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Single-circuit equivalent of a double-circuit line with parallel operation of circuits

***Abstract**. Analysis of modes in circuits with two-chain lines sometimes requires reducing them to an equivalent single-chain line. The solution to this problem for a single-line circuit is known, and there are corresponding expressions for determining the coefficients of an equivalent 4-pole in terms of the coefficients of the original 4-poles. However, in some cases, there is a need to analyze modes in a 3-phase formulation, where there may be schemes with parallel 3-phase circuits. In this case, the problem arises of reducing parallel independent 3-phase lines to a 3-phase equivalent. For this purpose, the modal method for determining matrix phase coefficients is used. A special approach is required when converting a 6-phase line (two circuits on the same support) to a 3-phase circuit, when it is necessary to consider the mutual influence of the circuits. This research aims to provide a solution to the problem of reducing a 6-phase line to a 3-phase equivalent in parallel operation of circuits. Calculations based on the proposed algorithm for the 3-phase equivalent of a 500 kV 6-phase line with a length of 500 km are presented. The results satisfy the fundamental property of the n-th order multipole coefficients.*

***Streszczenie.** Analizując mody w obwodach z liniami podwójnymi, czasami zachodzi potrzeba ich zredukowania do równoważnej linii jednotorowej. Rozwiązanie tego problemu dla obwodu jednoliniowego jest znane i istnieją odpowiednie wyrażenia umożliwiające określenie współczynników równoważnej sieci 4-portowej poprzez współczynniki oryginalnych sieci 4-portowych. Jednak w wielu przypadkach zachodzi potrzeba analizy trybów w układzie 3-fazowym i w tym przypadku mogą występować obwody z równoległymi obwodami 3-fazowymi. W tym przypadku pojawia się zadanie doprowadzenia równoległych niezależnych linii 3-fazowych do odpowiednika 3-fazowego. Do tych celów wykorzystuje się modalną metodę wyznaczania współczynników fazowych macierzy. Szczególnego podejścia wymaga zamiana linii 6-fazowej (dwa obwody na jednym wsporniku) na obwód 3-fazowy, gdy konieczne jest uwzględnienie wzajemnego wpływu obwodów. W artykule zaproponowano rozwiązanie problemu doprowadzenia linii 6-fazowej do odpowiednika 3-fazowego, gdy obwody pracują równolegle. Obliczenia z wykorzystaniem zaproponowanego algorytmu przedstawiono dla trójfazowego odpowiednika 6-fazowej linii 500 kV o długości 500 km. Wyniki obliczeń spełniają podstawową właściwość, jaką posiadają współczynniki sieci wieloportowej n-tego rzędu. (**Jednoprzewodowy odpowiednik linii dwutorowej z równoległą pracą obwodów**)*

Keywords: ultra-high voltage lines, double circuit power line, six phase power line
Słowa kluczowe: linie ultrawysokiego napięcia, dwuobwodowa linia energetyczna, sześciofazowa linia energetyczna

Introduction

The consolidation of energy systems together with an increase in the capacity of power plants and systems in general are accompanied by an increase in power flows transmitted along power lines. Without powerful high-voltage power lines, it is impossible to supply energy from modern large power plants, and it is impossible to create unified energy systems. Long-distance ultra-high voltage overhead lines, ensuring the transmission of large power flows over significant distances, have a significant impact on the reliability of the energy interconnection. The traditional solution to the reliability problem for long-distance overhead lines widely used in the world is the construction of double-circuit sectionalized lines. Double-circuit overhead lines not only solve reliability problems, but also increase throughput. Consideration of double-circuit supports led American specialists to the idea of creating six-phase power transmissions [1-4].

Double-circuit overhead power lines (OVL) are specific objects of power supply systems, which is associated with small distances between the conductors of adjacent circuits. Mutual electrostatic and electromagnetic connections between conductors have a significant impact on the parameters of not only transient, but also steady-state modes (SS), especially at different values of power flows in the circuits.

Calculations of modes in networks with double-circuit lines are carried out mainly using single-wire equivalent circuits (SEC), and the mutual influence of the circuits is not taken into account [5]. Consequently, considering the ongoing interest in these issues in the operation and design of DVLs, the development and implementation of a methodology for calculating UR in phase coordinates for asymmetrical DVLs in asymmetrical modes (in phases and in circuits) using a multiwire equivalent circuit (MCC) has been proposed [6, 7].

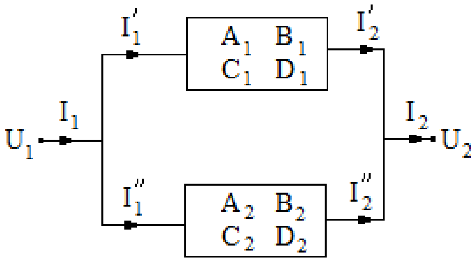
The relevance of the tasks of studying the modes of intersystem power lines in integrated electric power systems is confirmed by several studies. One of the widely studied problems is, for example, the asymmetry of inductance and capacitance of different phases for multi-circuit overhead power lines [8]. The authors of this work have proposed a technique for modeling power flows and electromagnetic fields of multi-circuit power transmission lines, in which conductors of several circuits of different voltage classes are placed on one support. The technique is based on the use of phase coordinates, which are the most natural description of three-phase power systems. The method is quite universal and can be used to solve these problems for multi-circuit overhead power lines of various designs.

The issues of optimal arrangement of phase sequences in multi-circuit lines are addressed in [9-12]. From the results of their study, it follows that the choice of the optimal phase sequence for multi-circuit lines is complex due to the presence of large asymmetry. One should also consider the mutual influence of circuits that arise in a system of circuits of different voltage classes suspended in the same support [13].

The purpose of this article is to develop a method for bringing a double-circuit line on different supports to a single-circuit 3-phase equivalent, as well as an algorithm for bringing a 6-phase line with circuits on one support to a 3-phase equivalent.

Algorithm for reducing a two-circuit line on different supports to a single-circuit 3-phase equivalent

When analyzing modes in circuits with double-circuit lines, sometimes there is a need to reduce them to an equivalent single-circuit line. The solution to this problem for a single-line diagram (Fig. 1) is available in [14].



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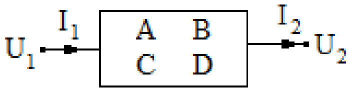


b)

Fig.1. Reducing a double-circuit line to a single-circuit equivalent for a single-line circuit: a – double-circuit line in the form of parallel 4-terminal networks; b – equivalent of a double-circuit line as a 4-port network

The corresponding relations for determining the coefficients of an equivalent 4-port network through the coefficients of the original 4-port networks have the form:

$$A = \frac{A_2 B_1 + A_1 B_2}{B_1 + B_2}, \qquad B = \frac{B_1 B_2}{B_1 + B_2},$$

$$C = C_1 + C_2 + \frac{DA}{B} - \frac{D_1 A_1}{B_1} - \frac{D_2 A_2}{B_2},$$

$$D = \frac{D_2 B_1 + D_1 B_2}{B_1 + B_2}.$$

However, in several cases, there is a need to analyze modes in a 3-phase formulation [15-25], and in this case there may be circuits with parallel 3-phase circuits, as, for example, shown in Fig. 2.

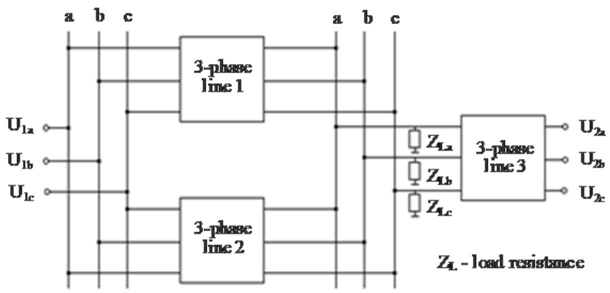
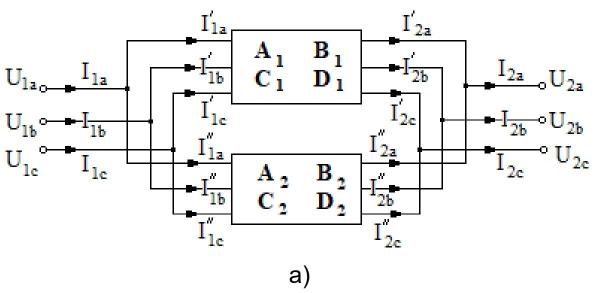


Fig.2. Design diagram with parallel 3-phase circuits

In this case, the task arises of bringing parallel independent 3-phase lines to a 3-phase equivalent.



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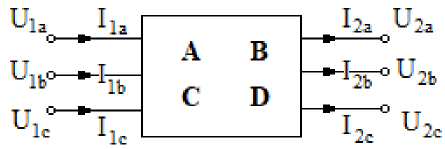


b)

Fig.3. Reduction of parallel independent 3-phase lines to a 3-phase equivalent: a – double-circuit line in the form of parallel 8-terminal networks; b – equivalent of a double-circuit line like an 8-port network

The relationship between the operating parameters at the ends of the line in phase coordinates for the circuit in Fig. 2a will be written

$$\begin{aligned} \mathbf{U}_1 &= \mathbf{A}_1\mathbf{U}_2 + \mathbf{B}_1\mathbf{I}'_2 \\ \mathbf{U}_1 &= \mathbf{A}_2\mathbf{U}_2 + \mathbf{B}_2\mathbf{I}''_2 \end{aligned} \tag{1}$$

$$\begin{aligned} \mathbf{I}'_1 &= \mathbf{C}_1\mathbf{U}_2 + \mathbf{D}_1\mathbf{I}'_2 \\ \mathbf{I}''_1 &= \mathbf{C}_2\mathbf{U}_2 + \mathbf{D}_2\mathbf{I}''_2 \end{aligned} \tag{2}$$

where

$$\mathbf{U}_1 = \begin{vmatrix} U_{1a} \\ U_{1b} \\ U_{1c} \end{vmatrix}, \quad \mathbf{U}_2 = \begin{vmatrix} U_{2a} \\ U_{2b} \\ U_{2c} \end{vmatrix}, \quad \mathbf{I}'_1 = \begin{vmatrix} I'_{1a} \\ I'_{1b} \\ I'_{1c} \end{vmatrix}, \quad \mathbf{I}''_1 = \begin{vmatrix} I''_{1a} \\ I''_{1b} \\ I''_{1c} \end{vmatrix}, \quad \mathbf{I}'_2 = \begin{vmatrix} I'_{2a} \\ I'_{2b} \\ I'_{2c} \end{vmatrix}, \quad \mathbf{I}''_2 = \begin{vmatrix} I''_{2a} \\ I''_{2b} \\ I''_{2c} \end{vmatrix}$$

- column vectors of 3-phase voltages and currents at the ends of the line;

$$\mathbf{A}_i = \begin{bmatrix} A_{iaa} & A_{iab} & A_{iac} \\ A_{iba} & A_{ibb} & A_{ibc} \\ A_{ica} & A_{icb} & A_{icc} \end{bmatrix}, \quad \mathbf{B}_i = \begin{bmatrix} B_{iaa} & B_{iab} & B_{iac} \\ B_{iba} & B_{ibb} & B_{ibc} \\ B_{ica} & B_{icb} & B_{icc} \end{bmatrix},$$

$$\mathbf{C}_i = \begin{bmatrix} C_{iaa} & C_{iab} & C_{iac} \\ C_{iba} & C_{ibb} & C_{ibc} \\ C_{ica} & C_{icb} & C_{icc} \end{bmatrix}, \quad \mathbf{D}_i = \begin{bmatrix} D_{iaa} & D_{iab} & D_{iac} \\ D_{iba} & D_{ibb} & C_{ibc} \\ D_{ica} & D_{icb} & D_{icc} \end{bmatrix}$$

- matrix phase coefficients, where $i = 1.2$.

The modal method for determining matrix phase coefficients is described for 3-phase lines in [26-27], and for n-wire lines in [28-30].

The relationship between the operating parameters at the ends of the line in phase coordinates for an equivalent 3-phase circuit in Fig. 2b will be written

$$\mathbf{U}_1 = \mathbf{A}\mathbf{U}_2 + \mathbf{B}\mathbf{I}_2 \tag{3}$$

$$\mathbf{I}_1 = \mathbf{C}\mathbf{U}_2 + \mathbf{D}\mathbf{I}_2 \tag{4}$$

where $\mathbf{I}_1 = \begin{vmatrix} I_{1a} \\ I_{1b} \\ I_{1c} \end{vmatrix}, \quad \mathbf{I}_2 = \begin{vmatrix} I_{2a} \\ I_{2b} \\ I_{2c} \end{vmatrix}$ - column vectors of total 3-phase currents at the ends of the line; $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ - matrix phase coefficients for equivalent circuit.

From (1) and (3) it follows

$$\mathbf{I}_2 = \mathbf{I}'_2 + \mathbf{I}''_2 = (\mathbf{B}_1^{-1} + \mathbf{B}_2^{-1})\mathbf{U}_1 + (\mathbf{B}_1^{-1}\mathbf{A}_1 + \mathbf{B}_2^{-1}\mathbf{A}_2)\mathbf{U}_2,$$

$$\mathbf{I}_2 = \mathbf{B}^{-1}\mathbf{U}_1 + \mathbf{B}^{-1}\mathbf{A}\mathbf{U}_2 .$$

From a comparison of these relations it follows

$$\mathbf{B}^{-1} = \mathbf{B}_1^{-1} + \mathbf{B}_2^{-1}$$

$$\mathbf{B}^{-1}\mathbf{A} = \mathbf{B}_1^{-1}\mathbf{A}_1 + \mathbf{B}_2^{-1}\mathbf{A}_2$$

And finally we have

$$\mathbf{B} = (\mathbf{B}_1^{-1} + \mathbf{B}_2^{-1})^{-1}$$

$$\mathbf{A} = \mathbf{B}(\mathbf{B}_1^{-1}\mathbf{A}_1 + \mathbf{B}_2^{-1}\mathbf{A}_2)$$

From (2) considering (1) and from (4) taking into account (3) we obtain:

$$\mathbf{I}_1 = \mathbf{I}_1' + \mathbf{I}_1'' = (\mathbf{D}_1\mathbf{B}_1^{-1} + \mathbf{D}_2\mathbf{B}_2^{-1})\mathbf{U}_1 + (\mathbf{C}_1 + \mathbf{C}_2 - \mathbf{D}_1\mathbf{B}_1^{-1}\mathbf{A}_1 - \mathbf{D}_2\mathbf{B}_2^{-1}\mathbf{A})\mathbf{U}_2$$

$$\mathbf{I}_1 = \mathbf{D}\mathbf{B}^{-1}\mathbf{U}_1 + (\mathbf{C} + \mathbf{D}\mathbf{B}^{-1}\mathbf{A})\mathbf{U}_2 .$$

From a comparison of the last relations, it follows

$$\mathbf{D}\mathbf{B}^{-1} = \mathbf{D}_1\mathbf{B}_1^{-1} + \mathbf{D}_2\mathbf{B}_2^{-1}$$

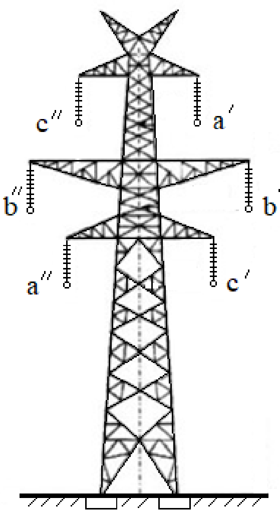
$$\mathbf{C} + \mathbf{D}\mathbf{B}^{-1}\mathbf{A} = \mathbf{C}_1 + \mathbf{C}_2 - \mathbf{D}_1\mathbf{B}_1^{-1}\mathbf{A}_1 - \mathbf{D}_2\mathbf{B}_2^{-1}\mathbf{A}$$

And as a result we have

$$\mathbf{D} = (\mathbf{D}_1\mathbf{B}_1^{-1} + \mathbf{D}_2\mathbf{B}_2^{-1})\mathbf{B}$$
$$\mathbf{C} = \mathbf{C}_1 + \mathbf{C}_2 + \mathbf{D}\mathbf{B}^{-1}\mathbf{A} - \mathbf{D}_1\mathbf{B}_1^{-1}\mathbf{A}_1 - \mathbf{D}_2\mathbf{B}_2^{-1}\mathbf{A} \qquad (5)$$

Algorithm for conducting a 6-phase line with circuits on one support to a 3-phase equivalent

A special approach is required when converting a 6-phase line (two circuits on one support, Fig. 4a, Fig. 4b) to a 3-phase circuit (Fig. 4c), when it is necessary to take into account the mutual influence of the circuits



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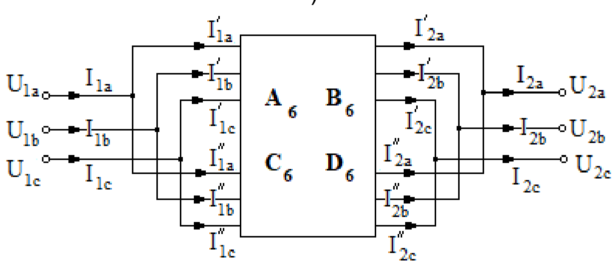

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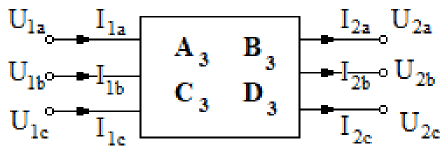

c)

Fig.4. Reducing a 6-phase line to a 3-phase equivalent: a – support; b – 6-phase line as a 12-port network; c – 3-phase equivalent for 6-phase line

The relationship between the operating parameters at the ends of the line in phase coordinates for a 6-phase line can be presented in the form

$$\mathbf{U}_{1(6)} = \mathbf{A}_6\mathbf{U}_{2(6)} + \mathbf{B}_6\mathbf{I}_{2(6)}$$
$$\mathbf{I}_{1(6)} = \mathbf{C}_6\mathbf{U}_{2(6)} + \mathbf{D}_6\mathbf{I}_{2(6)} \qquad (6)$$

where $\mathbf{U}_{1(6)} = \begin{vmatrix} U_{1a} \\ U_{1b} \\ U_{1c} \\ U_{1a} \\ U_{1b} \\ U_{1c} \end{vmatrix}$, $\mathbf{U}_{2(6)} = \begin{vmatrix} U_{2a} \\ U_{2b} \\ U_{2c} \\ U_{2a} \\ U_{2b} \\ U_{2c} \end{vmatrix}$, $I_{1(6)} = \begin{vmatrix} I'_{1a} \\ I'_{1b} \\ I'_{1c} \\ I''_{1a} \\ I''_{1b} \\ I''_{1c} \end{vmatrix}$,

$I_{2(6)} = \begin{vmatrix} I'_{2a} \\ I'_{2b} \\ I'_{2c} \\ I''_{2a} \\ I''_{2b} \\ I''_{2c} \end{vmatrix}$ - column vectors of 6-phase voltages and currents, respectively, at the beginning and end of the line

$$\mathbf{A}_6 = \begin{bmatrix} A_{a'a'} & A_{a'b'} & A_{a'c'} & A_{a'a''} & A_{a'b''} & A_{a'c''} \\ A_{b'a'} & A_{b'b'} & A_{b'c'} & A_{b'a''} & A_{b'b''} & A_{b'c''} \\ A_{c'a'} & A_{c'b'} & A_{c'c'} & A_{c'a''} & A_{c'b''} & A_{c'c''} \\ A_{a''a'} & A_{a''b'} & A_{a''c'} & A_{a''a''} & A_{a''b''} & A_{a''c''} \\ A_{b''a'} & A_{b''b'} & A_{b''c'} & A_{b''a''} & A_{b''b''} & A_{b''c''} \\ A_{c''a'} & A_{c''b'} & A_{c''c'} & A_{c''a''} & A_{c''b''} & A_{c''c''} \end{bmatrix}$$

$$\mathbf{B}_6 = \begin{bmatrix} B_{a'a'} & B_{a'b'} & B_{a'c'} & B_{a'a''} & B_{a'b''} & B_{a'c''} \\ B_{b'a'} & B_{b'b'} & B_{b'c'} & B_{b'a''} & B_{b'b''} & B_{b'c''} \\ B_{c'a'} & B_{c'b'} & B_{c'c'} & B_{c'a''} & B_{c'b''} & B_{c'c''} \\ B_{a''a'} & B_{a''b'} & B_{a''c'} & B_{a''a''} & B_{a''b''} & B_{a''c''} \\ B_{b''a'} & B_{b''b'} & B_{b''c'} & B_{b''a''} & B_{b''b''} & B_{b''c''} \\ B_{c''a'} & B_{c''b'} & B_{c''c'} & B_{c''a''} & B_{c''b''} & B_{c''c''} \end{bmatrix}$$

$$\mathbf{C}_6 = \begin{bmatrix} C_{a'a'} & C_{a'b'} & C_{a'c'} & C_{a'a''} & C_{a'b''} & C_{a'c''} \\ C_{b'a'} & C_{b'b'} & C_{b'c'} & C_{b'a''} & C_{b'b''} & C_{b'c''} \\ C_{c'a'} & C_{c'b'} & C_{c'c'} & C_{c'a''} & C_{c'b''} & C_{c'c''} \\ C_{a''a'} & C_{a''b'} & C_{a''c'} & C_{a''a''} & C_{a''b''} & C_{a''c''} \\ C_{b''a'} & C_{b''b'} & C_{b''c'} & C_{b''a''} & C_{b''b''} & C_{b''c''} \\ C_{c''a'} & C_{c''b'} & C_{c''c'} & C_{c''a''} & C_{c''b''} & C_{c''c''} \end{bmatrix}$$

$$\mathbf{D}_6 = \begin{bmatrix} D_{a'a'} & D_{a'b'} & D_{a'c'} & D_{a'a''} & C_{a'b''} & C_{a'c''} \\ C_{b'a'} & C_{b'b'} & C_{b'c'} & C_{b'a''} & C_{b'b''} & C_{b'c''} \\ C_{c'a'} & C_{c'b'} & C_{c'c'} & C_{c'a''} & C_{c'b''} & C_{c'c''} \\ C_{a''a'} & C_{a''b'} & C_{a''c'} & C_{a''a''} & C_{a''b''} & C_{a''c''} \\ C_{b''a'} & C_{b''b'} & C_{b''c'} & C_{b''a''} & C_{b''b''} & C_{b''c''} \\ C_{c''a'} & C_{c''b'} & C_{c''c'} & C_{c''a''} & C_{c''b''} & C_{c''c''} \end{bmatrix} -$$

matrix coefficients of 6-phase line.

Let's solve the problem of bringing a 6-phase line to a 3-phase equivalent with parallel operation of the circuits (Fig. 4b, Fig. 4c).

The line equations for the 3-phase equivalent are written

$$\begin{aligned} \mathbf{U_1} &= \mathbf{A_3 U_2} + \mathbf{B_3 I_2} \\ \mathbf{I_1} &= \mathbf{C_3 U_2} + \mathbf{D_3 I_2} \end{aligned} \tag{7}$$

From the first equation of system (6) it follows that

$$\mathbf{I_{2(6)}} = \mathbf{B_6^{-1} U_{1(6)}} - \mathbf{B_6^{-1} A_6 U_{2(6)}} \tag{8}$$

Let us introduce the following notation:

$$\mathbf{X_1} = \mathbf{B_6^{-1}} = \begin{bmatrix} \mathbf{X}_1' & \mathbf{X}_1''' \\ \mathbf{X}_1''' & \mathbf{X}_1'' \end{bmatrix}, \quad \mathbf{X}_2 = \mathbf{B_6^{-1}A}_6 = \begin{bmatrix} \mathbf{X}_2' & \mathbf{X}_2''' \\ \mathbf{X}_2''' & \mathbf{X}_2'' \end{bmatrix}$$

where $\mathbf{X}_1' \ldots \mathbf{X}_2''$ - corresponding 3rd order submatrices.

Let us rewrite equation (8) in expanded form

$$\begin{vmatrix} \mathbf{I}_2' \\ \mathbf{I}_2'' \end{vmatrix} = \begin{bmatrix} \mathbf{X}_1' & \mathbf{X}_1''' \\ \mathbf{X}_1''' & \mathbf{X}_1'' \end{bmatrix} \begin{vmatrix} \mathbf{U_1} \\ \mathbf{U_1} \end{vmatrix} - \begin{bmatrix} \mathbf{X}_2' & \mathbf{X}_2''' \\ \mathbf{X}_2''' & \mathbf{X}_2'' \end{bmatrix} \begin{vmatrix} \mathbf{U_2} \\ \mathbf{U_2} \end{vmatrix} \tag{9}$$

where $\mathbf{I}_2' = \begin{vmatrix} I_{2a}' \\ I_{2b}' \\ I_{2c}' \end{vmatrix}$, $\mathbf{I}_2'' = \begin{vmatrix} I_{2a}'' \\ I_{2b}'' \\ I_{2c}'' \end{vmatrix}$ - column vectors of 3-phase currents for the corresponding circuits.

When connecting circuits in parallel, the following conditions apply:

$$\begin{aligned} \mathbf{I_1} &= \mathbf{I_1'} + \mathbf{I_1''} \\ \mathbf{I_2} &= \mathbf{I_2'} + \mathbf{I}_2'' \end{aligned} \tag{10}$$

Taking into account (10) from (9), we find

$$\mathbf{I}_2 = \mathbf{X}_{1\Xi}\mathbf{U}_1 - \mathbf{X}_{2\Xi}\mathbf{U}_2 \tag{11}$$

where $\mathbf{X}_{1\Xi} = \mathbf{X}_1' + \mathbf{X}_1''' + \mathbf{X}_1''' + \mathbf{X}_1''$,

$\mathbf{X}_{2\Xi} = \mathbf{X}_2' + \mathbf{X}_2''' + \mathbf{X}_2''' + \mathbf{X}_2''$.

On the other hand, from the first equation of system (7) it follows

$$\mathbf{I}_2 = \mathbf{B}_3^{-1}\mathbf{U}_1 - \mathbf{B}_3^{-1}\mathbf{A_3}\mathbf{U}_2 \tag{12}$$

From a comparison of (11) and (12) we find

$\mathbf{B}_3^{-1} = \mathbf{X}_{1\Xi}$, $\quad \mathbf{B}_3^{-1}\mathbf{A}_3 = \mathbf{X}_{2\Xi}$.

And as a result we have

$$\begin{aligned} \mathbf{A}_3 &= \mathbf{X}_{1\Xi}^{-1}\mathbf{X}_{2\Xi} \\ \mathbf{B}_3 &= \mathbf{X}_{1\Xi}^{-1} \end{aligned} \tag{13}$$

Let us next find the coefficients $\mathbf{C}_3$ and $\mathbf{D}_3$ for an equivalent 3-port network. From the second equation of system (6), taking into account the first equation, we express the column vector of 6-phase currents at the beginning of the line through the column vectors of voltages at the ends of the line

$$\mathbf{I}_{1(6)} = \mathbf{D}_6\mathbf{B}_6^{-1}\mathbf{U}_{1(6)} + (\mathbf{C}_6 - \mathbf{D}_6\mathbf{B}_6^{-1}\mathbf{A}_6)\mathbf{U}_{2(6)} \tag{14}$$

Let us introduce the following notation:

$$\mathbf{X}_3 = \mathbf{D}_6\mathbf{B}_6^{-1} = \begin{bmatrix} \mathbf{X}_3' & \mathbf{X}_3''' \\ \mathbf{X}_3''' & \mathbf{X}_3'' \end{bmatrix},$$

$$\mathbf{X}_4 = \mathbf{C}_6 - \mathbf{D}_6\mathbf{B}_6^{-1}\mathbf{A}_6 = \begin{bmatrix} \mathbf{X}_4' & \mathbf{X}_4''' \\ \mathbf{X}_4''' & \mathbf{X}_4'' \end{bmatrix}$$

where $\mathbf{X}_3' \ldots \mathbf{X}_4''$ - corresponding 3rd order submatrices.

Let us rewrite equation (14) in expanded form

$$\begin{vmatrix} \mathbf{I}_1' \\ I_1'' \end{vmatrix} = \begin{bmatrix} \mathbf{X}_3' & \mathbf{X}_3''' \\ \mathbf{X}_3''' & \mathbf{X}_3'' \end{bmatrix} \begin{vmatrix} \mathbf{U}_1 \\ \mathbf{U}_1 \end{vmatrix} + \begin{bmatrix} \mathbf{X}_4' & \mathbf{X}_4''' \\ \mathbf{X}_4''' & \mathbf{X}_4'' \end{bmatrix} \begin{vmatrix} \mathbf{U}_2 \\ \mathbf{U}_2 \end{vmatrix} \tag{15}$$

where $\mathbf{I}_1' = \begin{vmatrix} I_{1a}' \\ I_{1b}' \\ I_{1c}' \end{vmatrix}$, $\mathbf{I}_1'' = \begin{vmatrix} I_{1a}'' \\ I_{1b}'' \\ I_{1c}'' \end{vmatrix}$ - column vectors of 3-phase currents for the corresponding circuits.

Taking into account the boundary conditions (6), reflecting the parallelism of the circuits, it follows from (15)

$$\mathbf{I}_1 = \mathbf{X}_{3\Xi}\mathbf{U}_1 + \mathbf{X}_{4\Xi}\mathbf{U}_2 \tag{16}$$

where $\mathbf{X}_{3\Xi} = \mathbf{X}_3' + \mathbf{X}_3''' + \mathbf{X}_3''' + \mathbf{X}_3''$,

$\mathbf{X}_{4\Xi} = \mathbf{X}_4' + \mathbf{X}_4''' + \mathbf{X}_4''' + \mathbf{X}_4''$.

From system (7) for the 3-phase equivalent it follows

$$\mathbf{I}_1 = \mathbf{D}_3\mathbf{B}_3^{-1}\mathbf{U}_1 + (\mathbf{C}_3 - \mathbf{D}_3\mathbf{B}_3^{-1}\mathbf{A}_3)\mathbf{U}_2 \tag{17}$$

From a comparison of (16) and (17) we find

$$\mathbf{D}_3\mathbf{B}_3^{-1} = \mathbf{X}_{3\Xi}, \qquad \mathbf{C}_3 - \mathbf{D}_3\mathbf{B}_3^{-1}\mathbf{A}_3 = \mathbf{X}_{4\Xi}$$

And as a result we have

$$\mathbf{C}_3 = \mathbf{X}_{4\Xi} + \mathbf{X}_{3\Xi}\mathbf{X}_{1\Xi}^{-1}\mathbf{X}_{2\Xi} \qquad (18)$$
$$\mathbf{D}_3 = \mathbf{X}_{3\Xi}\mathbf{X}_{1\Xi}^{-1}$$

The calculations carried out using the algorithm presented above for the 3-phase equivalent of a 6-phase 500 kV line with a length of 500 km, depicted in [31] and having a 3xAC-500 design, give the following results

$$\mathbf{A}_3 = \begin{vmatrix} 0.791+0.026i & -0.073+0.016i & -0.076+0.016i \\ -0.075+0.016i & 0.791+0.026i & -0.073+0.016i \\ -0.077+0.016i & -0.072+0.016i & 0.791+0.026i \end{vmatrix}$$

$$\mathbf{B}_3 = \begin{vmatrix} 23.229+177.418i & 18.75+113.824i & 18.762+113.89i \\ 18.754+113.833i & 23.229+177.418i & 18.764+113.896i \\ 18.764+113.896i & 18.762+113.89i & 23.241+177.454i \end{vmatrix}$$

$$\mathbf{C}_3 = \begin{vmatrix} -2.039*10^{-5}+3.124i*10^{-3} & -6.298*10^{-6}-8.321i*10^{-4} & -6.591*10^{-6}-8.354i*10^{-4} \\ -6.937*10^{-6}-8.351i*10^{-4} & -2.039*10^{-5}+3.124i*10^{-3} & -6.91*10^{-6}-8.368i*10^{-4} \\ -6.91*10^{-6}-8.368i*10^{-4} & -6.591*10^{-6}-8.354i*10^{-4} & -2.046*10^{-5}+3.125i*10^{-3} \end{vmatrix}$$

$$\mathbf{D}_3 = \begin{vmatrix} 0.791+0.026i & -0.073+0.016i & -0.072+0.016i \\ -0.075+0.016i & 0.791+0.026i & -0.077+0.016i \\ -0.073+0.016i & -0.076+0.016i & 0.791+0.026i \end{vmatrix}$$

The coefficients of n-th order multiport networks have the fundamental property

$$\mathbf{A}_n\mathbf{D}_n^T - \mathbf{B}_n\mathbf{C}_n^T = \mathbf{E}_n,$$

where $\mathbf{E}_n$ is the diagonal identity matrix; **T** - index indicating matrix transposition.

The above calculation data in relation to bringing a 6-phase line to a 3-phase equivalent with parameters for a 500 kV line shows that this fundamental property is satisfied

$$\mathbf{A}_3\mathbf{D}_3^T - \mathbf{B}_3\mathbf{C}_3^T = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}.$$

Conclusions

An algorithm is proposed for bringing a double-circuit line on different supports to a single-circuit 3-phase equivalent, as well as an algorithm for bringing a 6-phase line with circuits on one support to a 3-phase equivalent.

Calculations using the proposed algorithm are presented for the 3-phase equivalent of a 6-phase 500 kV line with a length of 500 km.

From the above it follows that the proposed algorithms will simplify calculations when analyzing the modes of multi-circuit intersystem power lines.

Conflict of Interest

The authors declare no conflict of interest.

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