinitely. At some point, factors such as over-density come into play and cause growth to slow down. Therefore, function (3) can be modified into a more real (not only theoretical) model of the increase in the number of vehicles. The following function can be proposed:

$$f(n) = qn(1 - n/N) \quad (9)$$

where: N - "large" positive constant defining the capacity of the set.

In this way, we replace the exponential mapping (6) with the logistic mapping [15].

$$n_{t+1} = f(n_t) = qn_t(1 - n_t/N) \quad (10)$$

As long as the size of the set of vehicles is small compared to N, the term n_t/N in formula (10) is irrelevant. The mapping (10) gives the same exponential evolution as the exponential mapping (6). As n_t approaches N, the part of the equation n_t/N becomes significant and the value of the expression ($1 - n_t/N$) starts to decrease. Then the growth rate slows down and the ex-pression ($1 - n_t/N$) causes the slowdown (what it would be for large values of n) when over-densification occurs. In the special case when the number n_t reaches a value equal to N, then the expression ($1 - n_t/N$) in formula (10) would assume a value equal to zero. This means that the size of the set in the next year n_{t+1} would be zero. If n_t exceeded the value of N, then n_{t+1} would take a negative value, which makes no sense in this case. It follows that the size of the set described by the logistic mapping (10) can never exceed the value N. Therefore, the number N determines the capacity of the set (environment) for this model. In this way, the meaning of the number N is explained. It should be noted that the logistic mapping (10) is nonlinear. Figures 1 and 2 show a comparison of exponential growth (6) and logistic growth (10), for the following conditions: growth rate parameter $q = 2$, initial set size $n_0 = 4000$, set capacity $N = 1\,000\,000$, time $t = 12$ years. The exponential line (Figure 1) shows the constant doubling of the number of vehicles, with exponential growth. The second curve (Figure 2) shows the growth predicted by logistic mapping (10). In this case, the logistic growth is different from the exponential growth, and for time $t \to 12$ the logistic growth becomes slower (excessive density), and the size of the set of vehicles is set at $n = 500\,000$.

Then, for the logistic mapping, instead of the set size n, you can enter the relative size of the set:

$$k = n/N \quad (11)$$

which is the ratio of the size of the set n at a given moment to the maximum possible size of the set N. Dividing both sides of equation (10) by N, the growth equation for kt will be:

$$k_{t+1} = f(k_t) = qk_t(1 - k_t) \quad (12)$$

where:

$$f(k) = qk(1 - k) \quad (13)$$

redefined function giving the mapping f.

Since the size of the set n takes values in the interval $0 \le n \le N$, the relative size of the set $k = n/N$ is limited to the interval:

$$0 \le k \le 1 \quad (14)$$

In this interval, the function $k(1 - k)$ takes a maximum value of 1/4 (for $k = 1/2$). In order for k_{t+1} from formula (12) not to exceed the value of 1, it should be assumed that the growth rate parameter cannot take values outside the range $0 \le q \le 4$. Therefore, the logistic mapping (12) will be analysed in the range $0 \le k \le 1$ for parameters meeting the condition $0 \le q \le 4$.

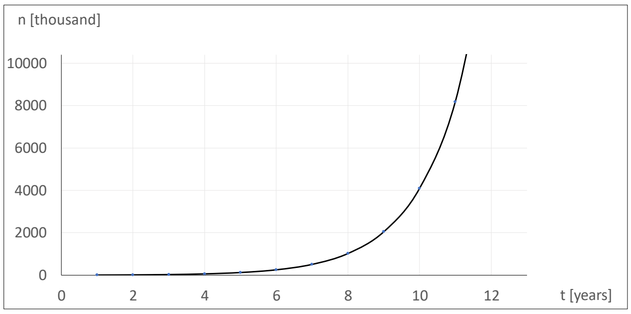

Fig. 1. Exponential growth, size of the set grows infinitely $q = 2$

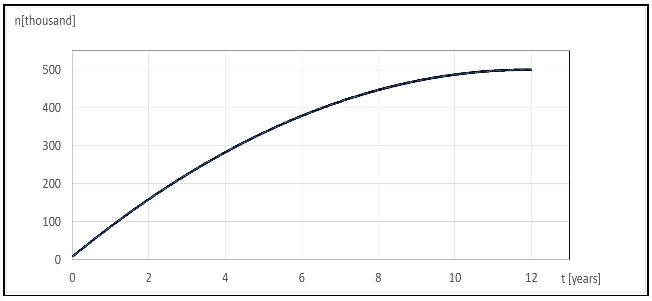

Fig. 2. Comparison of exponential and logistic growth, per parameter growth rate $q = 2$. Logistic growth, which becomes slower over time, and the size of the set eventually stabilizes at $n = 500\,000$

Figure 3 shows changes in the size of the logistic mapping set for $q = 0.8$ and two different initial values $k_0 = 0.1$ and $k_0 = 0.5$. In both cases, $k_t \to 0$ for $t \to \infty$. It follows that for $q < 1$, the size of the set, regardless of the initial value, tends to zero. This also follows from equation (12) because for $k_{t+1} \le qk_{t-1}$, $k_t \le q_tk_0$ if $q = 1$, then $k_t \to 0$ if $t \to \infty$.

Figure 4 shows the size of the set for the growth rate parameter $q = 1.5$ and the same two initial values ko. For $k_0 = 0.1$, the size of the set initially increases. For a larger initial size $k_0 = 0.5$, the set size initially decreases. However, in both cases, on a long time scale, the abundance is set at $k = 0.375$. The relative population sizes $k_t = n/N$ for the logistic mapping (12) shown in Figures 3 and 4 are determined for the time conditions $t \to 12$ and for $n \to 500\,000$. In both cases presented in Figures 3 and 4, the logistic mapping has a constant attractor, to which the population size tends: it is $k = 0$ for $q < 1$ and $k = 0.375$ for $q = 1.5$. For other values that t and n tend to, the attractor values will be different. If the size of the set at the initial moment is equal to the size of the permanent attractor $k_0 = k^*$, then it will have the same value $k_t = k^*$ for any time t. This can only be the case if:

$$f(k^*) = k^* \quad (15)$$

Any value of k^* that satisfies equation (15) is called a fixed point of the map f. These fixed points have a meaning analogous to the equilibrium position as in the case of a mechanical system, in the sense that when the starting point for the evolution of the system is a fixed point, then the state of the system does not change and the system remains at this point for any length of time [15]. If a mapping is given, we can find its fixed points by solving equation (15). For example, the fixed points of a logistic map-ping must satisfy the equation:

$$qk^*(l - k^*) = k^* \quad (16)$$

The solution is:

$$k^* = 0 \text{ or } k^* = (q\text{-}1)/q \quad (17)$$

The first solution is the fixed point $k^* = 0$, which was already described earlier. The second solution depends on the value of q. For $q < 1$ it has a negative value and is therefore irrelevant in these considerations. For $q = 1$, this solution coincides with the first solution $k^* = 0$. However, for $q > 1$ the solution gives a second fixed point. For ex-ample, for $q = 1.5$ we obtain, as before, a fixed point $k^* = 0.375$.

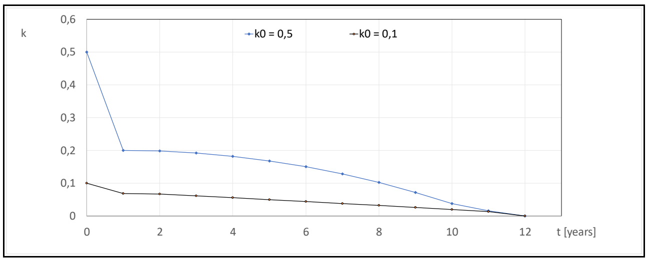

Fig. 3. Relative size of the set $k_t = n/N$ for mapping logistic (12) for two different initial values k0 and value growth rate parameter q = 0.8, for time $t \rightarrow 12$ and for $n \rightarrow 500\ 000$ the size of the set quickly approaches zero for both $k_0 = 0.1$ and $k_0 = 0.5$

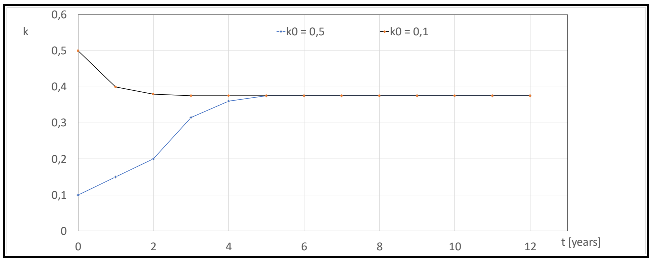

Fig. 4. Relative size of the set $k_t = n/N$ for mapping logistic (12) for two different initial values k_0 and value growth rate parameter q = 1.5, for time $t \rightarrow 12$ for $n \rightarrow 500\ 000$, the relative population size tends to a constant value of 0.375 (constant attractor)

The advantage of logistic mapping is that we can solve equation (15) and find fixed points in analytical form. However, the graphic form is interesting. To solve equation (15) graphically, two functions k and $f(k)$ should be graphed depending on the argument k – Fig. 5.

Fixed points correspond to the values of k^* for which both graphs intersect. If q is small, then the curve $y = f(k)$ lies below the line $y = k$, inclined at 45˚ to the kt axis, so that the only point of intersection is $k^* = 0$. This means that the only fixed point is $k^* = 0$. If q is large, then the curve $y = f(k)$ rises above the line $y = k$ and we get two points constantly.

The limiting value between these cases is when the slope of the curve $y = f(k)$ at point $k = 0$ is equal to q. If we increase q, this curve intersects the line $y = k$ (which has a slope equal to 1) for $q = 1$. Hence, for $q < 1$ we have only one fixed point $k^* = 0$, and for $q > 1$ we have two points constant, $k^* = 0$ and $k^* > 0$. The advantage of the graphical approach is that it can be applied to any other function $f(k)$, as long as its graph has a shape qualitatively similar to the curves shown in Figure 5 (bent "arc" up).

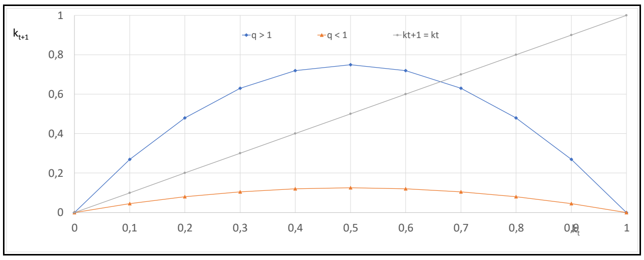

Fig. 5. Graph of the function $k_{t+1} = k_t$ (line inclined at 45˚) and the logistic mapping $f(k) = qk^*(l - k^*)$ (two curves) for two values $q < 1$ and $q > 1$. The fixed points of the logistic map lie at the intersection straight line with a curve. For $q < 1$ there is only one intersection point $k^* = 0$, for $q > 1$ there are two intersection points $k^* = 0$ and $k^* > 0$

Stability criterion

Because k^* is a fixed point of the logistic map (attractor), i.e. in a sense equivalent to the balance position. Therefore, if the initial size of the set is equal to k^*, it will remain so all the time. In order to consider k^* as a mapping attractor, it is necessary to additionally check whether k^* is a stable fixed point. This means that if the size of the set is close to k^*, as a result of evolution it will approach the value of k^* and not move away from it. This can be checked as follows. If kt is close to k^*, it can be written:

$$k_t = k^* + \varepsilon_t \quad (18)$$

where: ε_t – distance kt from the fixed point k^*.
If ε_t is a small quantity, an approximation can be used when calculating k_{t+1}

$$k_{t+1} = f(k_t) = f(k^* + \varepsilon_t) \approx f(k^*) + f'(k^*)\varepsilon_t = k^* + \Delta\varepsilon_t \quad (19)$$

where: Δ – value of the derivative of the function $f(k)$ at point k^* (k^* is a fixed point which means that $f(k^*) = k^*$).

$$\Delta = f'(k^*) \quad (20)$$

According to expression (18), we have $k_{t+1} = k^* + \varepsilon_t+1$. Comparing this with the last part of expression (19) we can write:

$$\varepsilon_{t+1} \approx \Delta\varepsilon_t \quad (21)$$

From relation (20) it follows that for $|\Delta| < 1$, when k_t is close to k^*, subsequent values get closer and closer to k^*. If $|\Delta| > 1$, then for k_t, close to k^*, subsequent values move away from k^*. In this way we obtain the stability criterion we are looking for. Therefore, the stability of fixed points can be written as follows. Let k^* be a fixed point of the mapping $k_{t+1} = f(k_t)$: $f(k^*) = k^*$. If $|f'(k^*)| < 1$, then k* is a stable fixed point and acts as an attractor. If $|f'(k^*)| > 1$, then k^* is an unstable fixed point and acts as a repulsive point.

Conclusions

The work uses logistic mapping to describe the course of the changes in the set of autonomous vehicles in an area, for example in an urban agglomeration. A model for analysing the year-to-year increase in the vehicle set is presented. The logistic mapping used for this purpose makes it possible to explain peculiar processes at the planned flow parameters.

When it comes to traveling in three dimensions instead of a two-dimensional system, the question arises whether this type of motion can be controlled at a high level of density of moving objects. In aviation, this is a relatively simple problem because the distances between units and objects are very large (measured in kilometres). When we have "flying vehicles" in an urban area, the traffic volume is high, so the distances between the individual vehicles much decrease (dramatically). It should be emphasis sized that movement of the vehicles moving in such conditions is not and cannot be chaotic. Otherwise, the vehicles would collide with each other.

Another issue. Since vehicles must move without collisions, it must be assumed that the parameters of their motion (velocity, momentum, energy, etc.) must be de-fined. These issues become particularly important when there are concepts of traveling without a driver (operator) - autonomous vehicles.

At the same time, it is necessary to take into account the issues such as maintaining appropriate separation between vehicles to avoid collisions, and thus it is possible to achieve driving velocities appropriate for given traffic conditions. Maintaining a smooth ride by limiting acceleration and braking, resulting in energy reduction, while at the same time it is possible to optimize the energy recovery process in the case of electric drives.

Moreover, in large urban agglomerations, restrictions are being introduced on the number of vehicles due to potential traffic jams or environmental pollution. Therefore, for example, various regulations apply to the purchase of vehicles, especially by private individuals - an example of such agglomeration is Singapore [18, 20].

Author: dr hab. inż. Dariusz Więckowski, Warsaw University of Technology, Faculty of Automotive and Construction Machinery Engineering; Narbutta st 84, 02-524 Warsaw, Poland; e-mail: dariusz.wieckowski@pw.edu.pl;

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