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Mathematical model of an induction motor with consideration of current displacement in rotor bars using Matlab/Simulink

***Abstract**. A mathematical model of an induction motor with a squirrel-cage rotor is proposed that takes into account the saturation of the magnetic system and the current displacement in the rotor bars. In order to take into account the saturation of the main magnetic flux, the main magnetization curve is used. To take into account the current displacement, the bars along with the short-circuited rings are partitioned into n layers in height. As a result n windings on the rotor are coupled to the main magnetic flux and covered by different magnetic leakage fluxes.*

***Streszczenie.** Zaproponowano model matematyczny silnika indukcyjnego z wirnikiem klatkowym, który uwzględnia nasycenie układu magnetycznego i przesunięcie prądu w prętach wirnika. Aby uwzględnić nasycenie głównego strumienia magnetycznego, wykorzystano główną krzywą magnesowania. Aby uwzględnić przesunięcie prądu, pręty wraz z zwartymi pierścieniami podzielono na n warstw wysokości. W rezultacie n uzwojeń na wirniku jest sprzężonych z głównym strumieniem magnetycznym i pokrytych różnymi strumieniami rozproszenia magnetycznego. (**Model matematyczny silnika indukcyjnego z uwzględnieniem przesunięcia prądu w prętach wirnika przy użyciu pakietu Matlab/Simulink**)*

Keywords: mathematical model, induction motor, current displacement, magnetic flux.
Słowa kluczowe: model matematyczny, silnik indukcyjny, przesunięcie prądu, strumień magnetyczny.

Introduction

Induction motors are an important component of a modern electric drive. They are widely used in the industrial sector in equipment such as electric furnaces, pumps, rolling mills, paper machines, combined metalworking machines, roller bearing drives, walking excavators, crane drives, elevators, wind tunnels, and are characterized by their ability to operate for a long time without technical maintenance, resulting in low operating and maintenance costs. They can be easily adapted to the requirements of different applications in size, power, and speed. These motors can operate at a variety of speeds and loads, making them suitable for use in a wide range of electric drives. Most machines operate with an efficiency of more than 85%, helping to reduce energy consumption and operating costs.

In modern conditions, the development of asynchronous drives requires the correct selection of an asynchronous motor and the development of a control system that would ensure the maximum possible efficiency of the drive system in general. In this situation, mathematical modeling of the electric drive system plays an important role. Obviously, the adequacy of the results of mathematical modeling depends on the chosen mathematical model of the IM, in particular, the consideration of those factors that affect the behavior of the electric drive in different operating modes. The main ones are the saturation of the magnetic circuit and the displacement of current in the bars of a short-circuited rotor, and since the technical literature contains a large number of ways to take these factors into account, an important problem is to analyze them to determine their suitability for solving specific problems.

Among the well-known mathematical modelling tools, the Matlab/Simulink software environment holds a leading position. It implements mathematical models of asynchronous motors with squirrel-cage and phase rotor in SimPowerSystem in the "Asynchronous machine" section. This block implements a three-phase induction machine (wound rotor, single squirrel-cage, or double squirrel-cage) that operates in generator or motor mode.

The model of an induction motor provides the possibility to set the parameters of the magnetic saturation curve of the stator and rotor iron (saturation of the mutual flux) without load. However, these mathematical models of the Matlab/Simulink 'Induction Machine' block are based on the classical single-phase T-shaped equivalent circuit [1], where the parameters have unchanged nameplate values, and the phenomenon of magnetic circuit saturation is either not taken into account at all or their saturated values are used [2].

Essentially, linear mathematical models are used. This approach is considered to be generally accepted, and a corresponding standard has even been developed [3]. However, in a number of operating cases of squirrel-cage induction motors, the displacement of currents in the bars is crucial, especially in the case of deep-groove motors. Obviously, the calculation results obtained under such conditions do not always meet the needs of practice in terms of accuracy, so new methods of artificially taking into account the phenomenon of current displacement and saturation using various coefficients appear in the technical literature, and the number of variants of such models is constantly growing.

One of the ways to take into account the skin-effect phenomenon in rotor bars is to represent the substitute winding circuit in the form of the equivalent circuit, but the problem is to determine the parameters of its elements. The most promising direction, in our opinion, is the method proposed in [5] and developed in [6, 7], the essence of which is to represent the rotor winding in the form of several, formed by dividing each rod together with short-circuiting rings in height into several elementary ones.

The goal of the paper is to improve the mathematical model of an induction motor with a squirrel-cage rotor in Matlab/Simulink regarding the influence of current displacement in the rotor bars and leakage fluxes in the rotor circuits.

Model description

The mathematical model of an induction machine is represented by equations in the state space of the fourth order - the electrical part, and the second order - the mechanical part. Electrical variables and rotor parameters reduced to the stator winding are indicated by dashes.

Equations of equilibrium of the stator equivalent circuits represented in the system of orthogonal axes *d, q,* which rotate at an arbitrary speed, have the form

$$\frac{d\psi_{ds}}{dt} = \omega\psi_{qs} - R_s i_{ds} + U_{ds} \;;$$

(1a) $$\frac{d\psi_{qs}}{dt} = -\omega\psi_{ds} - R_s i_{qs} + U_{qs}.$$

The short-circuited rotor winding in the axes d□□q is described by two equations

$$\frac{d\psi'_{dr}}{dt} = (\omega - \omega_r)\psi'_{qr} - R'_r i'_{dr}$$

(1b) $$\frac{d\psi'_{qr}}{dt} = -(\omega - \omega_r)\psi'_{dr} - R'_r i'_{qr}$$

where U_{ds}, U_{qs} are stator voltages in dq axes; i_{ds}, i_{qs} – stator currents in dq axes; i_{dr}, i_{qr} are rotor currents in dq axes; $\psi_{d.s}$, ψ_{qs} are flux linkages of the stator along the axes d, q, ψ'_{dr}, ψ'_{qr} are rotor flux linkages; R_s, R'_r – active resistances of the phase windings of the stator and the reduced rotor; ω is a reference frame angular velocity; ω_r- eelectrical angular velocity ($\omega_m \times p$), ω-ω_r=ω_2.
The electromagnetic moment of the IM is determined by the formula

$$M_e = 1.5p(\psi_{ds}i_{qs} - \psi_{qs}i_{ds})$$

The equation of motion of the rotor of an asynchronous motor is described by the following equation

$$\frac{d\omega_m}{dt} = \frac{p}{J}(M_e - M_m) \qquad (2)$$

where ω_m is an angular frequency of rotation of the rotor; J is the combined moment of inertia of the rotor along with the load on the shaft; M_e is the torque electromagnetic moment of ID; M_m is the mechanical load moment on the motor shaft.
In order to take into account the displacement of currents in the rotor rods, the slotted part of the bars, as well as the short-circuit rings, are divided into n layers by height [4, 5]. As a result, we will get n short-circuited windings on the rotor, which we will convert to three-phase windings [2]. Thus, the mathematical model considers m=n+1 three-phase windings (Fig. 1), between which there are mutual inductive connections both due to the main magnetic flux and scattering fluxes.

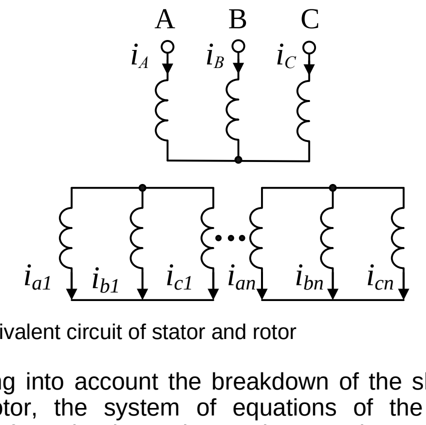

Fig. 1. Equivalent circuit of stator and rotor

Taking into account the breakdown of the slotted part of the rotor, the system of equations of the electrical balance of IM in the orthogonal axes d, q consists of equations (1) for the stator contours and 2n equations for the equivalent rotor loops:

$$\frac{d\psi'_{drk}}{dt} = (\omega - \omega_r)\psi'_{qrk} - R'_{rk} i'_{drk};$$

(3) $$\frac{d\psi'_{qrk}}{dt} = -(\omega - \omega_r)\psi'_{drk} - R'_{rk} i'_{qrk}, \ (k = 1,...,\nu).$$

The equation of mechanical equilibrium can be written in the form

(4) $$\frac{d\omega_r}{dt} = \frac{p}{J}\left(1{,}5p\left(\psi_{ds}i_{qs} - \psi_{qs}i_{ds}\right) - M_m\right).$$

The flux linkage of each winding has two components - the flux linkage with the working field, the lines of which cross the air gap, the flux linkage with the dissipation fields, the stator winding dissipation field determined only by the currents of the stator circuits, and the flux linkage of the rotor winding dissipation determined just by the rotor currents.

In addition, we will assume that the rotor winding is reduced to the stator winding by the number of phases and their turns. The current and voltage reduction factors are determined by the following formulas

$$k_i = 2m_s w_s k_s / z_2, \quad k_u = 4w_s k_s / z_2$$

where m_s – the number of phases of the stator winding; w_s, k_s – number of turns of the stator winding and its winding factor; z_2 is the number of rotor grooves .

The reduced active resistance and inductive scattering resistance of the rotor circuits are determined by the following formulas

$$r_r = r_c(4m_s(w_s k_s)^2 / z_2, \quad x_{\sigma r} = x_c(4m_s(w_s k_s)^2 / z_2$$

where r_c, x_c is active and resistance scattering of the rod.

Instead of currents and flux linkages of real IM circuits, we will consider currents and flux linkages transformed to orthogonal axes d, q contours At the same time, we will mark:

(5)
$$\Psi_{ds} = L_s i_{ds} + L_m\left(i'_{dr1} + \ldots + i'_{drn}\right)$$
$$\Psi_{qs} = L_s i_{qs} + L_m\left(i'_{qr1} + \ldots + i'_{qrn}\right)$$
$$\Psi'_{dr1} = L'_{r1} i'_{dr1} + L_m i_{ds}$$
$$\Psi'_{qr1} = L'_{r1} i'_{qr1} + L_m i_{qs}$$
$$\vdots$$
$$\Psi'_{drn} = L'_{rn} i'_{drn} + L_m i_{ds}$$
$$\Psi'_{qrn} = L'_{rn} i'_{qrn} + L_m i_{qs}$$

where L_s – total stator inductance: L_{r1},…,L_{rn} – dissipation inductance of the reduced rotor; L_m is the magnetization inductance.

The DE system (3) is supplemented with closed equations for the flux linkages of the circuits, which are determined based on the use of the magnetization curves of the main magnetic flux ψ_μ and by the dissipation currents of the stator windings $\psi_{\sigma s}$ and rotor $\psi_{\sigma r}$

$$\psi_\mu = \psi_\mu(i_\mu), \quad \psi_{\sigma s} = \psi_{\sigma s}(i_s), \quad \psi_{\sigma r} = \psi_{\sigma r}(i_r)$$

where

$$i_\mu = \sqrt{\left(i_{sd} + i_{rd}\right)^2 + \left(i_{sq} + i_{rq}\right)^2} \ ;$$

$$i_s = \sqrt{i_{sd}^2 + i_{sq}^2} \ ; \qquad i_r = \sqrt{i_{rd}^2 + i_{rq}^2} \ .$$

Due to the representation of each rotor rod by several elementary currents of the rotor circuits, they are defined as the sum of the currents of *n* bar elements.

$$i_{rx} = i_{r1x} + ... + i_{rnx}; \qquad i_{ry} = i_{r1y} + ... + i_{rny} \ .$$

The mutual inductive connection of the rotor contours along the groove scattering paths can be considered linear formed from the currents of the same phases, represented in the form

$$\vec{\Psi}_{nd} = L_n \vec{i}_d \ ; \quad \vec{\Psi}_{nq} = L_n \vec{i}_q .$$

where

$$L_n = \begin{bmatrix} L_{11} & \cdots & L_{1n} \\ \vdots & \vdots & \vdots \\ L_{n1} & \cdots & L_{nn} \end{bmatrix} \tag{6}$$

$$L_{\xi k} = \mu_0 k_z l \lambda_{\xi k} \, ,$$

where $\mu_0 = 4\pi \times 10^{-7}$ H/m; l is the calculated length of the rotor; $\lambda_{\xi k}$– own and mutual conductivities of the groove scattering paths of *n* layers of the rotor winding.

To calculate transient processes, it is necessary to integrate the nonlinear system of differential equations (3), (4) by one of the numerical methods. At a constant slip *s*, the steady-state regime is described by a nonlinear system of finite electrical equilibrium equations

$$-\omega_0 \psi_{sy} + R_s i_{sx} = u_{sx} \ ;$$

$$\omega_0 \psi_{sx} + R_s i_{sy} = u_{sy} \ ;$$

$$-s\omega_0 \psi_{1y} + R_1 i_{1x} = 0 \ ; \tag{7}$$

$$s\omega_0 \psi_{1x} + R_1 i_{1y} = 0 \ ,$$

$$-s\omega_0 \psi_{ny} + R_n i_{nx} = 0 \ \square$$

$$s\omega_0 \psi_{nx} + R_n i_{ny} = 0.$$

which is nonlinear due to the nonlinear dependence of fluxes on currents. Therefore, its solution requires the use of iterative methods.

The flux of each circuit depends on the currents of all circuits, and the solution of system (7) at a given slip value is the current vector $\vec{i}$ = (i_{sx}, i_{sy}, i_{1x}, i_{1y}, …, i_{nx}, i_{ny})* (the uppercase (*) means transposition). By setting the slip values *s* within 1.0 ≥ *s* > 0.0, the static characteristics can be calculated in the form of currents versus slip, which allow for the calculation of flux cohesion, electromagnetic torque, and power. Due to the nonlinear dependence of the flux coupling of the circuits on the currents caused by the saturation of the magnetic circuit of the IM, the system of algebraic equations (7) is nonlinear, so its solution requires the development of an appropriate algorithm.

Having formed the vectors of voltages $\vec{u} = (u_{sx}, u_{sy}, 0, \ldots, 0)^*$ and fluxes $\vec{\psi}$, we represent it in vector form

$$\vec{y}(\vec{\psi}, \vec{i}, s) = \vec{u} \ . \tag{8}$$

If the x-axis is aligned with the image vector of the supply voltage, which is commonly practiced, then $u_{sx} = U_m$; $u_{sy} = 0$, , where U_m is the amplitude value of the phase voltage. The solution to equation (8) at a given slip value *s* is the vector of loop currents. One of the ways to determine it is the differential method of continuation in the parameter [6]. To do this, in system (8), we multiply the vector $\vec{u}$ by the scalar parameter ε $(0 \le \varepsilon \le 1)$, and represent equation (8) in the form

$$\vec{y}(\vec{\psi}, \vec{i}, s) = \varepsilon \vec{u} \ . \tag{9}$$

We differentiate the system of finite equations (9) with respect to the parameter ε. As a result, we obtain the differential equation of the form

$$\begin{bmatrix} x_{sysx} & x_{sysy} & x_{sy1x} & x_{sy1y} & \cdots & x_{synx} & x_{syny} \\ -x_{sxsx} & -x_{sxsx} - r_s & -x_{sx1x} & -x_{sx1y} & \cdots & -x_{sxnx} & -x_{sxny} \\ sx_{1ysx} & sx_{1ysy} & sx_{1y1x} - r_1 & sx_{1y1y} & \cdots & sx_{1ynx} & sx_{1yny} \\ -sx_{1xsx} & -sx_{1ysy} & -sx_{1y1x} & -sx_{1y1y} - r_1 & \cdots & -sx_{1Yxnx} & -sx_{1yny} \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ sx_{nysx} & sx_{nysy} & sx_{ny1x} & sx_{ny1y} & \cdots & sx_{nynx} - r_n & sx_{nyny} \\ -sx_{nxsx} & -sx_{nxsy} & -sx_{nxsy} & -sx_{nx1y} & \cdots & -sx_{nxnx} & -sx_{nxny} - r_n \end{bmatrix} \times \begin{bmatrix} di_{sx}/dt \\ di_{sy}/dt \\ di_{r1x}/dt \\ di_{r1y}/dt \\ \vdots \\ di_{rnx}/dt \\ di_{srny}/dt \end{bmatrix} = \begin{bmatrix} U_m \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

Equation (10) is solved by the method of continuation by parameter, increasing the value of the applied voltage in proportion to the parameter ε. The value of the current vector $\vec{i}$ at a given voltage at each step is refined by the Newton method.

After reaching the steady-state mode by the method described above at a given slip $s = 1{,}0$, it is possible to perform iterative calculations for a number of consecutive slip values in the required range of changes in the mechanical characteristic. As the practice of calculations shows, the iterative process is convergent. In the event of a discrepancy, it is necessary, as at the first In the event of a discrepancy, it is necessary, as in the first stage, to exit by the parameter to the mode specified by slip.

The problem of calculating the steady-state mode at a given torque on the motor shaft is solved in two stages. At the first stage, we set a slip value that is probably close to the specified torque and calculate the coordinates of the mode, including the value corresponding to the numerical value of the slip, using the above electromagnetic torque formula. We substitute the obtained coordinate values into the steady-state equation and obtain the unbound states, which are reduced to zero step by step in the second stage of the calculation. In this case, the Jacobi matrix is the same as in the iterative refinement of the solution.

Research results

To evaluate the results of the changes made to the model of an induction motor with a squirrel-cage rotor, we simulated an induction motor in Matlab/Simulink 4AP160S4Y3 (P=15 kW, U = 220 V, I_n = 29.9 A, p_0 = 2, s_n=0.02, m_n=2.0, m_k=2.2, m_m=1.6). In Fig.2, the calculation results of current dependencies on time in three elements of the rotor bar in the starting mode with the rated torque on the shaft are presented.

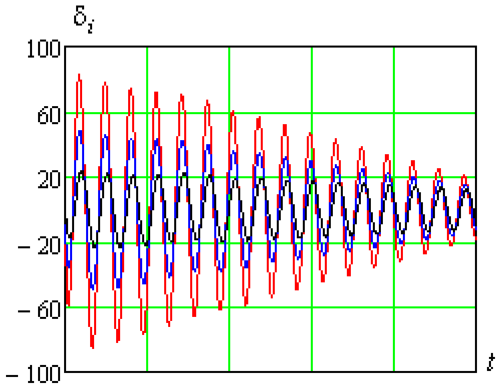

Fig. 2. An example of the results of calculating the time dependence of currents in three elements of the rotor bar in the starting mode with the rated torque on the shaft.

Figure 3 shows the calculated mechanical characteristics of the motor using the existing AD model in Matlab/Simulink and the improved model taking into account the influence of current displacement in the rotor bars.

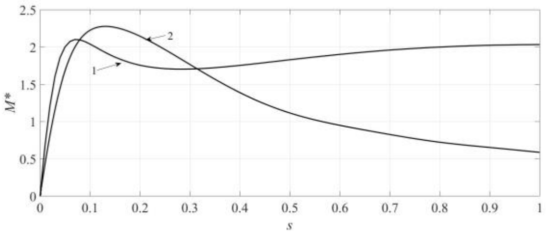

Fig. 3. Mechanical characteristics M(s). calculated taking into account the presentation of each bar with several elementary (1) and without it (2)

Conclusion

As it follows from the conducted studies, when choosing a replacement scheme for an asynchronous motor with a short-circuited rotor, it is necessary to keep in mind that the classic version of the T-shaped scheme for motors with deep grooves does not provide adequate calculation results. More accurate results can be obtained by equating a short-circuited winding with several, formed by dividing it into several layers in height.

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