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The effect of GRUCAD model modifications on its accuracy

***Abstract**. The paper focuses on a modification of the GRUCAD hysteresis model equations. The modification relies on a possible replacement of the Langevin function with a more general Brillouin function in three variants. The first case: only irreversible is given by Brillouin function; the second one: irreversible and reversible equations are given by Brillouin function; and the third case is like the second one but both parts use totally separated parameters. The proposed approach allows one to obtain better fitting capabilities for anisotropic soft magnetic materials, what is demonstrated on the example of hysteresis curves of grain-oriented electrical steel.*

***Streszczenie.** W artykule skupiono się na modyfikacji równań modelu histerezy GRUCAD. Modyfikacja polega na zastąpieniu funkcji Langevina bardziej ogólną funkcją Brillouina w 3 wariantach. Przypadek pierwszy: tylko nieodwracalność jest dana przez funkcję Brillouina; przypadek drugi: równania nieodwracalne i odwracalne podaje funkcja Brillouina; i trzeci przypadek: jak drugi przypadek, ale obie części używają całkowicie oddzielnych parametrów. Zaproponowane podejście pozwala uzyskać lepsze możliwości dopasowania anizotropowych miękkich materiałów magnetycznych, co pokazano na przykładzie krzywych histerezy stali elektrotechnicznej o ziarnie zorientowanym. (**Wpływ modyfikacji modelu GRUCAD na jego dokładność**)*

Keywords: Qualitative analysis, modelling, hysteresis loops, grain-oriented electrical steel, non-grain-oriented electrical steel
Słowa kluczowe: Analiza jakościowa, modelowanie, pętle histerezy, stal o ziarnach zorientowanych, stal o ziarnach niezorientowanych

Introduction

Since any description e.g. pertaining to magnetic hysteresis model is an approximation of real-life phenomena, an important stage in modelling is the qualitative analysis of different factors affecting the performance of developed approach.

Modifications of model equations may contain, for example, some extensions aimed at correcting its behaviour, or result from accounting some relevant physical phenomena (the effect of increased excitation frequency, mechanical strains and stresses, anisotropy or temperature) which previously were not taken into account and which were subject of other investigation [1].

Qualitative analysis of magnetic hysteresis loop models can be multi-faceted. On the one hand, it is important that from the model the losses may be computed accurately, on the other hand, it is crucial that the shape of hysteresis curve along with maintaining characteristic points such as the remanence, coercive or saturation are well reproduced. It is very important during designing magnetic circuit of electric machines and devices [2]. An exemplary modification may rely on the use of different elementary functions appearing in model equations. It is expected that model performance might be improved for different scenarios, moreover new knowledge on underlying physical principles would be gained. In the present paper we consider an modification of the GRUCAD model [3] which uses a more general description of the reversible and irreversible phenomena in comparison to the original approach.

Model description

For over thirty years the description advanced by Jiles and Atherton [5] has attracted a lot of attention of scientific community. This formalism is still very attractive for scientists and engineers alike. In the present paper, we focus on GRUCAD model which is considerable modification of Jiles-Atherton (JA) description proposed by the Brazilian GRUCAD [3,4]. The most important advantage of GRUCAD model is that it addresses a number of problems encountered in the original description, as pointed out in Refs. [6,7]. The crucial difference between the original JA formalism and the GRUCAD approach is that the latter model uses offsetting (shifting) from the anhysteretic curve along the H axis, and not along the M axis. This feature allows one to obtain quasi-static minor hysteresis loops without fragments with negative differential susceptibility, moreover it is correct from the point of energy balance relationships. To recall, the anhysteretic curve describes the state of global equilibrium in the thermodynamic sense.

One of most important advantage of GRUCAD description is that it is formulated as a *B*-input model, this feature facilitates the interpretation of results obtained in accordance with international standards. Standardized magnetic measurements are carried out with controlled polarization rate, yet it should be recalled that for most soft magnetic materials the difference between polarization and flux density may be neglected. Thus the model reflects real-life measurement conditions.

The GRUCAD model behaviour was the subject of analysis in some previously published papers, to mention e.g. a description of hysteresis curves in a permalloy core [8], soft magnetic composites [9,10], magnetocaloric LaFeCoSi alloys [11]. An extension aimed at consideration of the effect of excitation frequency was attempted for a nanocrystalline sample in [12], whereas Ref. [13] focused on model behaviour in the case of DC biased magnetization.

The set of equations used so far was:

(1) $$H_{\mathrm{an}} = \frac{B}{\mu_0} - M_s\left(\coth\lambda - \frac{1}{\lambda}\right),$$

(2) $$\lambda = \frac{H_{\mathrm{an}}(1-\alpha) + B\left(\frac{\alpha}{\mu_0}\right)}{a},$$

(3) $$\frac{\mathrm{d}H_h}{\mathrm{d}B} = \frac{H_{\mathrm{HS}}(\coth\lambda_H - 1/\lambda_H) - H_{\mathrm{h}}}{\gamma\delta},$$

(4) $$\lambda_H = \frac{H_{\mathrm{h}} + \delta H_{\mathrm{HS}}}{a},$$

(5) $$H = H_{\mathrm{an}} + H_{\mathrm{h}}$$

where α, a, γ, H_{HS} and M_{s} were model parameters. $\delta = \pm 1$ was used to distinguish the ascending and descending loop branches. $H_{\mathrm{an}} = H_{\mathrm{an}}(B)$ was the anhysteretic field strength, whereas $H_{\mathrm{h}} = H_{\mathrm{h}}(B)$ denoted the irreversible field strength, related to hysteresis, μ_0 was the permeability of free space, B was the magnetic flux density as input variable.

Different variants of equations

In the preceding section the expressions (1) and (2) were used as a complete description of the anhysteretic and expressions (3) and (4) as the hysteretic curve. It can be easily noticed that Eq. (1) and Eq. (3) availed of the Langevin function, $L(x) = \coth x - \frac{1}{x}$. .

The aim of the present paper is to introduce in that place a more general function, namely the Brillouin function

(6) $B_J(\lambda) = \frac{2J+1}{2J}\coth\left(\frac{2J+1}{2J}\lambda\right) - \frac{1}{2J}\coth\left(\frac{1}{2J}\lambda\right)$

in which an additional parameter *J* appears. In solid state physics it is interpreted as angular momentum quantum number. It takes either positive integer or half-integer values. Two limiting values are 0.5 (then the Brillouin function reduces to hyperbolic tangent) and ∞ (in practical computation $J \to 25$, then the Brillouin function approaches the Langevin function).

Exemplary shapes of curves reproduced with the Brillouin function in reduced units for different values of J parameter are depicted in Figure 1. Additionally in the Figure the dependence $y = \tanh x/3$ is shown. This dependence may be used instead of the Langevin function for smaller values of its argument, the advantage of this function is that it can be inverted analytically.

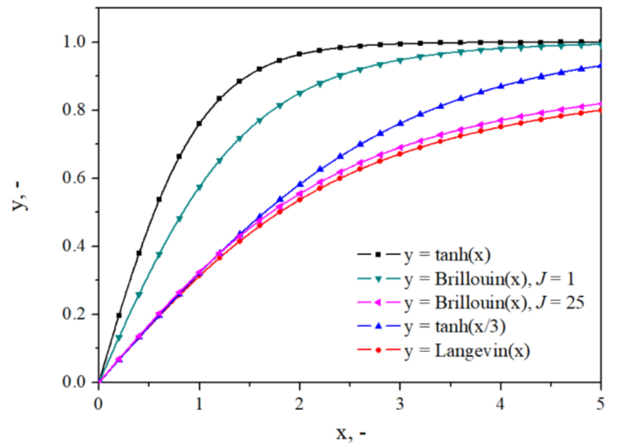

Fig. 1. The functions $L(x)$, $\tanh(x/3)$ and $B_J(x)$ for $J = 1.0$ and $J = 25$.

The modification considered in this paper bears some resemblance to the approaches described in Refs. [14,15]. The aforementioned papers considered the proper choice of angular momentum quantum number *J* in formula for the anhysteretic curve in the modified JA description might shed some light on the anisotropy class of the soft magnetic material being analyzed. The present paper applied the same concept to another model, which in our opinion is a much better choice for experts dealing with hysteresis modeling.

The replacement of Langevin function with the Brillouin function is considered in three possible cases. In the first attempt $B_J(x)$ replaced $L(x)$ in Eq. (3). The second case was when $B_J(x)$ was introduced in Eq. (1) and (3), finally the last case was the same as the second one, but the parameter a was split into two separate parameters appropriate for the relationship (2) pertaining to the reversible magnetization processes and to (4) related to irreversible ones. In this case the reversible and irreversible components are totally separated from each other.

The concept is to vary the value of parameter *J* and to find such a set of model parameters that yields the best match to the measured hysteresis curve.

Modelling

In the paper two types of electrical steel, differing in magnetic properties and morphology were considered. The justification for the choice stemmed from the fact that electrical steels are the most prevalent group of soft magnetic materials (around 80% of total produced volume are the non-grain-oriented (NO) electrical steels, used as core materials for rotating machines, whereas around 16% are grain-oriented (GO) steels, whose application target are magnetic circuits of power and distributions transformers).

The following two samples in the form of sheets were examined: the grade M330-35A (NO steel, 0.35 mm thick) and the grade ET120-27 (GO steel, 0.27 mm thick)).

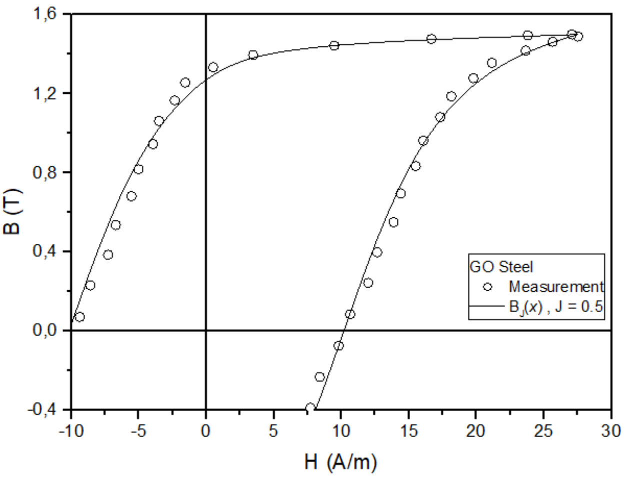

Fig. 2. The measured and the modeled hysteresis loop for the GO sample. Eq. (3) given as $B_J(x)$, $J = 0.5$.

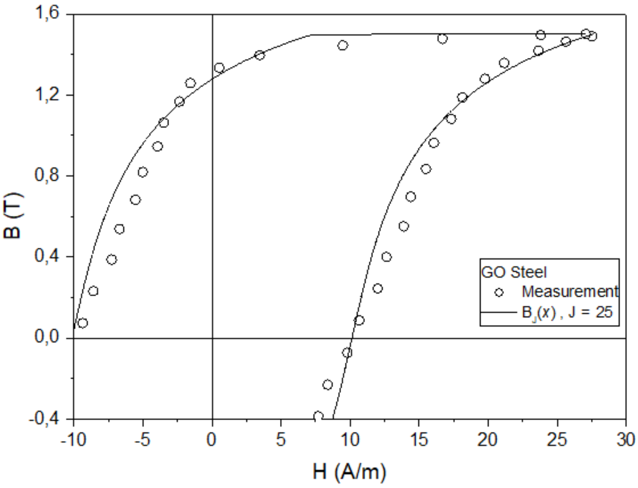

Fig. 3. The measured and the modeled hysteresis loop for the GO sample. Eq. (3) given as $B_J(x)$, $J = 25$.

Tables 1-3 contain the values of coercive field strength and remanence as well as modeling errors for the three considered cases. Taking into account that Brillouin function for $J = 25$ is almost equivalent to Langevin function (Figure 1) in Tables the results for the basic case, when Langevin function was used exclusively, were not included.

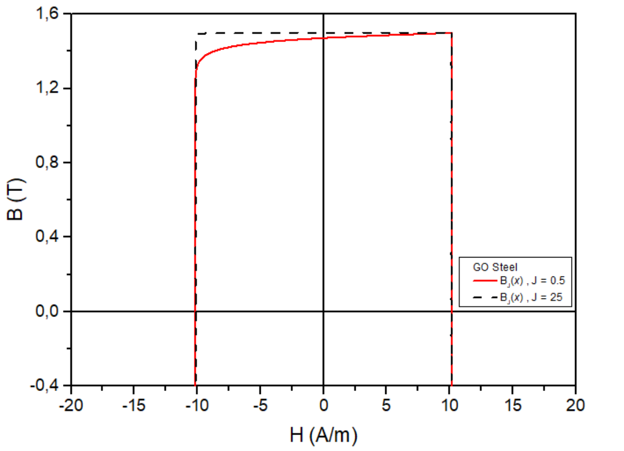

Fig. 4. A comparison of the shape of modeled H_h curve (Eqs. 3 and 4) for the grain-oriented sample. Eq. (3) given as $B_J(x)$ for two cases: J = 0.5 and J = 25.

Table 1. Values and percentage errors in chosen characteristic points for the grain-oriented steel. The hysteretic part, Eq. (3) is given as $B_J(x)$.

.	H_c [A/m]	B_r [T]	$\|\Delta H_c\|$	$\|\Delta B_r\|$	$\|\Delta E\|$
Meas.	10,00	1,30			
$B(x)$ J = 0.5	10,1	1,27	1,3%	2,3	0,1%
$B(x)$ J = 5	9,8	1,26	2,4%	2,4%	2,1%
$B(x)$ J = 10	10,3	1,26	3,3%	2,8%	1,8%
$B(x)$ J = 15	9,7	1,28	2,8%	1,0%	3,9%
$B(x)$ J = 25	10,1	1,28	1,3%	1,4%	2,4%

Table 2. Values and percentage errors in chosen characteristic points for the grain-oriented steel. The hysteretic and the anhysteretic parts are given as $B_J(x)$.

.	H_c [A/m]	B_r [T]	$\|\Delta H_c\|$	$\|\Delta B_r\|$	$\|\Delta E\|$
Meas.	10,0	1,30			
$B(x)$ J = 0.5	10,1	1,22	1,3%	5,6%	1,3%
$B(x)$ J = 5	10,2	1,24	2,0%	4,5%	0,5%
$B(x)$ J = 10	10,1	1,27	0,8%	1,9%	0,9%
$B(x)$ J = 15	10,2	1,25	1,9%	3,9%	2,9%
$B(x)$ J = 25	10,3	1,19	3,3%	8,3%	2,7%

Table 3. Values and percentage errors in chosen characteristic points for the grain-oriented steel. The hysteretic and the anhysteretic parts are given as $B_J(x)$. The parameter a from (4) is separated into independent a and a_h.

.	H_c [A/m]	B_r [T]	$\|\Delta H_c\|$	$\|\Delta B_r\|$	$\|\Delta E\|$
Meas.	10,0	1,30			
$B(x)$ J = 0.5	10,3	1,3	2,5%	2,4%	0,5%
$B(x)$ J = 5	10,2	1,2	1,8%	4,0%	0,7%
$B(x)$ J = 10	12,0	1,0	19,5%	22,6%	14.0%
$B(x)$ J = 15	10,2	1,3	1,7%	2,4%	0,3%
$B(x)$ J = 25	10,5	0,6	5,0%	56,8%	28,0%

Figures 5 and 6 depict the modeling results for the non-oriented steel. As in the previous case, two extreme cases of J value were considered for brevity. From the inspection of the figures a slight advantage where J = 0,5 is noticeable, in particular at the remanence point. This was confirmed by the values presented in Table 4. The situation changed during the second case was examined. In this case differences between J = 0,5 and J = 25 were more significant. There was a noticeable impact of the material grade on the results. In this case obtained results were more satisfactory. Similar trend was obtained for last considered case during the analysis of energy losses. As mentioned before, the evaluation of final results should be multi-faceted.

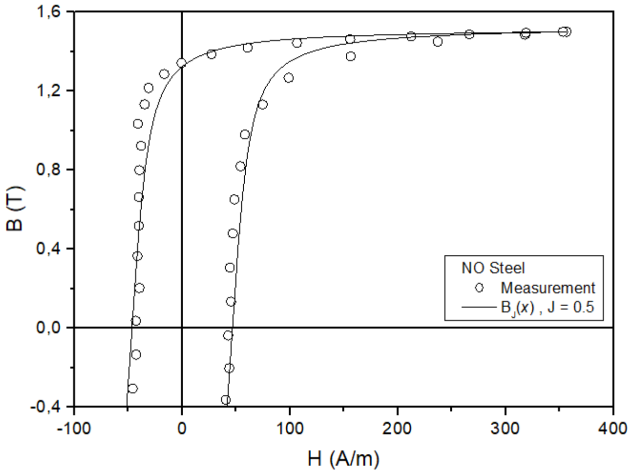

Fig. 5. The measured and the modeled hysteresis loop for the non-oriented sample. Eq. (3) given as $B_J(x)$, where J = 0.5.

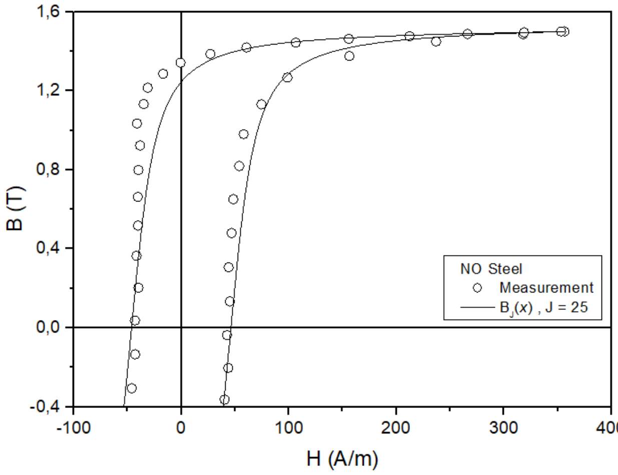

Fig. 6. The measured and the modeled hysteresis loop for the non-oriented sample. Eq. (3) given as $B_J(x)$, where J = 25.

Table 4. Values and percentage errors in chosen characteristic points for the non-oriented steel. The hysteretic part, Eq. 3 is given as $B_J(x)$.

.	H_c [A/m]	B_r [T]	$\|\Delta H_c\|$	$\|\Delta B_r\|$	$\|\Delta E\|$
Meas.	43,9	1,36			
$B(x)$ J = 0.5	46,6	1,33	6,0%	2,0%	6,0%
$B(x)$ J = 5	46,0	1,31	5,0%	3,0%	8,9%
$B(x)$ J = 10	46,9	1,22	7,0%	10,0%	4,6%
$B(x)$ J = 15	44,8	1,32	2,0%	3,0%	11%
$B(x)$ J = 25	46,2	1,25	5,3%	8,0%	7,4%

Table 5. Values and percentage errors in chosen characteristic points for the non-oriented steel. The hysteretic and the anhysteretic parts are given as $B_J(x)$.

.	H_c [A/m]	B_r [T]	$\|\Delta H_c\|$	$\|\Delta B_r\|$	$\|\Delta E\|$
Meas.	43,9	1,36			
$B(x)$ J = 0.5	44,1	1,15	0,5%	16%	16%
$B(x)$ J = 5	44,3	1,05	0,9%	23%	18%
$B(x)$ J = 10	44,8	1,07	1,9%	21%	11%
$B(x)$ J = 15	45,2	1,08	2,9%	21%	16%
$B(x)$ J = 25	48,3	1,36	9,9%	0%	2,0%

Table 6. Values and percentage errors in chosen characteristic points for the non-oriented steel. The hysteretic and the anhysteretic parts are given as $B_J(x)$. The parameter a from (4) is separated into independent a and a_h.

.	H_c [A/m]	B_r [T]	$\|\Delta H_c\|$	$\|\Delta B_r\|$	$\|\Delta E\|$
Meas.	43,9	1,36			
$B(x)$ J = 0.5	44,1	1,24	0,5%	8,8%	12,0%
$B(x)$ J = 5	44,8	1,36	1,9%	0,0%	9,5%
$B(x)$ J = 10	45,8	1,07	4,3%	21%	11,0%
$B(x)$ J = 15	45,7	1,04	4,1%	24%	8,3%
$B(x)$ J = 25	44,8	1,04	2,0%	24%	9,8%

For the non-oriented steel a Figure equivalent to Figure 4 (for the grain-oriented steel) is not shown, since the differences between the irreversible loop shapes were less distinct than in the previous considered case.

Conclusions

In the paper some modifications to the GRUCAD hysteresis model was proposed. The essential concept was to modify one or more of model equations. The Brillouin function was introduced in place of the Langevin function – Eq. (1) and (3). This approach allowed us to make the description more flexible and general, thus different anisotropy classes of soft magnetic materials could be considered. Model modification was verified using measurement data pertaining to two representative grades of grain-oriented electrical steel and non-oriented steel.

In the past some authors suggested the use of several curves given with the Langevin function in modeling [16,17]. In our approach we made yet one step ahead, assuming that the magnetization process might be well described with a combination of Brillouin functions, applied to different parts of the GRUCAD model. Our initial idea was that the same value of J parameter should be applied both in the description of reversible and irreversible parts. However we have found out that an improvement could be obtained when this restrictive condition was freed. The effect might be due to the existence of different phases in the material. Future work can be devoted to in-depth studies of the samples' morphology.

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