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Mathematical model of a compensated asynchronous motor as an element of a microgrid

***Abstract**. A mathematical model was developed to analyse electromagnetic and electromechanical transients and periodic modes of a compensated asynchronous motor. The model is formed in the normal Cauchy form and adapted to be used in computer mathematics software environments. The model provides the ability to take into account the circuit, mode and parametric features of a compensated induction motor as an element of a microgrid, the nonlinearity of the web-ampere characteristic of the main magnetic circuit of its magnetic core and the active power losses in it.*

***Streszczenie.** W artykule opracowano model matematyczny do analizy elektromagnetycznych i elektromechanicznych stanów nieustalonych oraz przebiegów okresowych kompensowanego silnika asynchronicznego. Model został utworzony w normalnej formie Cauchy'ego i przystosowany do użycia w komputerowych środowiskach. Model zapewnia możliwość uwzględnienia szczególnych charakterystyk kompensowanego silnika indukcyjnego jako elementu mikrosieci. (**Model matematyczny kompensowanego silnika asynchronicznego jako elementu mikrosieci**)*

Keywords: compensated asynchronous motor, mathematical model, microgrid.
Słowa kluczowe: kompensowany silnik asynchroniczny, model matematyczny, mikrosieć.

Introduction

An important direction of improving intelligent electric power distribution system (EPDS) (microgrid) and optimizing their modes and processes is the use of compensated asynchronous motors (AM) [1-4]. One of the important problems of the research of such type systems is the analysis of electromagnetic and electromechanical transient processes of EPDS during starts-up and self-starts-up of compensated AM and stable periodic processes. A progressive method of studying these systems is mathematical modeling using modern software packages and computer mathematics systems. The available specialized software packages do not allow for a sufficiently complete study of the transient processes of complex EPDSs that supply traditional and compensated AMs, taking into account circuit, mode, and parametric features, especially the nonlinearity of the characteristics of the elements of their electrical and magnetic circuits.

The presence in electric power distribution system of semiconductor frequency converters feeding AM causes the problem of electromagnetic compatibility of EPDS devices with conventional and compensated AMs. This problem receives special attention in branched EPDS networks with small levels of short-circuit currents accompanied by low-frequency transients [6, 11].

Papers [1-4] present the results of analyzing the influence of additional winding circuits and capacitor capacitance on the parameters of steady-state coordinates and starting characteristics of a compensated AM, but do not solve the problem of its being taken into account as an element of the electric power distribution system.

The goal of this paper is to solve the problem of mathematical modeling of compensated AM as an element of an electrical or electric power distribution system.

Formation of Mathematical Model

A. Mathematical model of compensated induction motor.

The compensated asynchronous motor differs from the classic one by the presence of an additional winding of the stator, which is connected in series with the main winding according to the scheme of a rotary autotransformer, with which the compensating capacitor is turned on in series [1-5].

To create a mathematical model of a compensated AM in this work, a scheme of AM was chosen in which the additional winding is made without a spatial offset of the axis relative to the main winding.

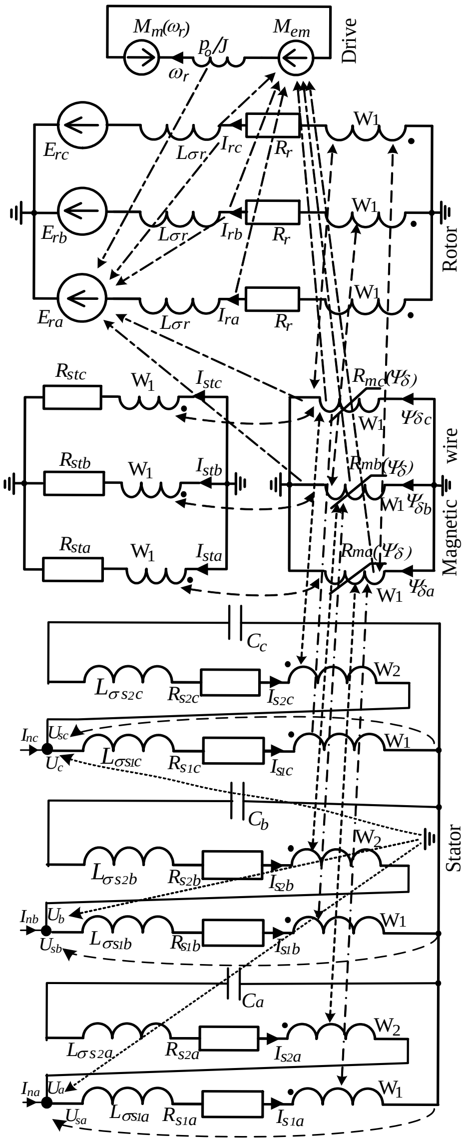

Fig. 1. Equivalent circuit of compensated AM

The equivalent circuit of the compensated AM, created based on the mathematical theory of the electromechanical converter using the equations of the conventional AM given in [7,8,9], is shown in Fig. 1.

The mathematical model of the induction motor is formed in the fixed phase coordinates of the stator and the slowed coordinates of the rotor as a projection of the spatial vectors of phase voltages, currents, fluxes, and magnetizing forces on the phase axes and active power losses in the magnetic circuit based on the approach to the creation of a mathematical model of an induction motor given in [8, 9].

The differential equations of the electric circuits of the main winding of the compensated AM are formed in the following form:

(1) $$u_{sa} = R_{s1a} i_{s1a} + L_{\sigma s1a} \frac{d\, i_{s1a}}{dt} + \frac{d\phi_{\delta a}}{dt},$$

(2) $$u_{sb} = R_{s1b} i_{s1b} + L_{\sigma s1b} \frac{d\, i_{s1b}}{dt} + \frac{d\phi_{\delta b}}{dt},$$

(3) $$u_{sc} = R_{s1c} i_{s1c} + L_{\sigma s1c} \frac{d\, i_{s1c}}{dt} + \frac{d\phi_{\delta c}}{dt},$$

where u_{sa}, u_{sb}, u_{sc} are phase voltages of the stator winding; $i_{s1a}, i_{s1b}, i_{s1c}$ – phase currents of the main winding of the stator, $\phi_{\delta a}, \phi_{\delta b}, \phi_{ca}$ are flux coupling of the stator phases reduced to the main winding from the main magnetic flux; $R_{s1a}, R_{s1b}, R_{s1c}, L_{\sigma s1a}, L_{\sigma s1b}, L_{\sigma s1c}$ are resistances and dissipation inductances of the winding of the stator phases

The differential equations of the electric circuits of the additional winding of the compensated AM are created in this form:

(4) $$u_{sa} = R_{s2a} i_{s2a} + L_{\sigma s2a} \frac{d\, i_{s2a}}{dt} + K_{12}^{-1} \frac{d\phi_{\delta a}}{dt} + u_{ca},$$

(5) $$u_{sb} = R_{s2b} i_{s2b} + L_{\sigma s2b} \frac{d\, i_{s2b}}{dt} + K_{12}^{-1} \frac{d\phi_{\delta b}}{dt} + u_{cb},$$

(6) $$u_{sc} = R_{s2c} i_{s2c} + L_{\sigma s2c} \frac{d\, i_{s2c}}{dt} + K_{12}^{-1} \frac{d\phi_{\delta c}}{dt} + u_{cc},$$

where $i_{s2a}, i_{s2b}, i_{s2c}$ – phase currents of the additional winding of the stator, $R_{s1a}, R_{s1b}, R_{s1c}, L_{\sigma s1a}, L_{\sigma s1b}, L_{\sigma s1c}$ – resistances and dissipation inductances of winding of stator phases; $K_{12}=W_1/W_2$ is the transformation coefficient between the main and additional windings of AM.

Differential equations of voltage drops u_{ca}, u_{cb}, u_{cc} on the compensating capacitors of AM phases have the following form:

(7) $$\frac{du_{ca}}{dt} = \frac{1}{C_a} i_{s2a},$$

(8) $$\frac{du_{cb}}{dt} = \frac{1}{C_b} i_{s2b},$$

(9) $$\frac{du_{cc}}{dt} = \frac{1}{C_c} i_{s2c}.$$

The differential equations of the AM rotor with a short-circuited winding have the following form:

(10) $$L_{\sigma r} \frac{di_{ra}}{dt} + \frac{d\phi_{\delta a}}{dt} + R_r i_{ra} + \left(L_{\sigma r}\left(i_{rb} - i_{rc}\right) + \phi_{\delta b} - \phi_{\delta c}\right)\frac{\omega_r}{\sqrt{3}} = 0,$$

(11) $$L_{\sigma r} \frac{di_{rb}}{dt} + \frac{d\phi_{\delta b}}{dt} + R_r i_{rb} + \left(L_{\sigma r}\left(i_{rc} - i_{ra}\right) + \phi_{\delta c} - \phi_{\delta a}\right)\frac{\omega_r}{\sqrt{3}} = 0,$$

(12) $$L_{\sigma r} \frac{di_{rc}}{dt} + \frac{d\phi_{\delta c}}{dt} + R_r i_{rc} + \left(L_{\sigma r}\left(i_{ra} - i_{rb}\right) + \phi_{\delta a} - \phi_{\delta b}\right)\frac{\omega_r}{\sqrt{3}} = 0,$$

where i_{ra}, i_{rb}, i_{rc} are rotor phase currents reduced to the stator winding; ω_r is angular speed of rotation of the rotor reduced to the pole section of the stator; R_r, $L_{\sigma r}$ are equivalent resistances and dispersion inductances of the phases of the rotor winding reduced to the stator winding, the values of which must be the same according to the imposed coordinate transformation conditions.

The active power losses in the magnetic core are taken into account by introducing additional circuits with a resistor in the substitute circuit of the AM electrical circuits, the quantitative value of which will reflect the losses by analogy with the losses in the transformer magnetic core [10]. The equations for these circuits (Fig. 1) are as follows:

(13) $$\frac{d\phi_{\delta a}}{dt} + R_{sta} i_{sta} = 0,$$

(14) $$\frac{d\phi_{\delta b}}{dt} + R_{stb} i_{stb} = 0,$$

(15) $$\frac{d\phi_{\delta c}}{dt} + R_{stc} i_{stc} = 0,$$

where $i_{sta}, i_{stb}, i_{stc}$ – currents in resistors R_{sta}, R_{stb}, R_{sc}, which take into account losses of active power in the AM magnetic circuit.

The state equations of the magnetic circuit written on the basis of Ampere's law have the following form:

(16) $$\frac{3}{2}\left(i_{s1a} + K_{12}^{-1} i_{s2a} + i_{ra} + i_{sta}\right) - \frac{i_m\left(\phi_\delta\right)}{\phi_\delta}\phi_{\delta a} = 0,$$

(17) $$\frac{3}{2}\left(i_{s1b} + K_{12}^{-1} i_{s2b} + i_{rb} + i_{stb}\right) - \frac{i_m\left(\phi_\delta\right)}{\phi_\delta}\phi_{\delta b} = 0,$$

(18) $$\frac{3}{2}\left(i_{s1c} + K_{12}^{-1} i_{s2c} + i_{rc} + i_{stc}\right) - \frac{i_m\left(\phi_\delta\right)}{\phi_\delta}\phi_{\delta c} = 0,$$

where $i_m\left(\phi_\delta\right)$ is the module of the magnetizing current spatial vector, which is determined from the web-ampere characteristic of the main magnetic circuit of the AM, $\phi_\delta = \sqrt{2\left(\phi_{\delta a}^2 + \phi_{\delta b}^2 + \phi_{\delta c}^2\right)/3}$ is the module of the space vector of the working flow coupling, $i_m\left(\phi_\delta\right)\phi_{\delta\nu} / \phi_\delta$, $\nu = a,b,c$ – projections of the magnetizing current spatial vector onto the phase axes.

Taking into account that the relation $i_m\left(\phi_\delta\right) / \phi_\delta$ in physical essence is a static magnetic resistance $R_m\left(\phi_\delta\right)$ of the main magnetic circuit AM, after determining the current expressions from equations (16–18). $i_{sta}, i_{stb}, i_{stc}$ in resistors R_{sta}, R_{stb}, R_{sc} and after substituting them into equations (13–15), we get

(19) $$\frac{d\phi_{\delta a}}{dt} = R_{sta}\left(i_{s1a} + K_{12}^{-1} i_{s2a} + i_{ra} - \frac{2}{3} R_m\left(\phi_\delta\right)\phi_{\delta a}\right),$$

(20) $$\frac{d\phi_{\delta b}}{dt} = R_{stb}\left(i_{s1b} + K_{12}^{-1} i_{s2b} + i_{rb} - \frac{2}{3} R_m\left(\phi_\delta\right)\phi_{\delta b}\right),$$

(21) $$\frac{d\phi_{\delta c}}{dt} = R_{stc}\left(i_{s1c} + K_{12}^{-1} i_{s2c} + i_{rc} - \frac{2}{3} R_m\left(\phi_\delta\right)\phi_{\delta c}\right).$$

The equation of motion of the AM rotor has the following form:

$$(22)\qquad \frac{J}{p_0}\frac{d\omega_r}{dt} = M_{em}(\bar{\phi}_\delta \times \bar{i}_r) - M_m(\omega_r)$$

where J, p_o – respectively, the moment of inertia of the AM rotor with the drive mechanism and the number of pole pairs and

$M_{em}\left(\bar{\phi}_\delta \times \bar{i}_r\right) = -p_0\left(\phi_{\delta a}\left(i_{rb}-i_{rc}\right)+\phi_{\delta b}\left(i_{rc}-i_{ra}\right)+\phi_{\delta c}\left(i_{ra}-i_{rb}\right)\right)/\sqrt{3}$

– electromagnetic moment AM, $M_m(\omega_r)$ is the mechanical moment of resistance of the drive mechanism.

The obtained system of differential equations of the AM state can be reduced to the normal Cauchy form by means of the corresponding identical transformations. To do this, in equations (1–6) and (10–12), we replace the derivatives of the flux linkages of the phases of the main magnetic circuit with the right-hand parts of equations (19–21). As a result, we get

$$(23)\qquad \frac{d\,i_{s1a}}{dt} = L_{\sigma s1a}^{-1}\begin{pmatrix} u_{sa}-\left(R_{s1a}+R_{sta}\right)i_{s1a}- \\ R_{sta}\left(K_{12}^{-1}i_{s2a}+i_{ra}-\frac{2}{3}R_m\left(\phi_\delta\right)\phi_{\delta a}\right)\end{pmatrix},$$

$$(24)\qquad \frac{d\,i_{s1b}}{dt} = L_{\sigma s1b}^{-1}\begin{pmatrix} u_{sb}-\left(R_{s1b}+R_{stb}\right)i_{s1b}- \\ R_{stb}\left(K_{12}^{-1}i_{s2b}+i_{rb}-\frac{2}{3}R_m\left(\phi_\delta\right)\phi_{\delta b}\right)\end{pmatrix},$$

$$(25)\qquad \frac{d\,i_{s1c}}{dt} = L_{\sigma s1c}^{-1}\begin{pmatrix} u_{sc}-\left(R_{s1c}+R_{stc}\right)i_{s1c}- \\ R_{stc}\left(K_{12}^{-1}i_{s2c}+i_{rc}-\frac{2}{3}R_m\left(\phi_\delta\right)\phi_{\delta c}\right)\end{pmatrix}$$

$$(26)\qquad \frac{d\,i_{s2a}}{dt} = L_{\sigma s2a}^{-1}\begin{pmatrix} u_{sa}-u_{ca}-\left(R_{s2a}+K_{12}^{-2}R_{sta}\right)i_{s2a}- \\ K_{12}^{-1}\left(i_{sa1}+i_{ra}-\frac{2}{3}R_m\left(\phi_\delta\right)\phi_{\delta a}\right)\end{pmatrix},$$

$$(27)\qquad \frac{d\,i_{s2b}}{dt} = L_{\sigma s2b}^{-1}\begin{pmatrix} u_{sb}-u_{cb}-\left(R_{s2b}+K_{12}^{-2}R_{stb}\right)i_{s2b}- \\ K_{12}^{-1}R_{stb}\left(i_{s1b}+i_{rb}-\frac{2}{3}R_m\left(\phi_\delta\right)\phi_{\delta b}\right)\end{pmatrix},$$

$$(28)\qquad \frac{d\,i_{s2c}}{dt} = L_{\sigma s2c}^{-1}\begin{pmatrix} u_{sc}-u_{cc}-\left(R_{s2c}+K_{12}^{-2}R_{stc}\right)i_{s2c}- \\ K_{12}^{-1}R_{stc}\left(i_{s1c}+i_{rc}-\frac{2}{3}R_m\left(\phi_\delta\right)\phi_{\delta c}\right)\end{pmatrix},$$

$$(29)\qquad \frac{di_{ra}}{dt} = L_{\sigma r}^{-1}\begin{pmatrix} R_{sta}\left(\frac{2}{3}R_m\left(\phi_\delta\right)\phi_{\delta a}-i_{s1a}-K_{12}^{-1}i_{s2a}\right)- \\ \left(R_r+R_{sta}\right)i_{ra}-\left(L_{\sigma r}\left(i_{rb}-i_{rc}\right)-\phi_{\delta b}+\phi_{\delta c}\right)\frac{\omega_r}{\sqrt{3}}\end{pmatrix},$$

$$(30)\qquad \frac{di_{rb}}{dt} = L_{\sigma r}^{-1}\begin{pmatrix} R_{stb}\left(\frac{2}{3}R_m\left(\phi_\delta\right)\phi_{\delta b}-i_{s1b}-K_{12}^{-1}i_{s2b}\right)- \\ \left(R_r+R_{stb}\right)i_{rb}-\left(L_{\sigma r}\left(i_{rc}-i_{ra}\right)-\phi_{\delta c}+\phi_{\delta a}\right)\frac{\omega_r}{\sqrt{3}}\end{pmatrix},$$

$$(31)\qquad \frac{di_{rc}}{dt} = L_{\sigma r}^{-1}\begin{pmatrix} R_{stc}\left(\frac{2}{3}R_m\left(\phi_\delta\right)\phi_{\delta c}-i_{s1c}-K_{12}^{-1}i_{s2c}\right)- \\ \left(R_r+R_{stc}\right)i_{rc}-\left(L_{\sigma r}\left(i_{ra}-i_{rb}\right)-\phi_{\delta a}+\phi_{\delta b}\right)\frac{\omega_r}{\sqrt{3}}\end{pmatrix}..$$

The system of differential equations (7-9) and (19-31) can be integrated in software environments. The advantage of the obtained system of equations is its representation in the normal Cauchy form, which ensures its implementation in modern computer mathematics systems.

The asynchronous motor connection with the EPDS is established through phase voltages u_{sa}, u_{sb}, u_{sc} winding and phase voltages of the network u_a, u_b, u_c according to the following formulas [9] (Fig. 1) $u_{sa} = \left(2u_a - u_b - u_c\right)/3$, $u_{sb} = \left(2u_b - u_a - u_c\right)/3$, $u_{sc} = \left(2u_c - u_a - u_b\right)/3$.

The mathematical model was tested for an asynchronous motor with the following parameters: U_n=380 V, P_n=30 kW, $cos\varphi$=0.91, f=50 Hz, p_o=1, $L_{\sigma s1}$ =1.11 mH, R_{s1}=0.16 Ohm, $L_{\sigma r}$=1.56 mH, R_r=0.08 Ohm. The Weberampere characteristic of the magnetic circuit is approximated by a polynomial $i_m\left(\phi_\delta\right) = 17{,}42\phi_\delta + 1{,}8\phi_\delta^5 + 0{,}74\phi_\delta^9$ from which the characteristic of static magnetic resistance is obtained $R_m\left(\phi_\delta\right) = 17{,}42 + 9\phi_\delta^4 + 6{,}7\phi_\delta^8$.

Resistances of losses of active power in the magnetic circuit are assumed to be constant $R_{sta} = R_{stb} = R_{stc} =$ 300 Ohm. The additional AM winding is selected with the following parameters $L_{\sigma s2}$=1.11 mH, R_{s2}=0.08 Ohm, C_a=C_b=C_c=55 μF and K_{12}=2. The motor is connected to an EPDS with a phase voltage of 220 V and internal resistance with parameters L=64 mH and R=0.01 Ohm. The mechanical characteristics of the drive mechanism are assumed to be ventilatory:

$$M_m\left(\omega_r\right) = 430\left(0{,}2+0{,}8\left(\omega_r/308\right)^2\right).$$

Figure 2 illustrates the shapes of the curves of the phase voltage EPDS U_a of phase A (a), the input current I_{na} of the AM (b) (fig. 1), and the dependence of the phase voltage on the input current (c) for the steady-state periodic mode of the AM with a selected value of the compensation capacitance with a value of C_a=C_b=C_c=55 μF, which provides a power factor $cos\,\varphi = 1$. This conclusion confirms the linear dependence of the phase voltage U_a on the input current I_{na} EPDS (c).

a)
b)
c)

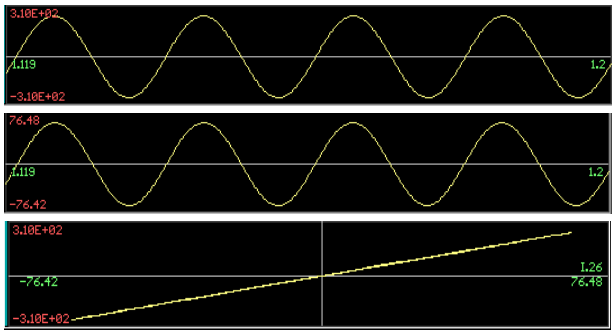

Fig. 2. Coordinates of the AM steady state

In Fig. 3 the forms of the AM coordinate curves during direct start are shown: the phase voltage of the motor, which in the steady-state mode has an amplitude value of 310 V (a), the voltage across the compensation capacitor is 460 V (b), the input current of the motor and the reduced main winding current of the rotor winding, which in the steady-state mode reach amplitude values of 76.4 and 76.6, respectively, reduced to the main winding flux from the main magnetic flux with an apparent flux of 0.94 Wb/sec (d), electromagnetic torque of the AM and drag torque with a steady-state value of 430 N*m (e), reduced to the pole section rotor angular velocity with a steady-state value 308 sec-1 (f).

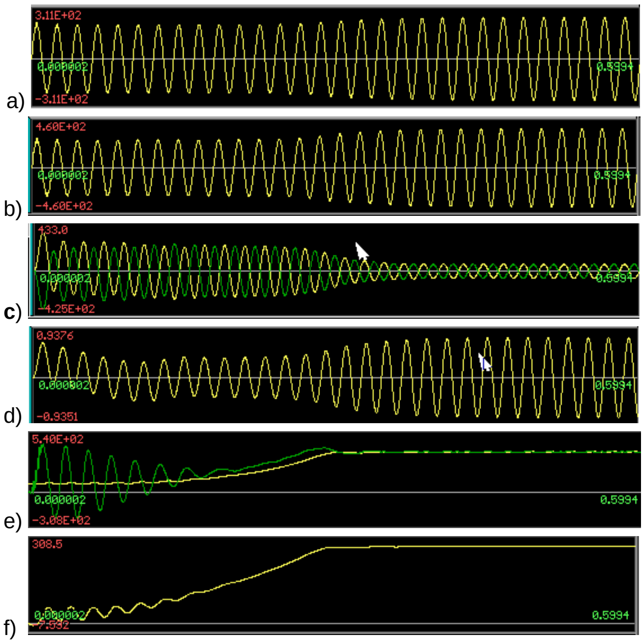

Fig. 3. Waveforms of the compensated AM mode during start-up

Conclusions

A universal mathematical model has been created for the analysis of transient processes and periodic modes in a compensated asynchronous motor as an element of an electrotechnical or electric power distribution system, taking into account the connection schemes of the stator winding and the parameters of its electric circuits, the nonlinearity of the magnetization characteristics of the magnetic circuit and the losses of active power in it. The model is formed in the normal Cauchy form and allows to expand the possibilities of research of transient processes in complex electric power distribution system and electrotechnical systems when solving specific problems of analysis and synthesis of such systems.

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