

## Bistream instability

**Abstract.** Research work in question is devoted to the study of the motion of a particle in a highly ionized plasma. For the simulation, we used the PIC method (Particle In Cell) considered to be the first method to have been developed historically for the simulation of the Vlasov Maxwell or Vlasov Poisson equations. The Particle-in-Cell (PIC) method is a numerical technique used to simulate the behaviour of charged particles (such as electrons and ions) in electromagnetic environments, such as plasmas and electron beams. This method is commonly used in plasma physics, particle beam simulation and other areas of physics where the interactions between charged particles and electromagnetic fields are crucial. The ESW are observed in satellite electric field waveform data as isolated sinusoids, i.e. as unipolar pulses (a positive electric field peak or a negative electron field peak), bipolar pulses (a single-cycle wave with a positive electron field peak followed by a negative peak, or vice versa) and tripolar pulses (a continuous wave composed of two positive electron field peaks with an intermediate negative peak, or vice versa). By their very nature as isolated pulses observed in waveform data, they are classified as non-linear waves. The one-dimensional electrostatic particle simulations we have performed involve two electron beams and an ion beam moving along a static magnetic field. The density ratio of the electron beams and the thermal velocities of the electron and ion beams are chosen to have bistream instability (bi-flux instability).

**Streszczenie.** historycznie do symulacji równań Vlasova-Maxwella lub Vlasova-Poissona. Metoda Particle-in-Cell (PIC) jest techniką numeryczną stosowaną do symulacji zachowania cząstek naładowanych (takich jak elektrony i jony) w środowiskach elektromagnetycznych, takich jak plazmy i wiązki elektronów. Ta metoda jest powszechnie stosowana w fizyce plazmy, symulacji wiązek cząstek i innych obszarach fizyki, w których oddziaływania między cząstkami naładowanymi a polami elektromagnetycznymi mają kluczowe znaczenie. ESW są obserwowane w danych przebiegu pola elektrycznego satelity jako izolowane sinusoidy, tj. jako impulsy unipolarne (dodatni szczyt pola elektrycznego lub ujemny szczyt pola elektronowego), impulsy bipolarne (fala jednocykliczna z dodatnim szczytem pola elektronowego, po którym następuje szczyt ujemny lub odwrotnie) i impulsy tripolarne (fala ciągła złożona z dwóch dodatnich szczytów pola elektronowego z pośrednim szczytem ujemnym lub odwrotnie). Ze względu na swoją naturę jako izolowane impulsy obserwowane w danych przebiegu, są klasyfikowane jako fale nieliniowe. Jednowymiarowe symulacje cząstek elektrostatycznych, które przeprowadziliśmy, obejmują dwie wiązki elektronów i wiązkę jonów poruszające się wzdłuż statycznego pola magnetycznego. Stosunek gęstości wiązek elektronów i prędkości termiczne wiązek elektronów i jonów są dobrane tak, aby miały niestabilność dwustrumieniową (niestabilność dwustrumieniową). (**Niestabilność dwustrumieniowa**)

**Keywords:** Highly ionized plasma, PIC approach, Bistream instability (ESWs).

**Słowa kluczowe:** Wysoce zjonizowana plazma, podejście PIC, niestabilność dwustrumieniowa (ESW).

### Introduction

Seven years of observations from the Geotail spacecraft have shown the existence of new types of plasma waves in the Earth's magnetosphere. When the Geotail spacecraft was flying in the plasma sheet boundary layer (PSBL), electrostatic solitary waves were always observed from the spacecraft in the form of pulses.

Electrostatic solitary waves (ESWs), sometimes called solitary electrostatic structures (ESSs), isolated electrostatic structures (IESs) and solitary bipolar pulses, have been reported in space plasmas since 1982.

These ESWs are observed in satellite electric field waveform data as isolated sinusoids, i.e. as unipolar pulses (a positive electric field peak or a negative electron field peak), bipolar pulses (a single-cycle wave with a positive electron field peak followed by a negative peak, or vice versa) and tripolar pulses (a continuous wave composed of two positive electron field peaks with an intermediate negative peak, or vice versa).

By their very nature as isolated pulses observed in waveform data, they are classified as non-linear waves.

Electrostatic solitary waves, also known as state modes or electromagnetic state waves refer to a phenomenon in which electromagnetic waves propagate within a structure, but with points of stability or immobility, creating specific patterns of electric and magnetic fields.

These waves occur when two identical waves move in opposite directions and interfere with each other.

Electrostatic standing waves generally form in waveguides, coaxial cables, resonant cavities and other similar structures.

The points where the electric and magnetic fields are immobile are called nodes, while the points where the fields oscillate at their maximum are called antinodes.

These waves have practical applications in various fields of electromagnetism, such as radio wave transmission, microwaves, antennas and electrical circuits.

Stationary modes allow electromagnetic energy to be concentrated in specific places, which can be used for transferring information, creating precise magnetic or electric fields, and other technical applications.

Electron beams are formed by various processes in space plasmas, such as particle reflection at shocks, magnetic reconnection, inductive electric field and parallel electric field of Alfvén kinetic waves.

Electron beams cause strong electrostatic instabilities, which lead to the excitation of Langmuir waves, electrostatic solitary waves (ESWs) and ion acoustic waves through various non-linear processes.

Electron beam instabilities are one of the most fundamental processes in space plasmas. Natural phenomena of wave-particle and wave-wave interactions, beam-plasma instabilities are often highly nonlinear. Such non-linear electron beam instabilities are too complicated for theoretical studies.

The aim of this paper is to provide an explanation of the non-linear processes associated with these observations of electrostatic waves and particles in space via one-dimensional electrostatic particle simulations using the Kempo 1 code.

Stationary electrostatic waves are governed by Maxwell's equations and the principles of electrostatics. They are generally not associated with the traditional notion of propagating waves, where energy travels through space. Instead, they represent a stable distribution of charges in a given space, resulting in specific electric field patterns that do not change with time.

ESWs are excited by two-stream electronic instabilities of particles along a static magnetic field.

Since only electrostatic wave-particle interactions in space plasma are treated, the basic equations are the equations of motion without collision terms and the Poisson equation.

The Poisson equation is solved instead of the full Maxwell equations to reduce the computational time, and the memory area required to perform the simulations is also reduced.

One-dimensional electrostatic particle simulations involving two electron beams and one ion beam moving along the static magnetic field is performed.

The density ratio of the electron beams and the thermal velocities of the electron and ion beams are very important.

The values of these parameters strongly affect the scattering processes of the electron beams and, consequently, different types of electrostatic waves can be generated.

We will study in this paper bistream instability with hot ions.

### Brief history and review of different works

Waveform data from the S3-3 spacecraft, launched in 1976 into a near-polar orbit, revealed for the first time the presence of small-amplitude unipolar pulses interpreted as double layers in the Earth's auroral acceleration region and indicated the presence of electric fields, parallel to the magnetic field [1].

The S3-3 data also showed the presence of solitary waves in the form of bipolar pulses parallel to the magnetic field, and these had no net change in potential between them. A few years later came the results from the Viking satellite [2], which took measurements in magnetospheric plasma (satellite apogee 13,527 km, perigee 817 km, inclination 98.8°).

Their data, similar to that from S3-3, revealed the presence of solitary waves (bipolar pulses) with no net potential drop and weak double layers (unipolar pulses) with a small net potential drop.

With the launch of the single Geotail mission in the early 1990s, primarily to study the magnetic tail, Matsumoto and al [3] were the first to name bipolar pulse ESWs and the first to establish the definitive association between the presence of an ESW and its FFT-derived spectrogram, equivalent to the upper frequency part of broadband electrostatic noise (BEN).

The first report of BEN was made by Scarf and al [4] at the intersections of plasma and neutral sheets by Gurnett and al [5] in the far magnetic tail. It has been described as abrupt and impulsive, with frequency changes on extremely short timescales.

The BEN emissions extended from the lowest frequencies around the lower hybrid frequency to around the local frequency of the electron plasma and exhibited a decreasing amplitude with increasing frequency by Gurnett and al [6].

This is the expected signature of a waveform pulse when transformed using a fast Fourier transform.

The first high-resolution measurements of BEN waveforms were presented by [3]. They used wave measurements from the Geotail spacecraft.

The data showed that BENs consist of a series of structures, which [7] called electrostatic solitary waves (ESWs). ESWs give electric fields with typical amplitudes of 100 mV/m and the pulse width on the time axis is 2-5 ms. The waveform spectra are broadband.

Comparison of orthogonal electric field measurements showed that ESWs are one-dimensional and that the transit time of these structures is much smaller than the GEOTAIL spin period (3 s).

The GEOTAIL spacecraft was launched on 24 July 1993 to study the characteristics and dynamics of the magnetosphere.

One of the main objectives of the GEOTAIL mission was

to study the plasma waves associated with the highly variable structure of the magnetosphere.

To this end, three different types of receivers have been integrated into the Geotail: Plasma Wave Instrument (PWI), a Scanning Frequency Analyser (SFA), a Multi-Channel Analyser (MCA) and a Waveform Sensor (WFC).

The first two receivers are spectral analysers and are complementary to each other; the SFA provides very good frequency resolution while the MCA gives better time resolution.

The WFC receiver can provide five simultaneous waveforms detected by two electrical and three magnetic antennas: this provides information on the phase of plasma waves.

The PWI instrument has provided a wealth of new data on the wave properties of the geomagnetic tail. Some of these data confirm the wave activity previously observed in the magnetotail, while some reveal completely new phenomena.

The WFC receiver displays waveforms other than just isolated pulses. This is illustrated in figure (1). This figure shows three types of BEN waves. It should be noted that the most frequently observed ESWs in the PSBL are ESWs that are composed of single isolated pulses like those of type A. The characteristic type B and type C waveforms are much less frequently observed in the PSBL.

For the simulation part, Omura and al [8] used a one-dimensional electrostatic particle code (Kempo 1) [9] to show that electron beams are capable of producing ESWs. Matsumoto and al [3], Omura and al [8] hypothesised that ESWs are BGK modes [10] or holes in electron phase space.

Electron phase space holes, contain positionally and velocity dependent electron depletions. We will perform in paragraph three, simulations of electrostatic solitary waves (ESWs) observed by GEOTAIL.

What is known is that ESWs are generated as a result of the non-linear coalescence of strong electrostatic waves excited by an electrostatic element. The instability is caused by two electron beams drifting relative to the ions.

As a necessary condition for ESW generation, the thermal velocity of the ions must be high enough to prevent the transition from electrostatic solitary waves to ionic acoustic waves.

Another condition is that the density of the drifting electron beam relative to the ions must be greater than 30% of the plasma density [11].

### Simulation model

The electrostatic particle code KEMPO1 [9] was used and the output files were modified (the original output files are very complicated). In our work, one-dimensional computer simulations were performed.

We will assume the existence of two electron beams and one ion beam which constitutes a neutralising background in the simulation system with the periodic boundary in the simulation system with the periodic boundary condition.

As an initial condition, all electron and ion beams have Maxwellian distribution functions given by the formula:

$$(1) f_s(v_x) = \frac{n_s}{\sqrt{2\pi}v_{ts}} \exp\left(-\frac{1}{2}\left(\frac{v_x - v_{ds}}{v_{ts}}\right)^2\right)$$

Where the subscripts  $-s = 1, 2$  represent the two electron beams,  $-s = i$  for the ion beam,  $-n$  number of particles,  $-v_d$  drift velocity,  $-v_t$  thermal rate,  $-v_x$  velocity in the Ox direction. The frequency of the electron plasma is defined as:

$$(2) \quad \omega_e = \sqrt{\frac{(n_1 + n_2)e^2}{m_e \epsilon_0}}$$

Where  $-e$ , electronic charge,  $-\epsilon_0$  the electrical permittivity of the vacuum,  $-m_e$  the mass of the electron respectively.

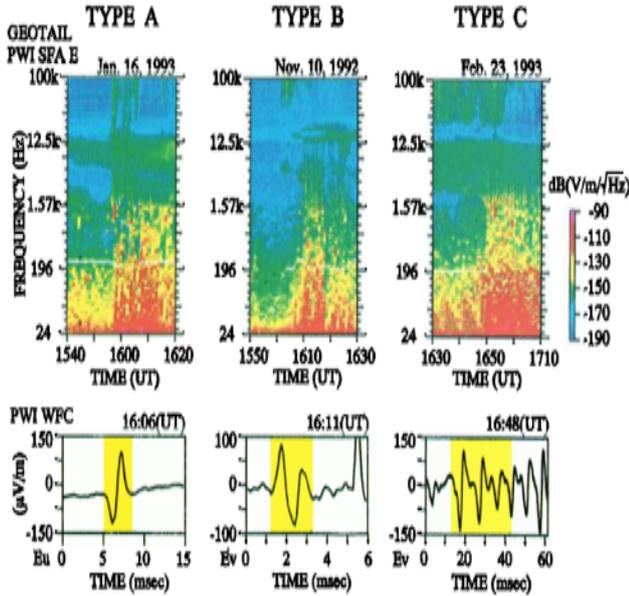


Fig.1. Three types of BEN waveforms and their respective dynamic frequency spectra. Type A is the most frequently observed in PSBL

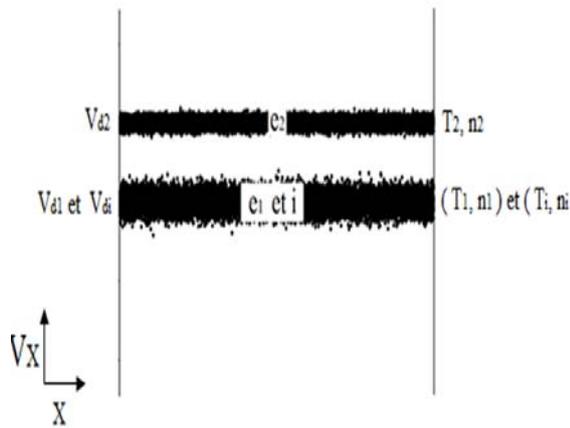


Fig .2. Simulation system

The density ratio  $R$  is defined as:

$$(3) \quad R = n_2 / (n_1 + n_2)$$

Where  $-n_1$  the density of the first majority electron beam,  $-n_2$  the density of the second minority electron beam.  $(n_1 + n_2)$  is also equal to the ion density.

The total density  $R$  takes a constant value for all the simulation.

Assume  $w_e = 1$ , so the frequencies are all normalised by the plasma frequency and all curves will have no units. The time  $t$  is taken equal to  $2\pi$  and will correspond to one period of plasma oscillation.

Some parameters used in the simulation are listed in table (1) :

Table 1. The common parameters used in the simulation

|                                 |                    |
|---------------------------------|--------------------|
| Grid spacing.                   | $\Delta x = 1.0$   |
| Step time.                      | $\Delta t = 0.025$ |
| Number of grid points           | $N_x = 1024$       |
| Total electron plasma frequency | $w_e = 1.0$        |
| Ion drift velocity              | $v_{di} = 0.0$     |
| Ion/electron mass ratio         | $m_i/m_e = 100$    |

The simulation system is one-dimensional (working along the  $x$ -axis) as it is presented in Fig. 2. The magnetic field is assumed to be in the  $Ox$  direction.

Only the electrostatic field  $E_x$  and the particle velocities in  $v_x$  are taken into account, the dynamics of the electrons and ions are therefore not affected by the static magnetic field.

Two electron beams are used, one consisting of background electrons (zero drift velocity) and the other of beam electrons (non-zero drift velocity).

The ions constitute the neutralising background with a drift velocity  $v_{di}$  (see figure (2)) equal to zero. Figure (2) shows the drift velocities of the different particles at the initial time (electrons  $vd_1$  and  $vd_2$  and ions  $vd_i$ ).

$v_{t1}$ ,  $v_{t2}$  and  $v_{ti}$  represent the thermal velocities of the electron and ion beams, respectively.

The velocities are normalised to the initial thermal velocities of the electron beam.

It should also be noted that the relative drift velocity between the electron beams is kept constant since  $v_{d2}-v_{d1} = 20.0$ .

The number of super particles used for the analyses with  $R = 0.5$  is 524288 for each electron beam.

Note that a large number of super particles must be used to avoid numerical particle scattering due to thermal fluctuations which are normally enhanced in particle codes [11].

### Bistream instability (bi-flux)

The bi-flux instability is a two-flux instability driven by two electron beams. We shall study in this paragraph the bi-flux instability with hot ions (we can bi-flux instability with cold ions).

The bistream instability with hot ions drifting with one of the electron beams is studied, a two-stream instability where the densities of the two electron beams are comparable and the thermal velocity of the electrons is low compared to the relative drift speed between the electron beams.

Omura and al [8] showed the parametric dependence of the bistream instability for ESW to form.

They established the formation of ESW for a density ratio  $R \geq 0.3$ . One of the electron beams travels at the same speed as the ion beam as will be shown as will be seen in table (2).

This table gives the values of the  $R$  ratio, the values of the electron and ion drift velocities, and the electron and ion thermal velocities.

Table.2. The bistream instability with a hot ion beam

| Bistream instability | $R$ | $v_{d1}$ | $v_{d2}$ | $v_{t1}$ | $v_{t2}$ | $v_{ti}$ |
|----------------------|-----|----------|----------|----------|----------|----------|
|                      | 0.5 | 0.0      | 20.0     | 1.0      | 1.0      | 2.0      |

We present our results in figures (4) and (6)) and those of [11] in figures (3) and (5).

We study the two-stream instability with hot ions ; all the parameters used to get into this condition are given in Table (2). Figures (4) and (6) show our results.

In figure (4), the left-hand panels show the  $x$ - $v_x$  space curves of the electrons, and the right-hand panels show the distribution functions  $f(v)$  at different times.

In the instability we are studying, a wave mode with a maximum growth rate grows to a saturation level to form a series of large vortices (positive potentials) that can trap the entire electron population.

These are also called electron holes and the potentials are called BGK potentials ; several theoretical works have shown that an electron hole corresponds to a BGK potential.

The formation of ESWs is found by the coalescence of trapped electron vortices, as shown in the  $x$ - $v_x$  phase plots (left-hand column) in Figure (4). At  $t = 410$  (left column of figure (4)), we see that two ESWs are about to merge with each other and give a larger and more intense electron hole. There is a process of formation and coalescence of BGK modes.

For the right-hand column, in this simulation without ion dynamics, the initial electron velocity distribution function has the known maxwellian form, at time  $t = 410$ , the electron velocity distribution function is asymmetrically scattered. Trapped electrons form the high-energy tail in the velocity distribution functions plotted in the right-hand column of figure (4), while untrapped electrons form the asymmetrically scattered non-Maxwellian velocity distribution. This asymmetry in the distribution function is due to the presence of ions that interfere with the trapping of electrons at the phase velocity close to the drift velocity of the ions. The asymmetry is due also to the kinetic motion scattering of the ions which have a much higher mass than the electrons ( $m_i/m_e = 100$ ); the kinetic energy of the ions is much higher than that of the electrons and the electrons are easily scattered by the kinetic motion of the ions.

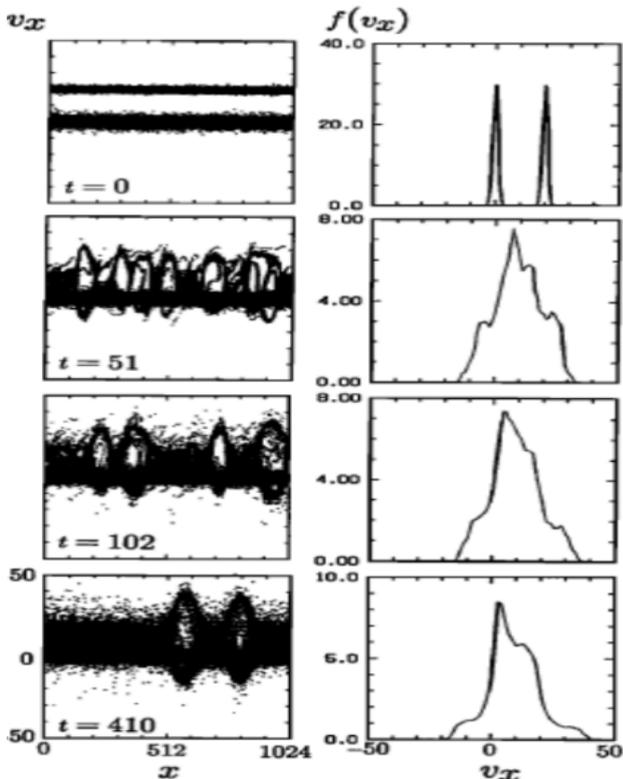


Fig.3. Time evolution of the bistream instability with a hot ion beam drifting with one of the electron beams : Left, electron and ion  $x$ - $v_x$  phase diagram ; Right, electron distribution function  $f(v_x)$  at times  $t = 0, 51, 102$  and  $410$  [11]

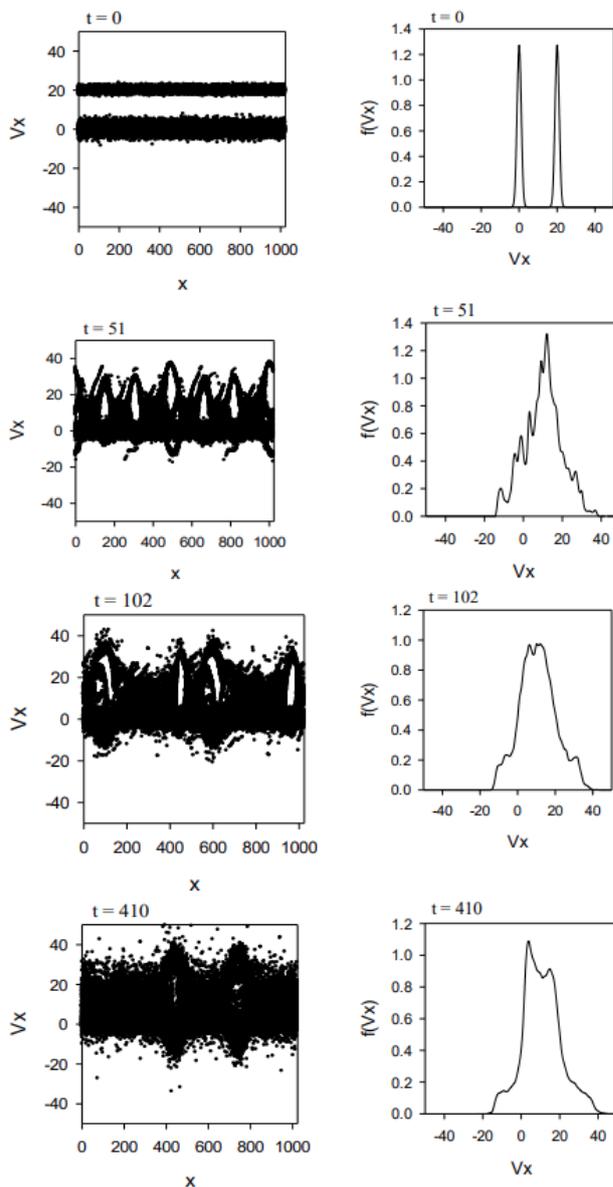


Fig.4. Time evolution of the bistream instability with a hot ion beam drifting with one of the electron beams: Left,  $x$ - $v_x$  phase diagram of electrons and ions; Right electron distribution function  $f(v_x)$  at  $t = 0, 51, 102$  and  $410$ . [Our results].

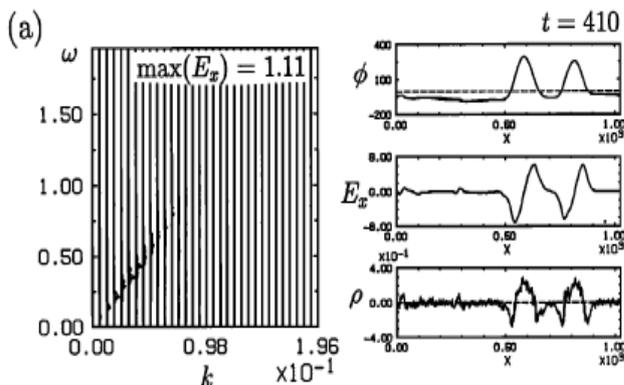


Fig.5. Left,  $w$ - $k$  spectrum for a hot ion beam drifting with one of the electron beams; Right, spatial profiles corresponding to the potential  $\phi$ , electric field  $E_x$  and charge density  $\rho$  at  $t = 410$  [11].

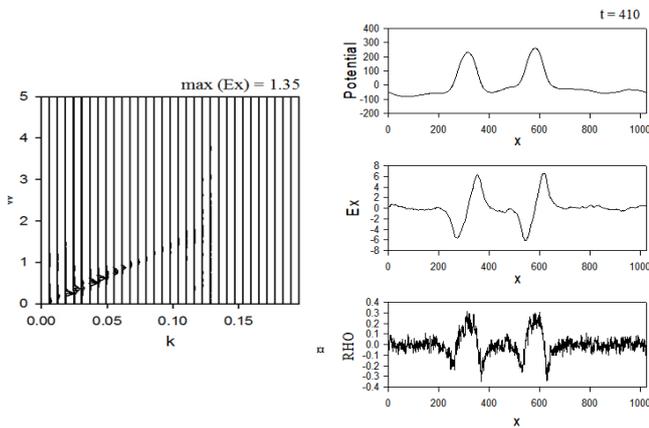


Fig.6. Left, w-k spectrum for a hot ion beam drifting with one of the electron beams: Right, spatial profiles corresponding to the potential  $\phi$ , electric field  $E_x$  and charge

The  $\omega$ - $k$  dispersion relations are equations that describe the relationship between the pulsation (angular frequency) of an electromagnetic wave or plasma wave (in the context of plasma physics) and its wave number  $k$  (wave vector in space). These dispersion relations play a crucial role in understanding the behaviour of waves in media such as plasmas.

The dispersion relation  $\omega$ - $k$  for these types of waves can be derived using Maxwell's equations and the equations of motion for the plasma particles. The specific form of the dispersion relation will depend on plasma parameters such as density, temperature and collision frequencies. In Figure (6), on the left, we show the w-k diagram, which is obtained using the Fourier transform for a fixed  $E_x$  field ( $x$ ,  $t$ ) in space ( $x = 0 \sim 1024$ ) and time ( $t = 0 \sim 409.6$ ). On the right, we show the spatial profile of the electrostatic potential  $\phi$ , the electric field  $E_x$  and the charge density  $\rho$  at  $t = 410$ .

Due to the ESW, the broadband frequency spectra found in the w-k diagram are observed. The electrostatic potential takes on positive values and is represented as holes, the electric field is Due to the ESW, the broadband frequency spectra found in the w-k diagram are observed. The electrostatic potential takes on positive values and is represented as holes, the electric field is given by an oscillating pulse (we have a plasma with an oscillation frequency).

If we compare our curves with those in [5], we can say that they are almost identical. The difference is due to the fact that we have modified the output files.

An important parameter for the ESW formation process is the ratio of the electron and ion temperatures. The temperature of the ions must be higher than that of the scattered untrapped electrons, so that the ionic acoustic mode does not exist in the system. The ratio  $R$  must also be greater than 0.3.

## Conclusion

In a plasma, there can be various types of instabilities that arise due to interactions between different particle populations (electrons and ions) and electromagnetic fields. These instabilities can affect the stability and behavior of the plasma. We study in this chapter two-stream instability. This instability occurs when two particle populations, such as ions and electrons, have different drift velocities. The relative motion between these two populations can lead to the generation of electromagnetic waves. These waves can grow and result in turbulent behavior within the plasma.

Electrostatic stationary waves involve stationary charge distributions and fixed electric field patterns. These phenomena are different from typical electromagnetic waves, as they do not involve the propagation of energy through space but rather the establishment of stationary charge configurations and electric fields.

We focused on the electrostatic potential structures along the magnetic field, which are often observed in the plasma sheet boundary layer (PSBL). The non-linear evolution of electron beam instabilities in a uniform periodic system was studied.

Since the ESW travels at relatively high speeds, the background plasma conditions at the ESW observation point are different from those in the generation region where the instability occurs. The medium considered is uniformly periodic.

In our one-dimensional computer simulations, the formation of BGK potentials has been confirmed by non-linear evolution of the electron beam instabilities. These BGK potentials are considered as electrostatic ESW potentials. The relationship  $\omega = f(k)$  describes how frequency (or pulsation) and wavenumber are related and can provide important information about the behaviour of the wave in a given medium.

Since the two-stream instability is a very strong instability, this instability develops to interact with almost all electrons. Therefore, ESWs generated by the two-stream instability have very large electric fields.

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