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Mathematical modelling of transient processes in a HVAC transmission line considering the effect lightning protection cables in the state of two-phase short-circuiting

***Abstract**. A mathematical model is developed of a fragment of an electric power transmission system whose key part is a long ultra-high voltage power line including lightning protection cables of distributed parameters. The model is based on a long line second-order partial derivative equation. The boundary conditions of the second and third types (Neuman and Poincare conditions) are used for the purposes of the equation. The line equations are solved by discretising partial derivatives using the straight-line method and then integrating the latter with the fourth-order Runge-Kutta method. Simulation results are given in a graphic format analysed in the state of two-phase short-circuiting.*

Streszczenie.** W artykule opracowano model matematyczny systemu przesyłania energii, którego elementem kluczowym jest długa linia elektroenergetyczna wysokiego napięcia z przewodami odgromowymi, analizowana, jako układ o parametrach rozłożonych. Opracowany model matematyczny opiera się na równaniu długiej linii drugiego rzędu o pochodnych cząstkowych. Do równania linii długiej wykorzystano warunki brzegowe drugiego i trzeciego rodzaju: Neumana i Poincarego. Równania linii rozwiązano drogą dyskretyzacji pochodnych cząstkowych metodą prostych z kolejnym całkowaniem w funkcji czasu za pomocą metody Runge-Kutty czwartego rzędu. Wyniki symulacji podane są w formie rysunków, które są analizowane w stanie zwarcia dwufazowego. **(Modelowanie matematyczne procesów nieustalonych w linii przesyłania HVAC z uwzględnieniem wpływu przewodów odgromowych w stanie dwufazowego zwarcia)

Słowa kluczowe: modelowanie matematyczne, linia energetyczna, przewody odgromowe, parametry rozłożone, procesy nieustalone.
Keywords: mathematical modelling, power line, lightning protection cables, distributed parameters, transients processes.

Introduction

Power lines are key parts of electricity transmission systems over considerable distances from the locations of energy generation to its end users. Their stability has immediate impact on the economy and welfare of virtually every country. This is already at the stage of design, therefore, that factors need to be address that potentially affect not only the lines but also other parts of power transmission systems. Transient electromagnetic states may be one of those factors.

Transient states in power lines can be caused by the turn-ons or turn-offs of energy sources as well as a range of emergency conditions. The analysis of these processes is of paramount importance to the stability and reliability of power supply, since calculating the parameters of equipment in power plants, stations, and lines, as well as the settings of relay protection devices and emergency automatic equipment depend on the nature of dynamic processes in a grid.

We address the highest (ultra-high) transmission lines. They normally cover substantial distances and are additionally provided with lightning protection cables. Therefore, complicated wave electromagnetic processes will occur in them as their mode of operation is switched. Full-scale experimentation in such facilities is virtually unfeasible, that's why we suggest basing such research on the mathematical modelling apparatus here.

A broad range of mathematical methods and approaches to the analysis of transient processes in electric power lines is currently in scientific use, yet too little attention is paid to additional factors that influence transient processes in the lines. This is particularly true of wave processes, since most studies ignore them and present mathematical line models in their circuit versions, that is, as concentrated parameter systems [1 – 3]. The presence of lightning protection cables is another major factor. According to the theory of electric power transmission, SEM will be induced in lightning protection cables in non-symmetrical modes of power line operation, which in turn affects the general electromagnetic condition of line phase wires. Unfortunately, this factor is commonly ignored in studies. The impact of lightning protection cables is sometimes taken into account when calculating the distributed parameters of power lines [4 – 6], however, such an approach doesn't provide for a real presentation of processes in lightning protection cables themselves.

It is therefore our objective to improve the methods of mathematical modelling of transient processes in ultra-high voltage transmission lines including lightning protection cables, also when the line is in the state of two-phase short-circuiting.

Mathematical model

A circuit diagram of a fragment of electric power transmission system is shown in Fig. 1. It consists of an electric power supply line, shown as a three-phase long line of distributed parameters including two lightning protection cables. The line operates under an equivalent, symmetrical active induction load. It comprises five wires: lightning protection cables T1, T2 and phases *A, B, C*. The equivalent active induction load is represented as r_H and L_H. The supply voltage is provided to the line's start. The lightning protection cables $T1$ and $T2$ are connected at the line's end and open at its start, thus forming an open electrical circuit with T2 grounded at the line's start.

Based on the laws of applied power engineering, an equation of electromagnetic state can be formulated:

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} = \left(\mathbf{LC}\right)^{-1}\left(\frac{\partial^2 \mathbf{u}}{\partial x^2} - \left(\mathbf{Lg} + \mathbf{rC}\right)\frac{\partial \mathbf{u}}{\partial t} - \mathbf{rgu}\right) \tag{1}$$

$$\frac{d\mathbf{i}_H}{dt} = \mathbf{L}_H^{-1}\left(\mathbf{u}_{EL}^H - \mathbf{r}_H \mathbf{i}_H\right), \tag{2}$$

where: $\mathbf{r}$, $\mathbf{L}$ – the square matrices of specific active resistances and inductances of the system of five independent circuits, $\mathbf{g}$, $\mathbf{C}$ – The square matrices of specific active conductances and capacitances of the system of five independent circuits, $\mathbf{r}_H$, $\mathbf{L}_H$ – the diagonal matrices of active resistances and inductances [7].

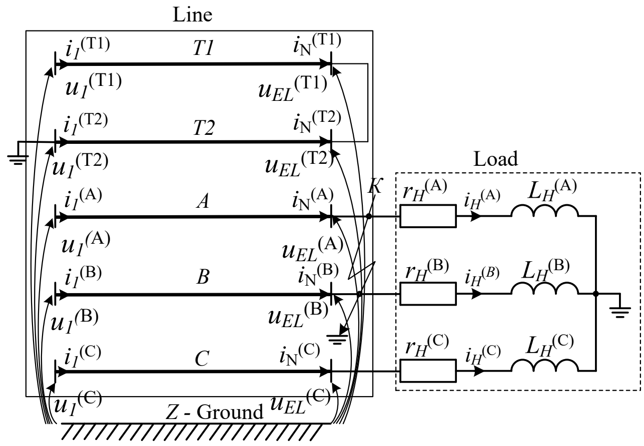

Fig.1. The circuit diagram of electric power line

Based on the laws of applied power engineering, an equation of electromagnetic state can be formulated:

$$(1)\qquad \frac{\partial^2 \mathbf{u}}{\partial t^2} = (\mathbf{LC})^{-1}\left(\frac{\partial^2 \mathbf{u}}{\partial x^2} - (\mathbf{Lg}+\mathbf{rC})\frac{\partial \mathbf{u}}{\partial t} - \mathbf{rgu}\right)$$

$$(2)\qquad \frac{d\mathbf{i}_H}{dt} = \mathbf{L}_H^{-1}\left(\mathbf{u}_{EL}^H - \mathbf{r}_H\mathbf{i}_H\right),$$

where: $\mathbf{r}$, $\mathbf{L}$ – the square matrices of specific active resistances and inductances of the system of five independent circuits, $\mathbf{g}$, $\mathbf{C}$ – The square matrices of specific active conductances and capacitances of the system of five independent circuits, $\mathbf{r}_H$, $\mathbf{L}_H$ – the diagonal matrices of active resistances and inductances [7].

The columnar vectors in (1) and (2) are as follows:

$$(3)\quad \mathbf{u}=\mathrm{col}\left(u^{(T1)},u^{(T2)},u^{(A)},u^{(B)},u^{(C)}\right),\ \mathbf{i}=\mathrm{col}\left(i^{(T1)},i^{(T2)},i^{(A)},i^{(B)},i^{(C)}\right);$$

$$(4)\quad \mathbf{i}_H=\mathrm{col}\left(i_H^{(A)},i_H^{(B)},i_H^{(C)}\right),\ \mathbf{u}_{EL}^H=\mathrm{col}\left(u_{EL}^{(A)},u_{EL}^{(B)},u_{EL}^{(C)}\right),$$

where $u^{(T1)}$, $u^{(T2)}$, $u^{(A)}$, $u^{(B)}$, $u^{(C)}$ – the voltages of $T1$, $T2$ with reference to earth and the phase voltages A, B, C, respectively; $i^{(T1)}$, $i^{(T2)}$, $i^{(A)}$, $i^{(B)}$, $i^{(C)}$ – the same for currents.

(1) describes transient wave electromagnetic processes in a long electric power supply line of distributed parameters. To solve it, boundary conditions are needed, especially voltages at the start and end of the electric power line. In our case, phase cable voltages at the start of the electric power line are known functions; these voltages are boundary conditions of the first type. As far as the voltages of lightning protection cables at the start and end of the line and of phase cables at its end are concerned, the latter are unknown due to the configuration of connections between lightning protection and phase cables and other parts of the power system fragment. To find these voltages, the boundary conditions of the second and third types (Neumann and Poincaré) are used) [8, 9, 10, 11].

The following equation serves as the boundary conditions fora (1):

$$(5)\qquad -\frac{\partial \mathbf{u}}{\partial x} = \mathbf{L}\frac{\partial \mathbf{i}}{\partial t} + \mathbf{ri}.$$

Discretising (1) and (5) with the straight-line method using the notion of central derivative [7] produces:

$$(6)\qquad \frac{d\mathbf{v}_j}{dt} = (\mathbf{LC})^{-1}\left(\frac{1}{(\Delta x)^2}\left(\mathbf{u}_{j-1} - 2\mathbf{u}_j + \mathbf{u}_{j+1}\right) - (\mathbf{Lg} + \mathbf{rC})\mathbf{v}_j - \mathbf{rgu}_j\right),\quad \frac{d\mathbf{u}_j}{dt} = \mathbf{v}_j,\quad j = 1,...,N;$$

$$(7)\qquad \frac{d\mathbf{i}_j}{dt} = \mathbf{L}^{-1}\left(\frac{1}{2\Delta x}\left(\mathbf{u}_{j-1} - \mathbf{u}_{j+1}\right) - \mathbf{ri}_j\right),\ j = 1,...,N,$$

where: Δx – discretisation step, N – the number of discretisation nodes.

(6) and (7) show that, in order to find the voltages of the first ($j = 1$) and last ($j = N$) discrete line nodes and the currents in the first and last discrete line branches, it's necessary to find the voltages of fictitious nodes at the start ($\mathbf{u}_0$) and end ($\mathbf{u}_{N+1}$) of the line [7].

We have seen the phase cable voltages at the line's start are known, $u_I^{(A)}$, $u_I^{(B)}$, $u_I^{(C)}$ – the supply voltages of the line. The voltage of the lightning protection cable $T2$ at the line's start is known too – equal to zero, $u_1^{(T2)} = 0$, since $T2$ at the start of the electric power line is earthed. Only the voltage of the lightning protection cable $T1$ is unknown to begin with, therefore. None of the voltages at the line's end is known.

The procedure of finding this voltage is detailed in [9]. The second and third-type boundary conditions are employed here. Only the formula for the zero voltage of the fictitious node will be given here:

$$(8)\quad u_0^{T1} = \frac{2\Delta x}{2C_{11}P_{11} + \Lambda_{11}}\left\{\frac{1}{2\Delta x}\left(\sum_{k=1}^{5}\left(\Lambda_{1,k}u_2^{(k)}\right) - \sum_{k=2}^{5}\left(\Lambda_{1,k}u_0^{(k)}\right)\right) + \sum_{k=1}^{5}\left(K_{1,k}i_1^{(k)}\right) - \Delta x\left(G_{11}v_1^{(T1)} - \sum_{m=T2}^{C}\left(g_{T1,m}v_1^{(m)} + C_{T1,m}\frac{dv_1^{(m)}}{dt}\right)\right) - \Delta x C_{11}\left(\frac{1}{(\Delta x)^2}\left(\sum_{k=1}^{5}\left(P_{1,k}i_1^{(k)} + P_{1,k}u_2^{(k)}\right)\right) - \frac{2}{(\Delta x)^2}\sum_{k=1}^{5}\left(P_{1,k}u_1^{(k)}\right) - \sum_{k=1}^{5}\left(F_{1,k}v_1^{(k)} + D_{1,k}u_1^{(k)}\right)\right)\right\},$$

where: $\Lambda = \mathrm{L}^{-1}$; $\mathrm{K} = \mathrm{L}^{-1}\mathrm{r}$; $\mathrm{P} = (\mathrm{LC})^{-1}$; $\mathrm{D} = \mathrm{Prg}$; $\mathrm{F} = \mathrm{P}(\mathrm{Lg} + \mathrm{rC})$; $g_{T1,m}$, $C_{T1,m}$ – Certain own and mutual active conductances and capacitances among $T1$ and $T2$, phases A, B, C of the line, and the earth Z; $u^{(T1,m)}$ – voltages among $T1$ and $T2$, respectively, phases A, B, C of the line, and the earth Z of the line's first discretisation node; C_{11}, G_{11} – matrix elements **C** and **G**; $m = T2, A, B, C, Z$; $k = 1,2,3,4,5$, k – the number of the matrix column.

To find $\mathbf{u}_{N+1}$ of the fictitious nodes at the end of the electric power line, [10] considered the boundary conditions of the second type to produce a universal expression. Since the idea of finding that voltage has not changed in the presence of lightning protection cables, we will skip the time-consuming mathematical transformations and only preset their final result:

$$(9)\quad \mathbf{u}_{N+1} = \mathbf{u}_{N-1} + 2\left(\mathbf{u}_{EL} - \mathbf{u}_N\right),\ \mathbf{u}_{EL} = \mathrm{col}\left(u_{EL}^{(T1)},u_{EL}^{(T2)},u_{EL}^{(A)},u_{EL}^{(B)},u_{EL}^{(C)}\right).$$

This expression allows for an autonomous utilisation of the power supply line's mathematical model founded on the long line equation for any configuration of the grid and the line connections. To this end, we compute the voltages ($u_{EL}^{(T1)}$, $u_{EL}^{(T2)}$), of the cables connected at the line's end and of the phase cables ($u_{EL}^{(A)}$, $u_{EL}^{(B)}$, $u_{EL}^{(C)}$), connected to the equivalent active induction load.

Since the lightning protection cables at the line's end are connected (see Fig. 1), their voltages at the line's end will be identical, i.e., $u_{EL}^{(T1)} = u_{EL}^{(T2)} = u_{EL}^{(T)}$. These are the formulas for the voltages at the line's end.

$$(10)\quad u_{EL}^{(T)} = \frac{1}{2}\left\{u_N^{(T2)} + u_N^{(T1)} - \Delta x\left(\sum_{m=T2,n=T1}^{C,C}\left(M_{T1,m}\frac{di_N^{(m)}}{dt} + M_{T2,n}\frac{di_N^{(n)}}{dt}\right) + 2r_Z\left(i_N^{(A)} + i_N^{(B)} + i_N^{(C)}\right)\right)\right\},$$

where: $m,\ n$ – the designations of lightning protection or phase cables ($n = T1,\ A,\ B,\ C$); $i_N^{(Z)}$ – the earthed current across the last discrete branch unit ($i_N^{(Z)} = i_N^{(A)} + i_N^{(B)} + i_N^{(C)}$).

The line's end voltages for phases *A, B, C* will be of the same type. This expression for the voltage at the end of an electric power line for phase *A* can serve as an example:

$$(12)\quad u_{EL}^{(A)} = -\frac{\Delta x L_A L_H^{(A)}}{\Delta x L_A - L_H^{(A)}}\left\{-\frac{1}{\Delta x L_A}\left[u_N^{(A)} + \Delta x\left(\sum_{q=T1}^{C}\left(M_{A,q}\frac{di_N^{(q)}}{dt}\right) + \left(r_A + r_Z\right)i_N^{(A)} + r_Z\sum_{q=B}^{C}\left(i_N^{(q)}\right)\right)\right] - \frac{r_H^{(A)} i_H^{(A)}}{L_H^{(A)}}\right\},$$

where: q – the designation of lightning protection or phase cables ($q = T1,\ T2,\ B,\ C$).

The following system of differentia equations: (6), (7) considering: (3), (4), (8) – (12), is jointly integrated,

Computer simulation results

The computer simulation followed the sequence below. At the instant $t = 0$ s, the controlled turn-on of an electric power line was imitated by a gradual turn-on of the line phases considering the starts of voltage function sinusoids beginning with zero at appropriate moments. As part of phase switching, phase *A* was turned on at $t = 0.009027$ s, B at $t = 0.005694$ s, and C at $t = 0.002361$ s. The turn-off was modelled as changeover commutations. On reaching the steady state at $t = 0.2$ s, a two-phase short-circuiting to the earth was simulated at the end of the electric power line.

An actual 750 kV power line between the stations of Zahidnoukrainska (Ukraine) and Albertirsha (Hungary) was adopted for the calculations. These are its parameters: $r_{0F} = 1.9\cdot10^{-5}$ Ω/m, $r_{0T} = 4.28\cdot10^{-4}$ Ω/m, $r_{0Z} = 5\cdot10^{-5}$ Ω/m, $L_{0F} = 1{,}647\cdot10^{-6}$ H/m, $L_{0T} = 2{,}405\cdot10^{-6}$ H/m, $M_{0FF} = 7.41\cdot10^{-7}$ H/m, $M_{0FT} = 7.4\cdot10^{-7}$ H/m, $M_{0TT} = 7.05\cdot10^{-7}$ Sm/m, $g_{0F} = 3{,}253\cdot10^{-11}$ Sm/m, $g_{0FF} = g_{0FT} = 3{,}253\cdot10^{-13}$ Sm/m, $g_{0T} \approx 0$ Sm/m, $g_{0TT} \approx 0$ Sm/m, $C_{0F} = 0.8647\cdot10^{-11}$ F/m, $C_{0FF} = 0.103\cdot10^{-11}$ F/m, $C_{0FT} = 0.0723\cdot10^{-11}$ F/m, $C_{0T} = 0.3501\cdot10^{-11}$ F/m, $C_{0TT} = 0.04162\cdot10^{-11}$ F/m.

The line was operated under a symmetrical, equivalent active induction load $r_H = 575$ Ω, $L_H = 0.95$ H. The simulation was carried out with the following values: $u_1^{(A)} = 635\ \sin(\omega t + 18^o)$ kV, $u_1^{(B)} = 635\ \sin(\omega t - 102^o)$ kV, $u_1^{(C)} = 635\ \sin(\omega t + 138^o)$ kV, $\omega = 314{,}15\ s^{-1}$. The discretisation step with the straight-line method was $\Delta x = l/20 = 23.8$ km. The electromagnetic state equations were integrated using the implicit Giro method of the second order.

The voltage and current functions highlighted yellow relate to phase *A*, green – phase *B*, red – phase *C*, and black, lightning protection cables.

Phase voltage functions in the middle of the power line are illustrated in Fig. 2. Note some minor overvoltages at turn-on in the middle of the line; in particular, the maximum momentary voltage of phase *A* is 690 kV, of phase *B:* 692 kV, and of phase *C:* 696 kV. The controlled commutation caused some minor overvoltages, reaching $1.08U_{MP}$ in phase C. In steady state before short-circuiting, the voltages had amplitudes of 620 kV, whereas the voltage amplitudes of phases *A* and *B* after the two-phase short-circuiting declined to 345 kV. The amplitude of phase *C* voltage raised a little to reach 630 kV.

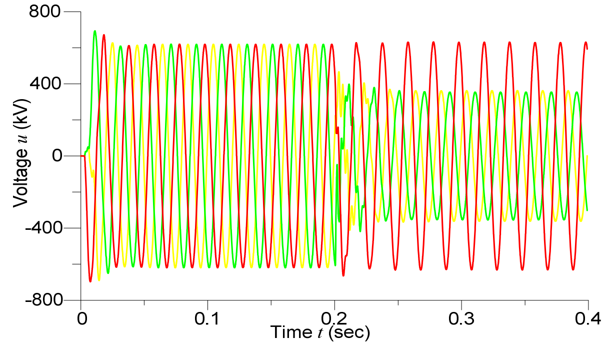

Fig.2. The time distribution of mid-line phase voltages

The functions of the line phase currents at the line's start are depicted in Fig. 3. It can be noted that at turn-on, the maximum phase A impact current was 1.84 kA, of phase *B* 1.94 kA, and phase C 1.55 kA. In the normal steady state, the current functions had an amplitude of 1.31 kA. Following an emergency state, the maximum phase impact currents had the following values: phase *A,* 7.55 kA, phase *B.* 5.81 kA, and phase C, 2.18 kA. In the steady emergency state, the amplitude of phase A current was 4.25 kA, of phase *B* 3.35 kA, and of phase *C,* 1.52 kA.

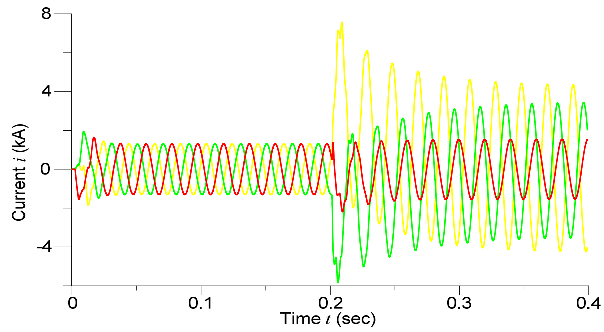

Fig.3. Phase current waveforms at the line's start

The time distribution of lightning protection cable *T*1 voltage function at the start of the line is shown in Fig. 4. *T*1's voltage at the line's start reaches a maximum of 210 kV at turn-on and assume the expected zero value in the normal steady state. After *T*1 is short-circuited, substantial overvoltages reaching 350 kV occur. In the steady emergency state, the voltage amplitude of *T*1 was up to 180 kV.

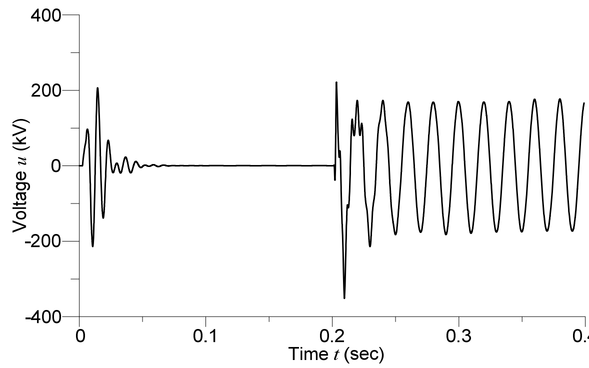

Fig.4. The time distribution of T1 voltage at the line's start

The time distribution of current function across two lightning protection cables, $T1$ and $T2$, at the line's start are presented in Figures 5 and 6. Figure 5 demonstrates the maximum touch current was 363 A as the line was turned on. On short-circuiting, due to the non-symmetrical line operation, current began flowing across the lightning protection cable again. The maximum impact current was 358 A. The current amplitude was 190 A in the steady emergency state. As far as the current across $T2$ in Fig. 6 is concerned, the maximum impact current was 450 A and 570 A after short-circuiting. The current amplitude was 330 A in the steady emergency state.

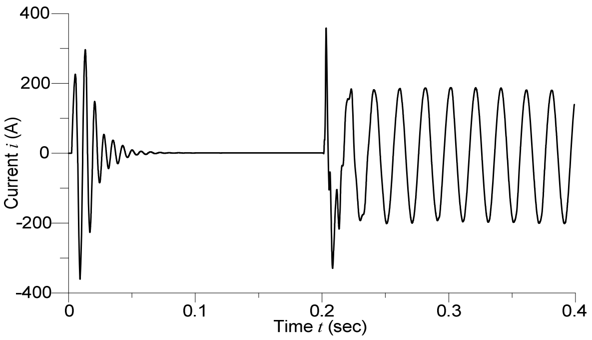

Fig.5. The time distribution of T1 current at the line's start

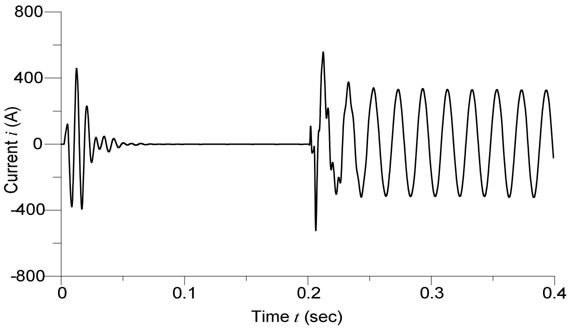

Fig.6. The time distribution of T2 current at the line's start

The time distributions of voltage and current functions across the lightning protection cables show the results of computer simulation fully corroborate the theoretical concept of electric power transmission in an ultra-high voltage power line.

Conclusion

1. The mathematical model of an electric power line including lightning protection cables, based on the long line equation solved by the application of Neumann and Poincaré boundary conditions, helps to recreate transient electromagnetic processes not only in the line's phase cables but also across lightning protection cables.
2. The results of computer simulation fully uphold the main assumptions of the electric energy transmission theory, which suggests a high adequacy of the mathematical model and the authenticity of the results.
3. The results of computer simulation show that, for situations like a controlled turn-on of a power line or asymmetrical emergency states, not only phase but also lightning protection cables need to be additionally taken into account.

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