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Observer-Based High Order Sliding Mode Control of a Continuous Chemical Reactor

***Abstract**. This paper deals with the design of an observer-based high order sliding mode control law for the Continuous Stirred Tank Reactor (CSTR). The concentration in the reactor is obtained via a nonlinear observer. The observer is coupled with a nonlinear controller designed based on feedback linearization and High-Order Sliding Mode (HOSM) for controlling the reactor temperature. The closed-loop stability of the combined observer–controller scheme is proved. Validating simulations are proposed to assess the efficiency of the proposed controller and its robustness against uncertainties on modelling parameters.*

Streszczenie.** Artykuł ten dotyczy projektowania opartego na obserwatorze prawa sterowania trybem ślizgowym wysokiego rzędu dla reaktora zbiornikowego z ciągłym mieszaniem (CSTR). Stężenie w reaktorze uzyskuje się za pomocą obserwatora nieliniowego. Obserwator jest połączony ze sterownikiem nieliniowym zaprojektowanym w oparciu o linearyzację ze sprzężeniem zwrotnym i tryb ślizgowy wysokiego rzędu (HOSM) do kontrolowania temperatury reaktora. Udowodniono stabilność w pętli zamkniętej połączonego schematu obserwator-kontroler. Proponuje się symulacje walidacyjne w celu oceny wydajności proponowanego sterownika i jego odporności na niepewności dotyczące parametrów modelowania.* (Sterowanie w trybie ślizgowym wysokiego rzędu w oparciu o obserwatora ciągłego reaktora chemicznego***)

Keywords: Non-linear observer, feedback linearizing controller, HOSM controller, global stability, chemical reactor.
Słowa kluczowe: Obserwator nieliniowy, regulator linearyzujący ze sprzężeniem zwrotnym, regulator HOSM, stabilność globalna

Introduction

Continuous Stirred Tank Reactors (CSTRs) are central components of many plants in the chemical and biochemical industry. These systems may exhibit highly nonlinear dynamics, multiplicity of equilibrium points and input constraints. Meanwhile control of an unstable equilibrium point of the reactor makes the problem worse [10].

Several researchers have investigated the problem of controlling CSTRs [6, 17, 18, 26, 28, 37, 39]. Self-tuning PIDs [26,39] robust controllers [18], adaptive- like control systems [37], digital control [17] and different kinds of nonlinear predictive controllers [6,28] have been successfully tested on this class of chemical systems. Furthermore, based on feedback linearization technique [22,32], several controllers either adaptive or non-adaptive have been proposed for the temperature control of CSTRs [1-3,11,14,23,25,35]. The CSTR is also known as an outstanding example for the application of neuro-predictive controllers [16], which are a sub-class of nonlinear predictive controllers. Moreover, fuzzy logic controllers are used in the control of CSTRs to generate either the control command directly [12,27] or control command increments [8,21].

During the last three decades, variable structure systems (VSS) and sliding mode control (SMC) have received significant interest and have become well-established research areas with great potential for practical applications. The theoretical development aspects of SMC are well documented in many books and research articles [7,15,19,24,36,38,40]. The principle of the sliding mode control is to forcibly constrain the system, by suitable control strategy, to stay on the sliding surface on which the system will exhibit desirable features. The advantages of the SMC are robustness, computation speed, compact implementation, controller order reduction, disturbance rejection, and insensitivity to parameter variations. The main disadvantage of the SMC strategy is the chattering phenomenon. SMC has been applied in many control fields which include robot control [30], motor control [5,41], flight control [31], control of power systems [9,29], and chemical process control [4].

In this paper, an observer-based high order sliding mode control law is designed to solve the problem of accurate trajectory tracking for the temperature of a continuous stirred tank reactor.

The SMC law for this class of continuous stirred tank reactor is obtained by designing a discontinuous feedback law using the input–output linearization of the system and by imposing an SMC action that stabilizes the output. The asymptotical stability of the closed-loop system is proved.

This article is organized as follows. The dynamic model of a class of CSTRs is presented in Section 2. In Section 3, a nonlinear observer for estimating the whole process state variables is constructed. In Sections 4 nonlinear controller based on output feedback linearization and high order sliding mode control law are introduced and their closed-loop stabilities, considering the observer dynamics, are established. Effectiveness of the proposed schemes is demonstrated by simulation in Section 5. Finally, the conclusion is given in Section 6.

2. Mathematical model of the reactor

The model for a first order, irreversible, exothermic reaction occurring in a continuous stirred tank reactor (CSTR) is given by:

(1) $\dot{T} = -\frac{UA}{\rho C_p V}\left(T - T_j\right) + \frac{F}{V}(T_i - T) - \frac{\Delta H}{\rho C_p} K(T) C_A$

(2) $\dot{C}_A = -K(T) C_A + \frac{F}{V}(C_{Ai} - C_A)$

(3) $y = T$

with: $K(T) = k_0 e^{-\frac{E}{RT}}$

where C_A is the outlet concentrations of the reactant A, C_{Ai} inlet concentration of the reactant A, T reactor outlet temperature, T_i reactor inlet temperature, T_j jacket temperature, F feed flow rate to reactor, U overall heat-transfer coefficient, A heat transfer surface area, C_p heat capacity of feed and product, E activation energy, R

universal gas constant, k_0 pre-exponential factor, ΔH heat of reaction and ρ density of mixture in reactor.

The model (1-3) has the following form:

$$(4) \quad \begin{cases} \dot{x} = f(x) + g(x)u \\ y = Cx \end{cases}$$

where: $x = [x_1 \ , \ x_2]^T = [T \ , \ C_A]^T$, $u = T_j$ and:

$$f(x) = \begin{bmatrix} -\alpha x_1 + r_1(x) + q(T_i - x_1) \\ r_2(x) + q(C_{Ai} - x_2) \end{bmatrix} \quad , \quad g(x) = \begin{bmatrix} \alpha \\ 0 \end{bmatrix}$$

$C = [1 \quad 0]$

With:

$r_1(x) = \beta K(x_1) x_2$, $r_2(x) = -K(x_1) x_2$, $\alpha = \frac{UA}{\rho C_p V}$,

$\beta = -\frac{\Delta H}{\rho C_p}$, $q = \frac{F}{V}$, $K(x_1) = k_0 e^{\frac{\gamma}{x_1}}$ and $\gamma = -\frac{E}{R}$

The reactor parameter values are given in Table 1 [10]

Table 1: Parameter values of the reactor.

Parameter	value	Unit
α	0.3	hr^{-1}
β	11.92	$m^3\ °K/kgmol$
k_0	34930800	hr^{-1}
γ	-5.9602×10^3	$°K$
q	1	hr^{-1}
T_i	298	$°K$
C_{Ai}	10	$Kgmol/m^3$

The steady state behaviour of the chemical reactor was studied at first and it is shown in Figure 1.

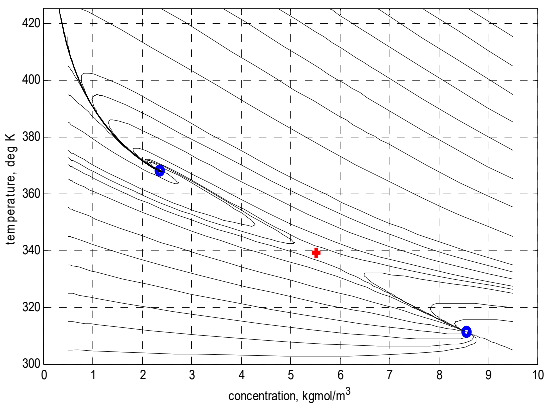


Fig. 1. Phase portrait of open-loop behaviour.

From Figure 1 it is clear, that the reactor has three steady states as shown below:

stable equilibrium point: $(x_1 \ , \ x_2) = (311.2 \ , 8.564)$

unstable equilibrium point: $(x_1 \ , \ x_2) = (339.1, 5.518 \ )$

stable equilibrium point: $(x_1 \ , \ x_2) = (368.1 \ , 2.359)$

For a safe operation under control, it is desired to operate the reactor at its middle (unstable) steady state.

3. Nonlinear observer design

The objective of this section is to introduce an observer for estimating the whole state vector.

A similar type of observer has been proposed by Daaou et al. [13]. The more general form of this type of observer has been considered by [34] and applied to several classes of reactors.

The following assumptions are needed:

Assumption 1. The function $K(x_1)$ is positive and bounded on $]0 \quad \infty[$:

$$\forall x_1 \in]0 \quad \infty[, \qquad \exists \mu_{K_{min}}, \mu_{K_{max}} \in]0 \quad +\infty[\ : \ \mu_{K_{min}} \leq K(x_1) \leq \mu_{K_{max}}$$

Assumption 2. The functions $r_1(x)$ and $r_2(x)$ are globally Lipschitz with respect to x

More precisely, there are constants μ_{r1} and μ_{r2} such that:

$$\forall x \in]0 \quad \infty[\times]0 \quad \infty[, \exists \mu_{r1} \in \mathbb{R}^+ : \ \|r_1(\hat{x}) - r_1(x)\| \leq \mu_{r1} \|\tilde{x}\|$$

$$\forall x \in]0 \quad \infty[\times]0 \quad \infty[, \exists \mu_{r2} \in \mathbb{R}^+ : \|r_2(\hat{x}) - r_2(x)\| \leq \mu_{r2} \|\tilde{x}\|$$

The following proposition can then be proved:

Proposition 1:

Under Assumptions 1 and 2, the dynamic system is given by:

$$(5) \quad \begin{cases} \begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{bmatrix} = \begin{bmatrix} -\alpha \hat{x}_1 + r_1(\hat{x}) + q(T_i - \hat{x}_1) \\ r_2(\hat{x}) + q(C_{Ai} - \hat{x}_2) \end{bmatrix} + \begin{bmatrix} \alpha \\ 0 \end{bmatrix} u - \begin{bmatrix} 1 & 0 \\ \varphi_1(x) & \varphi_2(x) \end{bmatrix} \begin{bmatrix} \theta & 0 \\ \theta^2 & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} \\ \hat{y} = C\hat{x} \end{cases}$$

where $(\hat{\ })$ stands for estimated variables and θ is a positive constant which determines the rate of observer convergence, with:

$$\tilde{x} = [\tilde{x}_1 \quad \tilde{x}_2]^T = [(\hat{x}_1 - x_1) \quad (\hat{x}_2 - x_2)]^T$$

$$(6) \quad \varphi_1(x) = \frac{\alpha + \frac{\gamma}{x_1^2} \beta K(x_1) x_2 + q x_1}{\beta K(x_1)}$$

$$(7) \quad \varphi_2(x) = \frac{1}{\beta K(x_1)}$$

is an asymptotic dynamic observer for the switched nonlinear system (4).

Proof.

Let $\tilde{x}_1 = \hat{x}_1 - x_1$ and $\tilde{x}_2 = \hat{x}_2 - x_2$. Then we have:

$$(8) \quad \dot{\tilde{x}}_1 = -(\alpha + q + \theta)\tilde{x}_1 + r_1(\hat{x}) - r_1(x)$$

$$(9) \quad \dot{\tilde{x}}_2 = -q\tilde{x}_2 + r_2(\hat{x}) - r_2(x) - \left(\theta\varphi_1(x) + \theta^2\varphi_2(x)\right)\tilde{x}_1$$

Define $V_o(\tilde{x}) = \frac{1}{2}(\tilde{x}_1^2 + \tilde{x}_2^2)$ as the Lyapunov candidate function, then its time derivative is:

$$(10) \quad \dot{V}_o(\tilde{x}) = \tilde{x}_1 \dot{\tilde{x}}_1 + \tilde{x}_2 \dot{\tilde{x}}_2$$

$$(11) \quad \dot{V}_o(\tilde{x}) = \tilde{x}_1[-(\alpha + q + \theta)\tilde{x}_1 + r_1(\hat{x}) - r_1(x)] + \tilde{x}_2[-q\tilde{x}_2 + r_2(\hat{x}) - r_2(x) - (\theta\varphi_1(x) - \theta^2\varphi_2(x))\,\tilde{x}_1]$$

$$(12) \quad \dot{V}_o(\tilde{x}) = -(\alpha + q + \theta)\tilde{x}_1^2 - q\tilde{x}_2^2 + \tilde{x}_1[r_1(\hat{x}) - r_1(x)] + \tilde{x}_2[r_2(\hat{x}) - r_2(x)] - \left[\left(\theta\varphi_1(x) + \theta^2\varphi_2(x)\right)\right]\tilde{x}_1\tilde{x}_2$$

Using assumption (2), we obtain:

$$(13) \quad \dot{V}_o(\tilde{x}) \leq -(\alpha + q + \theta)\tilde{x}_1^2 - q\tilde{x}_2^2 + \mu_{r1}\tilde{x}_1\|\tilde{x}\| + \mu_{r2}\tilde{x}_2\|\tilde{x}\| - \left[\left(\theta\varphi_1(x) + \theta^2\varphi_2(x)\right)\right]\tilde{x}_1\tilde{x}_2$$

Knowing that $\|\tilde{x}_1\| \leq \|\tilde{x}\|$ and $\|\tilde{x}_2\| \leq \|\tilde{x}\|$, the above inequality becomes:

$$(14) \quad \dot{V}_o(\tilde{x}) \leq -(\alpha + q + \theta)\|\tilde{x}\|^2 - q\|\tilde{x}\|^2 + \mu_{r1}\|\tilde{x}\|^2 + \mu_{r2}\|\tilde{x}\|^2 - \left[\left(\theta\varphi_1(x) + \theta^2\varphi_2(x)\right)\right]\|\tilde{x}\|^2$$

(15) $\dot{V}_o(\tilde{x}) \leq -[\alpha + q + \theta + q - \mu_{r1} - \mu_{r2} + \theta\varphi_1(x) + \theta^2\varphi_2(x)]\|\tilde{x}\|^2$

Using the following assumption (Assumption 1) The function $K(x_1)$ is bounded, i.e.
$\mu_{K_{min}} \leq K(x_1) \leq \mu_{K_{max}}$

Then the functions $\varphi_1(x)$ and $\varphi_2(x)$ are positives and bounders on $]0 \quad \infty[$:

$\mu_{\varphi1_{min}} \leq \varphi_1(x) \leq \mu_{\varphi1_{max}}$, $\mu_{\varphi2_{min}} \leq \varphi_2(x) \leq \mu_{\varphi2_{max}}$

and inequality (15) becomes:

(16) $\dot{V}_o(\tilde{x}) \leq -\left[\alpha + q + \theta + q - \mu_{r1} - \mu_{r2} + \theta^2\mu_{\varphi1_{min}} + \theta\mu_{\varphi2_{min}}\right]\|\tilde{x}\|^2$

(17)
$\dot{V}_o(\tilde{x}) \leq -[\theta - \mu_r + \eta_o]\|\tilde{x}\|^2$

where:
$\mu_r = \mu_{r1} + \mu_{r2}$ and $\eta_o = \alpha + 2q + +\theta^2\mu_{\varphi1_{min}} + \theta\mu_{\varphi2_{min}}$

Then if we choose $\theta \geq \mu_r$ the estimation error converges exponentially to zero. This completes the proof of Proposition 1.

4. Input–output feedback linearization controller design
In this section a temperature controller based on input–output linearization is designed. The linearization condition that permits to verify if a nonlinear system admits an input output linearization is the relative degree order of the system [20,33].
The relative degree of an output is the number of times that it is necessary to derive the output to reveal the input u.
(18) $\dot{y} = L_f(x_1) + L_g(x_1)u$

with:
(19) $L_f(x_1) = -\alpha x_1 + r_1(x) + q(T_i - x_1)$
(20) $L_g(x_1) = \alpha$

Then, the relative degree of the system is equal to 1. The relation between system input and the linearizing signal is given below:
(21) $u = \frac{v - L_f(x_1)}{L_g(x_1)}$
where $v_1 = \dot{y}$.

It should be noted that the state variables are estimated using an observer, then the equation (21) becomes:
(22) $u = \frac{v - \hat{L}_f(x_1)}{\hat{L}_g(x_1)}$
With:
(23) $v = \dot{y} - \hat{L}_f(x_1) + L_f(x_1)$

where:
(24) $\hat{L}_f(x_1) = \alpha\hat{x}_1 + r_1(\hat{x}) + q(T_i - \hat{x}_1) - \theta\tilde{x}_1$

We choose v such that the system is closed loop stable and achieve a desired setpoint on temperature.

(25) $v = -\delta_1 e_1 - \delta_2 e_2 + \dot{x}_1^r$

where:
$e_1 = \int (x_1 - x_1^r)dt, \quad e_2 = x_1 - x_1^r$

x_1^r is the desired reactor temperature.
The closed-loop error dynamics and observer are given by:

(26) $$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = -\delta_1 e_1 - \delta_2 e_2 + \left(\hat{L}_f(x_1) - L_f(x_1)\right) \\ \dot{\tilde{x}}_1 = -(\alpha + q + \theta)\tilde{x}_1 + r_1(\hat{x}) - r_1(x) \\ \dot{\tilde{x}}_2 = -q\tilde{x}_2 + r_2(\hat{x}) - r_2(x) - \left(\theta\varphi_1(x) + \theta^2\varphi_2(x)\right)\tilde{x}_1 \end{cases}$$

Proposition 2:
Consider the control law stated in (22) and the observer (5) , if Assumptions 1 and 2 are satisfied and by selecting δ_1 and δ_2 such that all roots of the polynomial $s^2 + \delta_2 s + \delta_1$ lie in the open left-hand side of the complex plane then the closed loop system described by (26) is globally asymptotically stable.

Proof:
Consider the following Lyapunov function candidate:

(27) $V(\tilde{x}, e) = V_0(\tilde{x}) + V_c(e)$
where $V_o(\tilde{x}) = \frac{1}{2}(\tilde{x}_1^2 + \tilde{x}_2^2)$ and $V_c(e) = \frac{1}{2}(e_1^2 + e_2^2)$

The time derivative of the Lyapunov function $V(\tilde{x}, e)$ is

(28) $\dot{V}(\tilde{x}, e) = \tilde{x}_1\dot{\tilde{x}}_1 + \tilde{x}_2\dot{\tilde{x}}_2 + e_1 e_2 + e_2\dot{e}_2$

We have: $\dot{V}_o(\tilde{x}) \leq -[\theta - \mu_r + \eta_o]\|\tilde{x}\|^2$, then:

(29) $\dot{V}(\tilde{x}, e) \leq -[\theta - \mu_r + \eta_o]\|\tilde{x}\|^2 + e_1 e_2 - \delta_1 e_1 e_2 - \delta_2 e_2^2 + e_2(-(\alpha + q + \theta)\tilde{x}_1 + r_1(\hat{x}) - r_1(x))$

(30) $\dot{V}(\tilde{x}, e) \leq -[\theta - \mu_r + \eta_o]\|\tilde{x}\|^2 - e^T A_c e + e_2(-(\alpha + q + \theta)\tilde{x}_1 + r_1(\hat{x}) - r_1(x))$

with $A_c = \begin{bmatrix} 0 & -1 \\ \delta_1 & \delta_2 \end{bmatrix} > 0$

(31) $\dot{V}(\tilde{x}, e) \leq -[\theta - \mu_r + \eta_o]\|\tilde{x}\|^2 - \lambda_{A_c}^{\min}\|e\|^2 - (\alpha + q + \theta)e_2\tilde{x}_1 + e_2(r_1(\hat{x}) - r_1(x))$

where $\lambda_{A_c}^{\min}$ is the minimum eigenvalue of A_c. Using assumption (2), inequality (31) becomes:

(32) $\dot{V}(\tilde{x}, e) \leq -[\theta - \mu_r + \eta_o]\|\tilde{x}\|^2 - \lambda_{A_c}^{\min}\|e\|^2 - (\alpha + q + \theta)e_2\tilde{x}_1 + \mu_{r1}e_2\|\tilde{x}\|$

(33) $\dot{V}(\tilde{x}, e) \leq -[\theta - \mu_r + \eta_o]\|\tilde{x}\|^2 - \lambda_{A_c}^{\min}\|e\|^2 - (\alpha + q + \theta)\|e\|\|\tilde{x}\| + \mu_{r1}\|e\|\|\tilde{x}\|$

Inequality (33) can be rewritten as:
(34) $\dot{V}(\eta) \leq -\eta^T \Gamma \eta$

with $\eta = [\tilde{x} \quad e]^T$, $\Gamma = \begin{bmatrix} [\theta - \mu_r + \eta_o] & (\alpha + q + \theta) \\ -\mu_{r1} & \lambda_{A_c}^{\min} \end{bmatrix}$

If $\lambda_{A_c}^{\min} > -\frac{\mu_{r1}(\alpha + q + \theta)}{[\theta - \mu_r + \eta_o]}$ then $\Gamma > 0$ and $\dot{V}(\eta) \leq 0$.
Consequently, asymptotical stability of the closed-loop system is established.

5. High order sliding mode control laws
It is a well-known fact that feedback linearization controllers are not robust to changes in the parameters of the system and to disturbances acting on the system. Therefore, we will use a technique that makes the proposed feedback linearization controller robust.
In the following, we present a control strategy which essentially combines this method with the SMC approach leading to more robust results.

Proposition 3:
Considering the following control law:

(35) $u = \frac{1}{\hat{L}_g(x_1)}\left(-a|S|^p sign(S) + \dot{x}_1^r - \hat{L}_f(x_1)\right),\ a > 0$, p an odd number ($p = 2n+1$), $n \in Z^+$

where S is the sliding variable and the observer (5), if Assumptions 1 and 2 are satisfied then the closed loop system is globally asymptotically stable.

Proof:
The observer and closed-loop error dynamics are given by:

$$(36)\ \begin{cases} \dot{S} = -a|S|^p sign(S) + \left(\hat{L}_f(x_1) - L_f(x_1)\right) \\ \dot{\tilde{x}}_1 = -(\alpha + q + \theta)\tilde{x}_1 + r_1(\hat{x}) - r_1(x) \\ \dot{\tilde{x}}_2 = -q\tilde{x}_2 + r_2(\hat{x}) - r_2(x) - \left(\theta\varphi_1(x) + \theta^2\varphi_2(x)\right)\tilde{x}_1 \end{cases}$$

with:
$S = e = x_1 - x_1^r$
To analyse closed-loop stability, we introduce the Lyapunov function:
(37) $V(\tilde{x}, S) = \frac{1}{2}(\tilde{x}_1^2 + \tilde{x}_2^2) + \frac{1}{2}S^2$

Differentiating $V(\tilde{x}, e)$
(38) $\dot{V}(\tilde{x}, S) = \tilde{x}_1\dot{\tilde{x}}_1 + \tilde{x}_2\dot{\tilde{x}}_2 + S\dot{S}$

we have:
$\dot{V}_o(\tilde{x}) = \tilde{x}_1\dot{\tilde{x}}_1 + \tilde{x}_2\dot{\tilde{x}}_2 \leq -[\theta - \mu_r + \eta_o]\|\tilde{x}\|^2$, then:

(39) $\dot{V}(\tilde{x}, S) \leq$
$-[\theta - \mu_r + \eta_o]\|\tilde{x}\|^2 + S\left[-a|S|^p sign(S)\left(\hat{L}_f(x_1) - L_f(x_1)\right)\right]$

(40) $\dot{V}(\tilde{x}, S) \leq -[\theta - \mu_r + \eta_o]\|\tilde{x}\|^2 + S[-a|S|^p sign(S) - (\alpha + q + \theta)\tilde{x}_1 + r_1(\hat{x}) - r_1(x)]$

(41) $\dot{V}(\tilde{x}, S) \leq -[\theta - \mu_r + \eta_o]\|\tilde{x}\|^2 - a|S|^2 - S(\alpha + q + \theta)\tilde{x}_1 + S[r_1(\hat{x}) - r_1(x)]$

By using the Lipschitz condition of assumptions (2), we obtain:
(42) $\dot{V}(\tilde{x}, S) \leq -[\theta - \mu_r + \eta_o]\|\tilde{x}\|^2 - a|S|^2 - (\alpha + q + \theta - \mu_{r1})|S|\|\tilde{x}\|$

Choose θ such that $\theta > \mu_r$. Hence:
(43) $\dot{V}(\tilde{x}, S) \leq 0$
This implies that the tracking errors $(x_1 - x_1^r)$ converge asymptotically to zero as $t \to +\infty$.

6. Simulation results:

Numerical simulations for the closed-loop system were performed in order to show the effectiveness of the proposed scheme. The reactor parameters are given in Table 1. In simulations the following values are chosen for I/O linearizing controller parameters $\delta_1 = 16.5$ and $\delta_2 = 8.5$, and similarly for high order sliding-mode controller parameters $a = 6$ and $p = 3$
The states initial conditions were set to:

$$x(0) = [\,340\ ;\ 5.5]$$

and parameter θ is equal to 5. The temperature set point has the form:

$$x_1^r = 400(1 - 0.5\exp(-0.5t))[°K]$$

The transient observer performances for two designed controllers are shown in Figures 2 and 3. According to these figures, the observer is capable of estimating the process state variables with a fast rate of convergence. The temperature transient responses for two controllers and their corresponding control actions are shown in Figures 4 and 5.

As can be seen, the desired trajectory is followed almost without error after 0.5h and 8h for HOSM and I/O linearizing controller respectively.

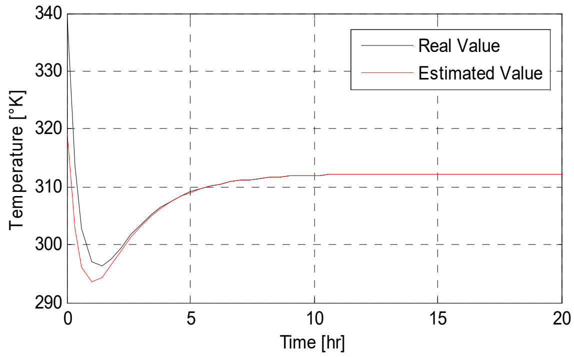


Fig. 2. Actual and estimated reactor temperature

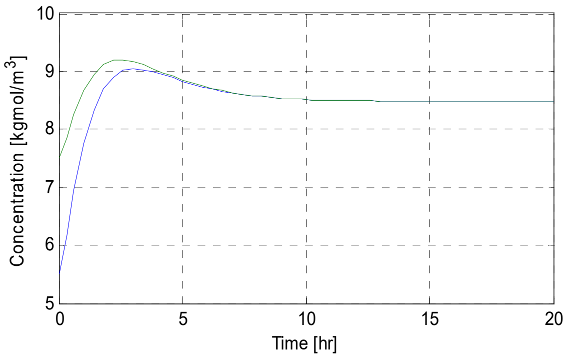


Fig. 3. Actual and estimated Concentration

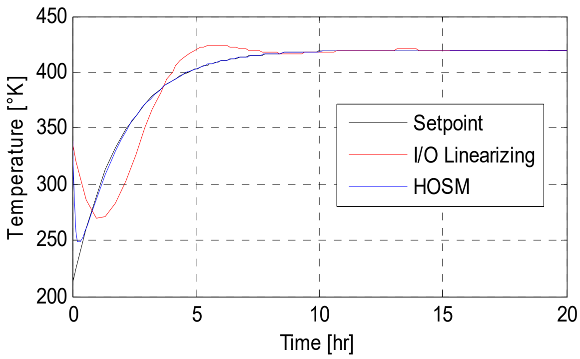


Fig. 4. Time response for the reactor temperature

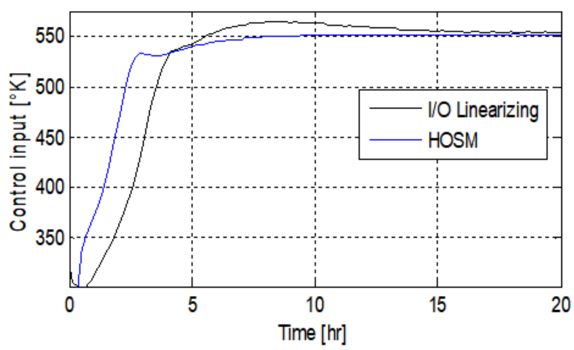
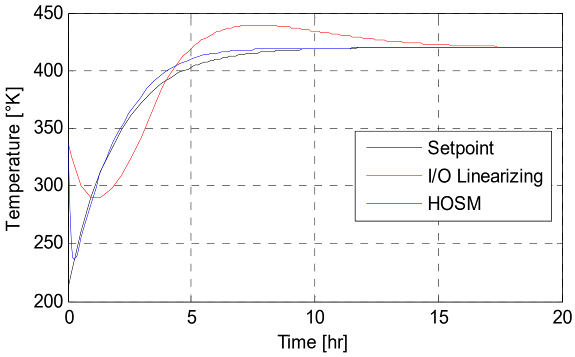


Fig. 5. Control inputs

Now, we examine the robustness of the proposed controllers for load rejection and model mismatch. Figures 6 and 7 show the performances of controllers for 6% increase in the inlet temperature. It is evident from these figures that HOSM controller performs better both in transient response and steady state.



Fig/ 6. Time response for the reactor temperature for 6% increase in the inlet temperature

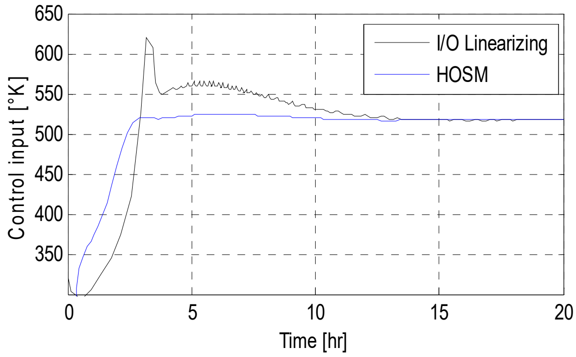


Fig. 7. Control inputs for 6% increase in the inlet temperature

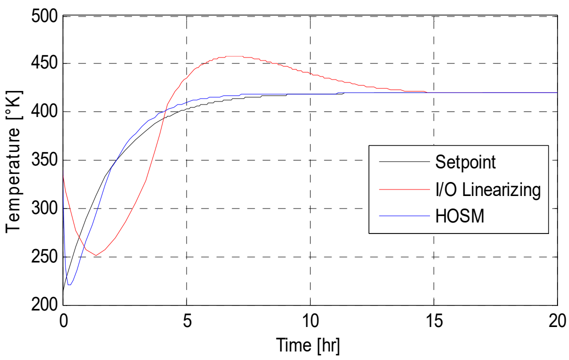


Fig. 8. Time response for the reactor temperature for −15% error in the heat transfer coefficient

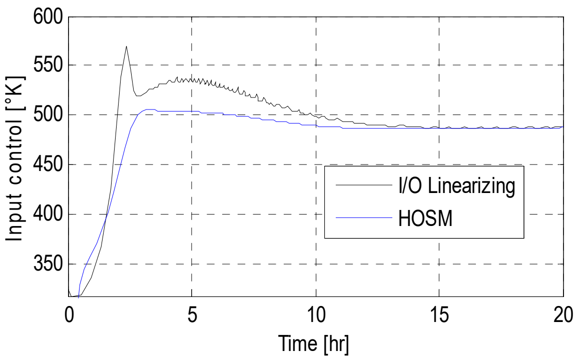


Fig. 9. Control inputs for −15% error in the heat transfer coefficient

The performances of the system are simulated when the parameters of the system are assumed to be unknown exactly. Figures 8 and 9 illustrate the controllers' responses for 15% decrease in the heat transfer coefficient. According to these figures, HOSM outperforms the feedback linearizing controller.

Therefore, it can be concluded that the proposed control schemes are robust to changes in the parameters and to disturbances acting on the system.

7. Conclusion

In this paper, we present a robust observer based on a nonlinear control scheme for a continuous stirred chemical reactor (CSTR). In order to perform the estimation, we proposed a high-gain observer that robustly estimates the whole state vector based on the available measurements. The proposed observer offers the advantage of only one tuning parameter θ. This observer is coupled with two nonlinear controllers. The controllers are constructed through feedback linearization and high order sliding mode (HOSM) techniques for temperature reactor control. The closed-loop stability of the combined observer–controller scheme is proved.

Through numerical simulations, we illustrated the feasibility of the designed control system. Moreover, the proposed control exhibits a satisfactory performance when used with disturbance and dynamics uncertainty.

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REFERENCES

[1] B.W. Bequette, "Process Control Modeling, Design and Simulation". Prentice-Hall, Upper Saddle River, NJ., 2003.
[2] Yu, D.L., Chang, T.K., Yu, D.W., "A stable self-learning PID control for multivariable time varying systems", Control Engineering Practice, Vol.15, no.12, pp. 1577-1587, 2007.
[3] C.R. Madhuranthakam, A. Elkamel, H. Budman, "Optimal tuning of PID controllers for FOPTD, SOPTD and SOPTD with lead processes" Chemical Engineering and Processing: Process Intensification, Vol. 47, no. 2, pp.251-264, 2008.
[4] L. Feng, J.L. Wang and E.K. Poh, Improved robust model predictive control with structured uncertainty, Journal of Process Control, Vol.17, no.8, pp. 683-688, 2007.
[5] W. Wu, "Adaptive-like control methodologies for a CSTR system with dynamic actuator constraints", Journal of Process Control, Vol. 13, pp. 525-537, 2003.
[6] M. P. Di Ciccio, M. Bottini, and P. Pepe, "Digital Control of a Continuous Stirred Tank Reactor", Mathematical Problems in Engineering, Vol. 2011, Article ID 439785,2011.
[7] J. Prakash and R. Senthil, "Design of Observer Based Nonlinear Model Predictive Controller for a Continuous Stirred Tank Reactor" Journal of Process Control, Vol. 18, no. 5, pp. 504-514, 2008.
[8] R K Al-Seyab, and Y. Cao, "Differential Recurrent Neural Network based Predictive Control", Computers and Chemical Engineering, Vol. 32, no. 7, pp. 1533-1545, 2008.
[9] Khalil, H.K., "Nonlinear Systems", second ed. Prentice-Hall, Upper Saddle River, NJ., 1996.
[10] Slotine, E., Li, W., "Applied Nonlinear Control". Prentice-Hall, New Jersey, 1991.
[11] Limqueco, L.C., Kantor, J.C., "Nonlinear output feedback control of and exothermic reactor", Computers and Chemical Engineering Vol. 14, pp. 427-437, 1990.
[12] Adebekun, A.K., Schork, F.J., "On the global stabilization of nth order reactions", Chemical Engineering Communications Vol. 100, pp. 47-59, 1991a.
[13] Adebekun, A.K., Schork, F.J., "On the robust global stabilization of nth order reactions", Chemical Engineering Communications Vol. 101 no.1, pp. 1-15, 1991b.
[14] Adebekun, K., The robust global stabilization of a stirred tank reactor, A.I.Ch.E. Journal Vol. 38, pp. 651–659, 1992.
[15] Kosanovich, K.A., Piovoso, M.J., Rokhlenko, V., Guez, A.," Nonlinear adaptive control with parameter estimation of a CSTR", Journal of Process Control Vol. 5, pp. 133–148, 1995.

[16] Tyner, D., Soroush, M., Grady, C.G., "Adaptive temperature control of multiproduct jacketed reactors", Industrial and Engineering Chemistry Research Vol. 38, pp. 4337–4344, 1999.

[17] Bouhamida, M., Daaou, B., A. Mansouri, M. Chenafa, "Observer-Based Input-Output Linearization Control of a Multivariable Continuous Chemical Reactor" J. Korean Math. Soc. Vol. 49, No. 3, pp. 641–658, 2012.

[18] Daaou, B., A. Mansouri, M. Bouhamida, M. Chenafa, "Development of Linearizing Feedback Control with a Variable Structure Observer for Continuous Stirred Tank Reactors" Chinese Journal of Chemical Engineering, Vol. 20 N° 3, pp. 1-5, 2012.

[19] H. Demuth, M. Beale and M. Hagan, "Neural networks toolbox 5, user's guide", The MathWorks, Online, 2007.

[20] Y.-Y. Cao and P. M. Frank, "Analysis and synthesis of nonlinear time-delay systems via fuzzy control approach," IEEE Transactions on Fuzzy Systems, vol. 8, no. 2, pp. 200–211, 2000.

[21] Y. Oysal, Y. Becerikli and A. F. Konar, "Modified descend curvature based fixed form fuzzy optimal control of nonlinear dynamical systems". Computers and Chemical Engineering, Vol. 30, pp. 878-888, 2006.

[22] K. Belarbi, F. Titel, W. Bourebia and K. Benmahammed, "Design of Mamdani fuzzy logic controllers with rule base minimisation using genetic algorithm" Engineering Applications of Artificial Intelligence, Vol. 18, pp. 875-880, 2005.

[23] C. Karakuzu "Retraction notice to: Fuzzy controller training using particle swarm optimization for nonlinear system control" ISA Transactions, Vol. 47, no. 2, pp. 229-239, 2008.

[24] Bartolini G, Zolezzi T. "Variable structure systems nonlinear in the control law", IEEE Trans Autom Control, Vol. 30, 681–4. 1985.

[25] DeCarlo RA, Zak SH, Matthews GP. "Variable structure control of nonlinear multivariable systems: a tutorial", Proceedings of the IEEE, Vol. 76, no. 3, pp. 212-232, 1988.

[26] Utkin VI. "Sliding modes in control and optimization", Springer-Verlag, Berlin,1992.

[27] Hung JY, Gao W, Hung JC. "Variable structure control: a survey", IEEE Trans Ind Electron Vol. 40, no. 1, pp. 2–22, 1993.

[28] Zinober ASI. "Variable structure and Lyapunov control", Springer-Verlag, Berlin, 1994.

[29] Young KD, Utkin VI, Ozguner U. "A control engineer's guide to sliding mode control", IEEE Trans Control Syst Technol, Vol. 7, no. 3, pp. 328-342, 1999.

[30] Koshkouei AJ, Zinober ASI., "Robust frequency shaping sliding mode control", IEE Proc—Control Theory Appl, Vol. 147, no. 3, pp. 312-320, 2000.

[31] Sira-Ramirez H, Ahmad S, Zribi M., "Dynamical feedback control of robotic manipulators with joint flexibility", IEEE Trans Syst Man Cybernet, Vol. 22, no. 4, pp. 736-747, 1992.

[32] Zribi M, Sira-Ramirez H, Ngai A. "Static and dynamic sliding mode control schemes for a permanent magnet stepper motor", International Journal of Control, Vol. 74, pp. 103-117, 2001.

[33] Alrifai MT, Zribi M, Sira-Ramirez H. "Static and dynamic sliding mode control of variable reluctance motors", International Journal of Control, Vol. 77, pp. 1171-1188, 2004.

[34] Sira-Ramirez H, Zribi M, Ahmad S. "Dynamical sliding mode control approach for vertical flight regulation in helicopters", IEE Proc. Control Theory Appl, Vol.141, pp. 19-24, 1994.

[35] Bengiamin NN, Chan WC., "Variable structure control of electric power generation", IEEE Trans Power Apparatus Syst, Vol. 101, pp. 376-380, 1982.

[36] Shtessel Y, Zinober ASI, Shkolnikov I. "Sliding mode control of boost and buck-boost power converters using the dynamic sliding manifold", Int J Robust Nonlin Control, Vol. 13, pp. 1285-1298, 2003.

[37] Aguilar R., Martinez-Guerra R., Maya-Yescas R.," Temperature Regulation via PI High-Order Sliding-Mode Controller Design: Application to a Class of Chemical Reactor, International Journal of Chemical Reactor Engineering , Vol. 7, 2009.

[38] Daaou, B., A. Mansouri, M. Bouhamida, M. Chenafa, "A Robust Nonlinear Observer for State Variables Estimation in Multi-Input Multi-Output Chemical Reactors," International Journal of Chemical Reactor Engineering: Vol. 6, A86, 2008.

[39] Soroush, M., "Nonlinear state-observer design with application to reactors", Chemical Engineering Science, Vol. 52, pp. 387, 1997.

[40] Isidori, A., "Nonlinear Control Systems", Springer-Verlag, New York, 1989.

[41] Slotine, J.-J. E., & Hedrick, J. K. "Robust input–output feedback linearization", International Journal of Control, Vol. 57, pp. 1133– 1139, 1993.