

# A Backstepping Sliding Mode Control of DC-DC Buck Converter

**Abstract.** In this paper, a Backstepping Sliding Mode controller is designed for the DC-DC buck converter under inherent nonlinearity and time variation. The proposed controller design is based on the integration both the merits of Backstepping control method and sliding mode control method (SMC). Based on Lyapunov synthesis approach, the derived controller can closely track change in reference output voltage and guarantees stability converge. Simulation results demonstrate the effectiveness and robustness of the proposed combination approach.

**Streszczenie.** W tym artykule kontroler trybu ślizgowego z krokiem wstecznym został zaprojektowany dla przetwornicy buck DC-DC w warunkach nieodłącznej nieliniowości i zmian w czasie. Proponowana konstrukcja sterownika opiera się na integracji zalet metody sterowania Backstepping i metody sterowania w trybie ślizgowym (SMC). W oparciu o podejście syntezy Lyapunowa, wyprowadzony kontroler może dokładnie śledzić zmiany referencyjnego napięcia wyjściowego i gwarantuje zbliżenie stabilności. Wyniki symulacji wykazują skuteczność i solidność proponowanego podejścia łączonego. (Sterowanie trybem ślizgowym z krokiem wstecznym przetwornicy buck DC-DC)

**Keywords:** Backstepping control, sliding mode control, buck converter.

**Słowa kluczowe:** Backstep, tryb przesuwania, konwerter buck.

## Introduction

The DC-DC power converters have been commonly utilized in the power supply equipment of most electronic systems. The buck type of DC-DC converters essentially operates as step-down power converters, they are used in many applications such as computers, renewable energy systems, mobile phones and in many other commercial devices [1-5]. Due to their nonlinear and time-varying nature, primarily attributed to their inherent switching operation, the control system design for DC-DC buck converters presents a significant challenge. The emphasis in the field of dc/dc power converters has primarily revolved around employing linear control theories. However, these methods can only guarantee stability around a specific operating point and in the presence of minimal disturbances. Nevertheless, in cases where the dc/dc power converter experiences considerable disturbances or operates across wide voltage ranges the fixed nominal operating point becomes unattainable leading to deterioration in control effectiveness.

Nonlinear control methods are considered superior options for controlling dc/dc converters as they effectively improve the controller's performance in large signal conditions and enhance their dynamic response.

One of the most popular control techniques that are suited to control the DC-DC buck converter is the sliding mode control (SMC) technique [6-8], owing to its numerous advantages, including exceptional robustness against external disturbances and its simplicity in implementation.

Backstepping control technique [9-11] is a nonlinear method that involves a systematic construction of both feedback control laws and Lyapunov synthesis approach, the stabilization problem becomes much easier with the construction of the Lyapunov function whose derivative can be made negative definite by a variety of control laws for the closed loop system, this is one of the major advantages of the backstepping control.

In [1] an adaptive backstepping control strategy is developed to regulate a DC-DC buck converter, and the obtained outcomes are contrasted with a PSO-based PID technique, in [2] an adaptive backstepping control approach is specifically designed to regulate a DC-DC buck converter that is supplying power to an unknown load while operating with an unknown input voltage, in [3] wherein a modified sliding function is utilized. The sliding surface is defined as a proportional-integral (PI) function of the conventional

sliding function used in sliding mode control (SMC). In [4] a proposed method for a DC-DC buck converter with mismatched time-varying disturbance. This method utilizes a generalized proportional integral observer (GPIO) combined with dynamic prescribed performance sliding mode control (DPPSMC).

This paper introduces a novel technique called backstepping-sliding mode control (BSMC) for effectively controlling a DC-DC buck converter. This technique combines the advantages of the two aforementioned nonlinear control techniques. The simulation results demonstrate the performance and effectiveness of the proposed controller to accurately track variations in the reference voltage. Furthermore, they illustrate the robustness of the controller in the presence of the input voltage and load resistance disturbances, all while maintaining system stability.

The structure of this paper is as follows: Section II introduces a proposed state space averaged model, followed by Section III where a backstepping-sliding mode control is designed. Simulation results are presented in Section IV, and finally, Section V concludes the paper.

## State space averaging of DC-DC buck converter

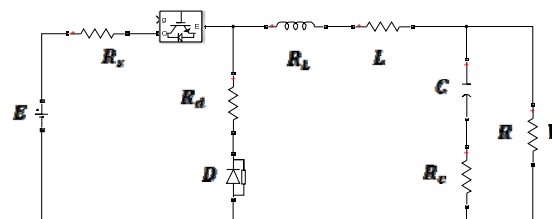


Fig. 1. Buck converter circuit.

In Fig.1, the circuit diagram of a DC-DC buck converter is depicted. The components in the converter are represented with non-ideal characteristics. Specifically,  $R_c$ ,  $R_L$ ,  $R_s$  and  $R_d$ .

$$(1) \quad \begin{cases} \frac{dV_c}{dt} = \frac{1}{(R_c + R)C} V_c + \frac{R}{(R_c + R)C} i_L \\ \frac{di_L}{dt} = -\frac{R}{(R_c + R)L} V_c - \left( \frac{RR_c}{(R_c + R)L} + \frac{R_s + R_L}{L} \right) i_L + \frac{E}{L} \mu \end{cases}$$

By simplifying the averaged model, it is possible to express Eq.1. in the following form:

$$(2) \quad \begin{cases} \dot{x}_1 = \lambda_1 x_1 + \lambda_2 x_2 \\ \dot{x}_2 = \lambda_3 x_1 + \lambda_4 x_2 + \lambda_5 \mu \end{cases}$$

Note that  $\lambda_1, \lambda_2, \lambda_3, \lambda_4$  and  $\lambda_5$  are the parameters of the buck converter defined by the following equations:

$$(3) \quad \lambda_1 = \frac{1}{(R_c + R)C}$$

$$(4) \quad \lambda_2 = \frac{R}{(R_c + R)C}$$

$$(5) \quad \lambda_3 = \frac{R}{(R_c + R)L}$$

$$(6) \quad \lambda_4 = -\frac{R_c R}{(R_c + R)} - \frac{(R_L + R_s)}{L}$$

$$(7) \quad \lambda_5 = \frac{E}{L}$$

The variables  $x_1$  and  $x_2$  correspond to the capacitor voltage  $v_c$  and inductor current  $i_L$ , respectively. The symbols  $R_c, R_L, R_s$ , and  $R$  represent the resistance values of the capacitor, inductor, switch/diode, and load, respectively. Additionally,  $E$  represents the value of the external source voltage.

Note that  $\lambda_2$  is always superior than zero if the load resistance  $R \neq 0$  and  $\lambda_5$  is always greater than zero if  $E \neq 0$ .

To attain satisfactory performance, an integral term is incorporated to attenuate steady-state errors. The state equation is provided as follows:

$$(8) \quad \dot{\xi} = x_1 - v_d$$

Where  $v_d$  represents the desired output voltage.

### Backstepping sliding mode control

In the following section, the design procedures are outlined.

**Step1:** Considering Eq. 8.,  $x_1$  is regarded as a virtual input. Let us choose the Lyapunov function as follows:

$$(9) \quad V_1 = \frac{1}{2} \xi^2$$

Next, the virtual control can be expressed as follows:

$$(10) \quad x_1 = \gamma_0 = -\rho_0 \xi + v_d$$

Differentiate  $V_1$  with respect to time to obtain:

$$(11) \quad \dot{V}_1 = \xi \dot{\xi} = -\rho_0 \xi^2 + \xi(x_1 - \gamma_0)$$

It is worth noting that  $\rho_0$  serves as the design parameter, ensuring that the resulting expression is negative definite:

Differentiating yields:

$$(12) \quad \dot{\gamma}_0 = -\rho_0 \dot{\xi} + \dot{v}_d = -\rho_0(x_1 - v_d) + \dot{v}_d$$

$$(13) \quad \begin{aligned} \dot{\gamma}_0 &= -\rho_0(\dot{x}_1 - \dot{v}_d) + \dot{v}_d \\ &= -\rho_0(\lambda_1 x_1 + \lambda_2 x_2) + \rho_0 \dot{v}_d + \dot{v}_d \end{aligned}$$

**Step 2** involves expressing the derivative of the error  $(x_1 - \gamma_0)$  as follows:

$$(14) \quad \dot{x}_1 - \dot{\gamma}_0 = \lambda_1 x_1 + \lambda_2 x_2 - \dot{\gamma}_0$$

The selection for the augmented Lyapunov function is as the following manner:

$$(15) \quad V_2 = V_1 + \frac{1}{2}(x_1 - \gamma_0)^2$$

By utilizing Eq. (11) and Eq. (14), the derivative of  $V_2$  can be expressed as follows:

$$(16) \quad \begin{aligned} \dot{V}_2 &= \dot{V}_1 + (x_1 - \gamma_0)(\dot{x}_1 - \dot{\gamma}_0) \\ &= -\rho_0 \xi^2 + \xi(x_1 - \gamma_0) + (x_1 - \gamma_0)(\dot{x}_1 - \dot{\gamma}_0) \\ &= -\rho_0 \xi^2 + (x_1 - \gamma_0)(\xi + \lambda_1 x_1 + \lambda_2 x_2 - \dot{\gamma}_0) \end{aligned}$$

The design of the virtual control for can be formulated as follows:

$$(17) \quad x_2 = \gamma_1 = \frac{1}{\theta_2} [-\rho_1(x_1 - \gamma_0) - \xi - \lambda_1 x_1 + \dot{\gamma}_0]$$

To ensure that  $\dot{V}_2$  is negative definite:

$$(18) \quad \begin{aligned} \dot{V}_2 &= -\rho_0 \xi^2 + (x_1 - \gamma_0)(\xi + \lambda_1 x_1 + \lambda_2 x_2 - \dot{\gamma}_0 + \lambda_2 \gamma_1 - \lambda_2 \gamma_1) \\ &= -\rho_0 \xi^2 + (x_1 - \gamma_0) \\ &[\xi + \lambda_1 x_1 + \lambda_2 x_2 - \dot{\gamma}_0 + (-\rho_1(x_1 - \gamma_0) - \xi - \lambda_1 x_1 + \dot{\gamma}_0) - \lambda_2 \gamma_1] \\ &= -\rho_0 \xi^2 + (x_1 - \gamma_0)[- \rho_1(x_1 - \gamma_0) + \lambda_2(x_2 - \gamma_1)] \\ &= -\rho_0 \xi^2 - \rho_1(x_1 - \gamma_0)^2 + \lambda_2(x_1 - \gamma_0)(x_2 - \gamma_1) \end{aligned}$$

where  $\rho_1 > 0$  is a design parameter.

**Step 3:** The time derivative of  $\gamma_1$  can be derived from equation (17) as follows:

$$(19) \quad \begin{aligned} \dot{\gamma}_1 &= \frac{1}{\lambda_2} [-\rho_1(\dot{x}_1 - \dot{\gamma}_0) - \dot{\xi} - \lambda_1 \dot{x}_1 + \dot{\gamma}_0] \\ &= \frac{1}{\lambda_2} [(\rho_1 \dot{\gamma}_0 - (x_1 - v_d) + \dot{\gamma}_0) + (-\rho_1 - \lambda_1)(\lambda_1 x_1 + \lambda_2 x_2)] \end{aligned}$$

The expression for the derivative of  $(x_1 - \gamma_1)$  can be represented as:

$$(20) \quad \dot{x}_1 - \dot{\gamma}_1 = \lambda_3 x_1 + \lambda_4 x_2 + \lambda_5 \mu - \dot{\gamma}_1$$

To proceed, we will define a sliding surface  $S$ , select a Lyapunov function  $V_3$ , and design a feedback law  $\mu$  with the objective of ensuring that  $\dot{V}_3$  is negative definite.

$$(21) \quad S = x_2 - \gamma_1$$

$$(22) \quad V_3 = V_2 + \frac{1}{2} S^2$$

The derivative of the Lyapunov function  $V_3$ , using Eq.18., Eq. 19., and Eq. 20., is elucidated as:

$$(23) \quad \begin{aligned} \dot{V}_3 &= \dot{V}_2 + S \dot{S} \\ &= -\rho_0 \xi^2 - \rho_1(x_2 - \gamma_1)^2 + \lambda_2(x_1 - \gamma_0)(x_2 - \gamma_1) \\ &+ S(\dot{x}_2 - \dot{\gamma}_1) \\ &= -\rho_0 \xi^2 - \rho_1(x_1 - \gamma_0)^2 \\ &+ S[\lambda_2(x_1 - \gamma_0) + \lambda_3 x_1 + \lambda_4 x_2 + \lambda_5 \mu - \dot{\gamma}_1] \end{aligned}$$

The feedback law, designed to nullify the indefinite term in  $\dot{V}_3$ , is expressed as:

$$(24) \quad \mu = \frac{1}{\lambda_5} [-\lambda_2(x_1 - \gamma_0) - \lambda_3 x_1 - \lambda_4 x_2 + \dot{\gamma}_1 - K_1 S - K_2 \text{sign}(S)]$$

Where  $K_1 \geq 0$  and  $K_2 \geq 0$  are design parameters.

With the designed control input  $\mu$ , the derivative of Lyapunov function  $V_3$  is:

$$(25) \quad \begin{aligned} \dot{V}_3 &= -\rho_0 \xi^2 - \rho_1(x_1 - \gamma_0)^2 - K_1 S^2 - K_2 S \text{sign}(S) \\ &= -\rho_0 \xi^2 - \rho_1(x_1 - \gamma_0)^2 - K_1 S^2 - K_2 |S| \end{aligned}$$

which is negative definite.

## Simulation results

The backstepping-sliding mode controller, designed based on the state space averaged model of the buck converter, was simulated using MATLAB on a digital computer. The simulation results were obtained by executing the simulation under the following conditions:

- Setpoint changes from 8 volts to 12 volts.
- Step disturbances the load resistance.
- Step disturbances in the power supply.

The specifications of the converter are mentioned in Table 1, and the design parameters of the proposed controller are given in Table 2.

Table 1. Parameters of the DC-DC buck converter.

Parameter name	Symbol	Value	Unit
Power supply	$E$	20	V
Inductance	$L$	92	$\mu\text{H}$
Capacitance	$C$	220	$\mu\text{F}$
Resistance	$R$	12	$\Omega$
Inductor resistance	$R_L$	74	m $\Omega$
Capacitor ESR	$R_c$	70	m $\Omega$
Diode resistance	$R_d$	30	m $\Omega$
Switching resistance	$R_s$	44	m $\Omega$

Table 2. Design parameters of the controller.

Symbol	Value
$\rho_0$	120
$\rho_1$	60000
$K_1$	50000
$K_2$	2000

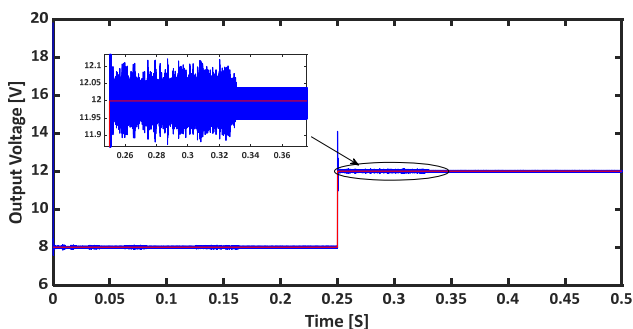


Fig. 2. Output voltage response

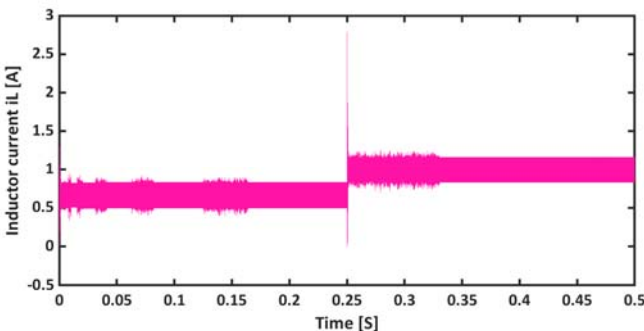


Fig. 3. Inductor current response

### Response to set point changes

To examine the effectiveness of the proposed Backstepping sliding mode controller (BSMC) in regulating the output voltage to a desired value, the voltage reference value was altered from 8 Volts to 12 Volts precisely at 0.25 seconds. This change was made to evaluate the controller's capability to track and maintain the desired output voltage level. Fig.2. illustrates the behavior of the output voltage. It is evident from the figure that the proposed controller performs effectively in tracking the reference output voltage

( $V_{ref}$ ) when there is a change in the setpoint. The controller demonstrates a commendable performance in maintaining the desired output voltage level.

### Response to load changes

The simulation results of the BSMC are provided for the output voltage and the inductor current (in Fig.5. and Fig.6. respectively) with load resistance changing from  $R=12\Omega$  to  $R=8\Omega$  during the time  $[0.4, 0.6]$  s and from  $R=8\Omega$  to  $R=15\Omega$  during the time interval  $[0.6, 0.75]$  s, during the time interval  $[0.75, 1]$  s  $R=12\Omega$ . Observations reveal that the output voltage displays approximately 0.45V overshoots during load disturbances. Nevertheless, it is noteworthy that the proposed controller exhibits commendable robustness in the presence of such disturbances. Despite the overshoots, the controller effectively maintains stability and demonstrates resilience in handling these types of disturbances.

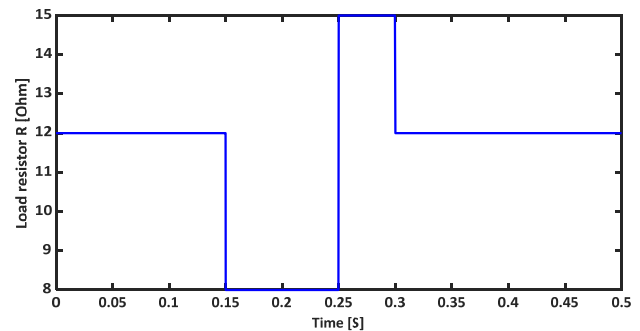


Fig. 4. Load resistance variations.

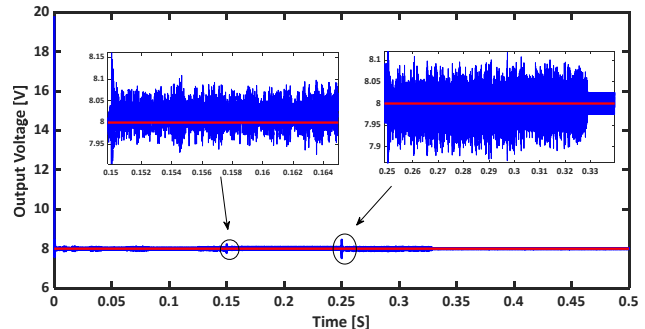


Fig. 5. Output voltage response

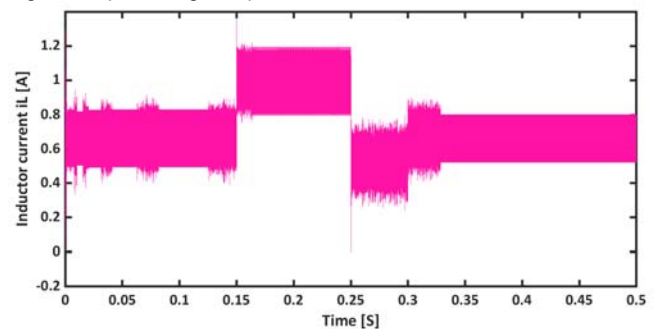


Fig. 6. Inductor current response.

### Response to external input voltage changes

In this section, the performance analysis of the backstepping sliding mode controller (BSMC) is conducted under voltage source disturbances. Specifically, the power supply is altered from 20V to 18V within the time interval of  $[0.2, 0.3]$  s. Figure 8 visually represents the behavior of the BSMC during this period. Notably, the output voltage response exhibits approximately (+0.05V) overshoots during the disturbance. However, it is evident that the proposed controller effectively tracks the desired output voltage, displaying its robust performance even in the presence of source voltage disturbances.

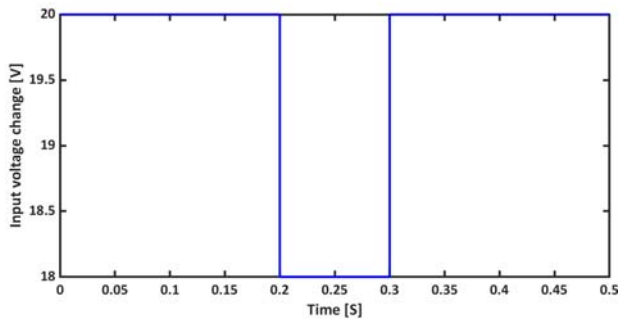


Fig. 7. Input voltage variations.

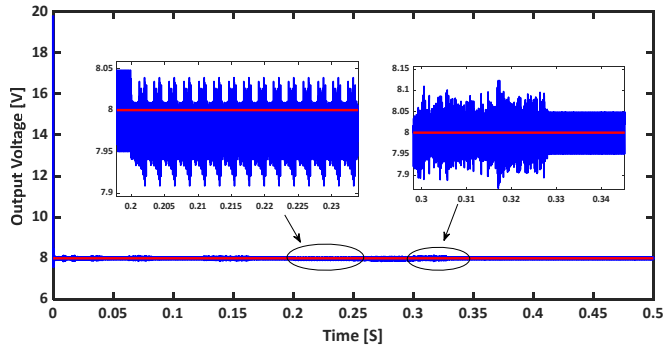


Fig. 8. Output voltage response.

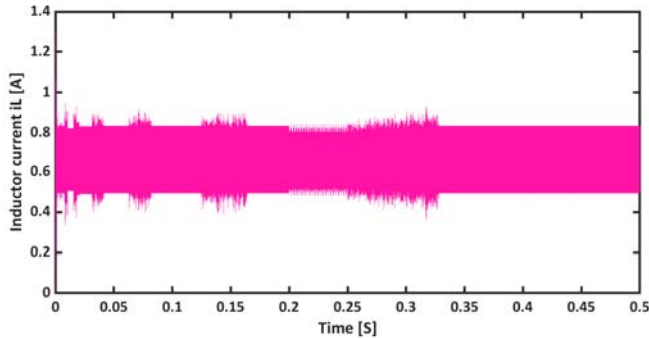


Fig. 9. Inductor current response.

## Conclusion

This paper introduces a proposed control strategy known as Backstepping Sliding Mode Controller (BSMC), which aims to regulate the output voltage of a DC/DC buck converter. The (BSMC) is a hybrid control technique that integrates both the merits of the backstepping control and the sliding mode control technique. The control technique utilized in this approach relies on Lyapunov synthesis, which effectively addresses the stability issue. The proposed control technique was subjected to simulation tests involving load resistance and input voltage perturbations. The response of the controller exhibited robustness against these disturbances, demonstrating excellent performance in accurately tracking the desired voltage reference.

**Authors:** I. BOUSSADIA, K. SAOUDI and I. GRICHE is with Department of Electrical Engineering, Faculty of Sciences and Applied Sciences, University, Bouira, Algeria e-mail: i.boussadia@univ-bouira.dz, k.saoudi@univ-bouira.dz, issam.grich@univ-bouira.dz, Z.Bouchama, University of Bordj Bou Arreridj, Algeria e-mail:bouchama\_ziad@yahoo.fr, I. ALISKAN, Yildiz Technical University, Turkey, e-mail: ialiskan@yildiz.edu.tr, M.AYAD, University of Setif1, Algeria, e-mail: m.ayad@univ-setif.dz.

## REFERENCES

- [1] SA Saadat, SM Ghamari, Adaptive backstepping controller design on Buck converter with a novel improved identification method, IET Control Theory, 16 (2022), No. 5, 485-495.
- [2] AH Mary, AH Miry, MH Miry, System uncertainties estimation based adaptive robust backstepping control for DC DC buck converter, International Journal of Electrical and Computer Engineering (IJECE), 11(2021), No. 1,347-355.
- [3] BB Naik, AJ Mehta, Sliding mode controller with modified sliding function for DC-DC Buck Converter, ISA transactions, 70 (2017), 279-287.
- [4] J Wang, J Rong, Dynamic prescribed performance sliding mode control for DC-DC buck converter system with mismatched time-varying disturbances, ISA transactions, 129, Part B (2022), 546-557.
- [5] N Zerroug, MN Harmas, S Benagoune, Z. Bouchama, K. Zahar, DSP-based implementation of fast terminal synergetic control for a DC-DC Buck converter, Journal of the Franklin Institute, 355 (2018), No. 5, 2329-2343.
- [6] Siew-Chong Tan, Y. M. Lai, C. K. Tse and M. K. H. Cheung, An adaptive sliding mode controller for buck converter in continuous conduction mode, Nineteenth Annual IEEE Applied Power Electronics Conference and Exposition, (2004), APEC 04, Anaheim, CA, USA, No. 3, 1395-1400.
- [7] JF Tsai, YP Chen, Sliding mode control and stability analysis of buck DC-DC converter, International Journal of Electronics, 94 (2007), No. 3, 209-222.
- [8] S. Ding, W. X. Zheng, J. Sun and J. Wang, Second-Order Sliding-Mode Controller Design and Its Implementation for Buck Converters, IEEE Transactions on Industrial Informatics, 14 (2018), No. 5, 1990-2000.
- [9] C Robles Algarin, J Taborda Giraldo, Omar Rodríguez Álvarez, Fuzzy logic based MPPT controller for a PV system, Energies, 10 (2017), No. 12, 2036.
- [10] Ahmed M. Kassem, MPPT control design and performance improvements of a PV generator powered DC motor-pump system based on artificial neural networks, International Journal of Electrical Power & Energy, 43(2012), No. 1, 90-98.
- [11] B. ChittiBabu, S. R. Samantaray, N. Saraogi, M. V. Ashwin Kumar, R. Sriharsha and S. Karmaker, Synchronous Buck Converter based PV Energy System for Portable Applications, IEEE Technology Students' Symposium, Kharagpur, India, (2011), 335-340
- [12] J. Xiao, A. Peterchev, J. Zhang and S. Sanders, An ultra-low-power digitally controlled buck converter IC for cellular phone applications, Nineteenth Annual IEEE Applied Power Electronics Conference and Exposition, (2004). APEC '04., Anaheim, CA, USA, , No. 1, 383-391.
- [13] Y. S. Lee, S. J. Wang and S. Y. R. Hui, Modeling, analysis, and application of buck converters in discontinuous-input-voltage mode operation, IEEE Transactions on Power Electronics, 12 (1997), No. 2, 350-360.
- [14] O. Diouri, A. Gaga, S. Senhaji, M. Ouazzani Jamil, Design and PIL Test of High Performance MPPT Controller Based on P&O-Backstepping Applied to DC-DC Converter, Journal of Robotics and Control (JRC), 3(2022), No. 4, 431-438.
- [15] Abjadi, N.R., Goudarzian, A.R., Arab Markadeh, G.R. et al. Reduced-Order Backstepping Controller for POESLL DC-DC Converter Based on Pulse Width Modulation. Iran J Sci Technol Trans Electr Eng 43 (2019), 219-228.
- [16] T. K. Nizami and C. Mahanta, "Adaptive backstepping control for DC-DC buck converters using Chebyshev neural network, Annual IEEE India Conference (INDICON), Pune, India, 2014, 1-5.