

Sliding Mode Control Restart Strategy for Rotating Induction Machine Using Time-Varying Switching Line

Abstract. In this paper, an induction motor control strategy is considered with special aim to provide a method of fast restart of a rotating machine after a power loss. A sliding mode control with time-varying sliding line is derived in order to ensure the robustness to parameters' changes during the whole control process. It is shown, how to find the optimal parameters for the control law in order to obtain the shortest possible regulation time taking into account the control voltage limitations. The presented strategy is verified with use of a simulation example and its robustness is proven.

Streszczenie. W tej pracy przedstawiona została metoda szybkiego restartu obracającego się silnika indukcyjnego po utracie zasilania. Zaprojektowano algorytm sterowania ślizgowego z ruchomą krzywą przełączeń, by zapewnić odporność na zmiany parametrów podczas całego procesu sterowania. Pokazano sposób wyznaczania optymalnych parametrów pozwalających na osiągnięcie najkrótszego możliwego czasu sterowania po uwzględnieniu ograniczeń napięcia. Prezentowana strategia została zweryfikowana z użyciem przykładu symulacyjnego, który potwierdza jej odporność na odchylenia wartości parametrów. (Metoda ponownego rozruchu wirującego silnika indukcyjnego z wykorzystaniem sterowania ślizgowego z ruchomą krzywą przełączeń)

Keywords: sliding mode control, time-variant sliding mode, field-oriented control, induction motor restart

Słowa kluczowe: sterowanie ślizgowe, sterowanie ślizgowe z ruchomą krzywą przełączeń, sterowanie polowo-zorientowane, ponowny rozruch silnika indukcyjnego

Introduction

Induction motor restart is a crucial issue in drive systems in which there is a significant level of probability of disturbing an on-going process, for example by temporary power loss. This problem is well-known in some industrial cases. Some special method has to be used in order to repower the rotating induction motor, as the remaining magnetic flux of the rotor will cause a significant amount of current flowing through the motor in case of plugging it directly to AC power supply [4], [8].

One example of restart method is the *speed-search* or *flying-start* algorithm implemented in some industrial inverters from KEB company [1]. The main idea behind this solution is to find the synchronous speed of a rotating induction machine by increasing the supply voltage frequency to the maximum value while keeping its amplitude relatively low. When the measured phase currents are equal to zero, the synchronous speed is found and the drive can proceed to its standard control algorithm. Similar algorithms, based on the restored power measurement, have also been presented [2], [3].

Mentioned solution is applicable to variable frequency drives (VFDs). There is not, however, an universal method for restarting a rotating induction motor using field-oriented control (FOC). In industrial solutions and drives created for electric vehicles this problem is omitted by diligent tuning of the PID controllers, so that the transient states were acceptable.

Some methods to eliminate the inrush current have been studied. In more advanced control strategies, feedforward can be used to decrease or eliminate the starting current. The control law is designed in such a way, that the feedforward control signal is equal to the value of the electromotive force (EMF) being the consequence of the residual magnetic flux [8], [9], [10]. When the residual magnetic flux is already negligible, the motor start-up can be easily performed. However, if the restart must be performed shortly after voltage loss, a good knowledge of current machine state is necessary. The necessary information can be obtained using flux observer or flux estimator.

In this article a new restart strategy is proposed based on the sliding mode control (SMC) for stator currents. The inrush current elimination is achieved due to equivalent control based on full-order induction motor model. The q-axis rotor

flux, although being frequently omitted as it becomes equal to zero in the steady state of FOC, is included in the control strategy for better performance. The switching action of the used SMC makes the proposed strategy robust and ensures correct operation even when the induction motor parameters change during operation, e.g. due to generated heat. Finally, a time-varying sliding line is defined to provide robustness during the whole control process.

Problem Description

The aim of this paper is to deliver a time-optimal restart strategy of an induction motor after a temporary power loss. For this purpose, a full model of induction machine is necessary. The following model is described in a dq0 frame rotating with synchronous speed ω_e . Its position is measured in positive trigonometric direction, meaning that it rotates counter clockwise.

$$(1) \quad J \frac{d\omega}{dt} = T_e - T_L - b\omega$$

$$(2) \quad L_\sigma \frac{d}{dt} i_{ds} = - \left(R_s + \frac{L_m^2}{L_r^2} R_r \right) i_{ds} + L_\sigma \omega_e i_{qs} + \frac{L_m R_r}{L_r^2} \psi_{dr} + \frac{L_m}{L_r} p \omega \psi_{qr} + u_d$$

$$(3) \quad L_\sigma \frac{d}{dt} i_{qs} = - \left(R_s + \frac{L_m^2}{L_r^2} R_r \right) i_{qs} - L_\sigma \omega_e i_{ds} + \frac{L_m R_r}{L_r^2} \psi_{qr} - \frac{L_m}{L_r} p \omega \psi_{dr} + u_q$$

$$(4) \quad \frac{d}{dt} \psi_{dr} = - \frac{R_r}{L_r} \psi_{dr} + \omega_{sl} \psi_{qr} + \frac{L_m R_r}{L_r} i_{ds}$$

$$(5) \quad \frac{d}{dt} \psi_{qr} = - \frac{R_r}{L_r} \psi_{qr} - \omega_{sl} \psi_{dr} + \frac{L_m R_r}{L_r} i_{qs}$$

$$(6) \quad T_e = \frac{3}{2} p \left(\vec{i}_s \times \vec{\psi}_r \right)$$

The following symbols have been used in the above equations:

- i_{ds}, i_{qs} – stator currents in d, q axes respectively [A],
 ψ_{dr}, ψ_{qr} – d and q coordinate of rotor flux vector [Wb],
 R_s, R_r – stator and rotor resistance [Ω],
 L_s, L_r – stator and rotor inductance [H],
 L_m – magnetizing inductance [H],
 L_σ – leakage inductance coefficient [H],
 ω_{sl} – slip frequency [$\frac{\text{rad}}{\text{s}}$],
 ω_e – electrical synchronous speed [$\frac{\text{rad}}{\text{s}}$],
 ω – mechanical angular speed of the motor [$\frac{\text{rad}}{\text{s}}$],
 p – number of pole pairs [-],
 J – moment of inertia [$\text{kg} \cdot \text{m}^2$],
 T_e – electromagnetic torque [Nm],
 T_L – load torque [Nm],
 b – bearing torque coefficient [$\frac{\text{Nm} \cdot \text{s}}{\text{rad}}$].

Leakage inductance is given by $L_\sigma = \frac{L_r L_s - L_m^2}{L_r}$ and slip frequency is calculated as follows $\omega_{sl} = \omega_e - p\omega$.

The structure of the control system resembles the standard indirect FOC structure. Instead of the PI controllers, the sliding mode controllers are used to control d- and q-axis current. Full block diagram of the control system is presented in the picture below.

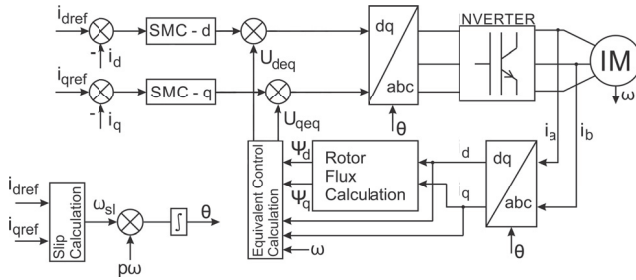


Fig. 1. Block diagram of the control system.

When considering FOC, equations (4) and (5) are very often simplified by assuming $\psi_{qr} = 0$. Under this assumption, it follows from (5) that $\omega_{sl} = \frac{L_m R_r}{L_r} \frac{i_{qs}}{i_{dr}}$. For $\psi_{qr} = 0$ it is also true, that $\psi_{dr} = L_m i_{ds}$, thus $\omega_{sl} = \frac{R_r}{L_r} \frac{i_{qs}}{i_{ds}}$.

Using this value of slip frequency to calculate synchronous speed in the following manner $\omega_e = p\omega + \omega_{sl}$ is very common, since it simplifies the control algorithm and makes it possible to use two separate PI controllers for d- and q-axis currents. However, this reasoning is only true in the steady state when both $\frac{d}{dt}\psi_{dr}$ and $\frac{d}{dt}\psi_{qr}$ are indeed equal to zero.

The control strategy presented in this paper is more general and remains accurate and applicable also when $\psi_q \neq 0$. Nevertheless, the formula for ω_{sl} mentioned above will be used, as the formula (6) is significantly simplified by such assumption and the value of electromagnetic torque depends only on the stator currents.

Sliding Mode Controller

The control law presented in this paper consists of three parts. The equivalent control, which can be treated as a feed-forward control, is its main part.

The remaining two parts are the switching function, which ensures system's robustness by keeping its representative point on the sliding surface, and the component connected with time-varying sliding line, which causes the occurrence of the sliding motion from the very beginning of the control process. This makes the trajectory of the system's representative point predictable and also enables fast disturbance attenuation.

Generally, the SMC requires the designer to choose the sliding surface. In case of the second or higher order systems, the description of the object's dynamics is used for this purpose. The considered object is described with two vector equations. However, equations (4) and (5) will be used only for flux estimation and the variables ψ_{qr} and ψ_{dr} will not be directly controlled. In the presented strategy, only the stator current vector is of interest.

Hence, there is only one vector equation, which will be used to design the sliding surface. This will be done by dividing control strategy into two separate parts for d- and q-axis. The proper reasoning is presented in the next sections. For better understanding, one may refer to [5], where a similar solution was presented.

A. Sliding Surface Selection

For the d-axis, the sliding surface is chosen as follows

$$(7) \quad s_d(t) = \begin{cases} e_d(t) + A + Bt, & t \leq t_0 \\ e_d(t), & t > t_0 \end{cases},$$

where $e_d = i_{dref} - i_d$ and i_{dref} is the reference value of d-axis current, which will later be assumed to be constant. Letters A and B denote two constants, which will be chosen later and will determine the behaviour of time-varying sliding line. Time t_0 denotes the moment, in which the desired sliding surface is reached. Similar reasoning was presented in [7].

For the other axis, the sliding surface is defined similarly

$$(8) \quad s_q(t) = \begin{cases} e_q(t) + C + Dt, & t \leq t_0 \\ e_q(t), & t > t_0 \end{cases},$$

where $e_q = i_{qref} - i_q$ and i_{qref} is the reference value of q-axis current. Parameters C and D correspond to A and B from the previous equation.

It is desired, that $s_d = 0$ and $s_q = 0$ over all of the control process.

B. Equivalent Control and Lyapunov Function Analysis

To ensure system's stability, a Lyapunov function $V_d(s) = L_\sigma \frac{s_d^2}{2}$ will be considered. For $t \leq t_0$, its derivative is equal to

$$(9) \quad \begin{aligned} \dot{V}_d(s_d) &= L_\sigma s \dot{s} = s \left(L_\sigma \frac{d}{dt} i_{dref} - L_\sigma \frac{d}{dt} i_{ds} + L_\sigma B \right) = \\ &= \left(L_\sigma \frac{d}{dt} i_{dref} + \left(R_s + \frac{L_m^2}{L_r^2} R_r \right) i_{ds} - L_\sigma \omega_e i_{qs} \right. \\ &\quad \left. - \frac{L_m R_r}{L_r^2} \psi_{dr} - \frac{L_m}{L_r} p \omega \psi_{qr} - u_d + L_\sigma B \right). \end{aligned}$$

One can easily verify, that for $t > t_0$, the component $L_\sigma B$ is eliminated. However, no other change is made.

The control law, which ensures systems stability and robustness, consists of three elements, as it was stated before. First one is the equivalent control u_{deq} and the other two, the switching element u_{dsw} and the element corresponding to the time-varying sliding line u_{dsl} , keep the representative point on the desired trajectory.

The equivalent control is defined as follows

$$(10) \quad u_{deq} = L_\sigma \frac{d}{dt} i_{dre} + \left(R_s + \frac{L_m^2}{L_r^2} R_r \right) i_{ds} - L_\sigma \omega_e i_{qs} - \frac{L_m R_r}{L_r^2} \psi_{dr} - \frac{L_m}{L_r} p \omega \psi_{qr}.$$

In the time period $[0, t_0]$, additional element added to the control law is equal to

$$(11) \quad u_{dst} = L_\sigma B.$$

The sign function will be used for the switching component of the control law defined as

$$(12) \quad u_{dsw} = \gamma_d \text{sgn}(s_d),$$

where γ_d is a positive value responsible for disturbance attenuation and systems robustness to parameters' changes.

This number has to be chosen carefully, since if it is too small, the desired robustness will not be achieved. Great values of γ_d will, however, increase the chattering. The latter can be avoided, for example, by using a different, continuous switching function [5] or using more advanced methods [6], but this topic will not be discussed in this paper.

In the end, the control law is defined as

$$(13) \quad u_d = u_{deq} + u_{dsw} + \begin{cases} L_\sigma B, & t \leq t_0 \\ 0, & t > t_0 \end{cases}.$$

Plugging (13) into (9), one gets

$$(14) \quad \dot{V}(s_d) = -\gamma_d s_d \text{sgn}(s_d)$$

for any time t , so the derivative of the chosen Lyapunov function is negative semi-definite and the system is stable.

Following the same scheme, the control law for the q-axis can be easily derived. From the analysis of the Lyapunov function, one gets the equivalent control described as follows

$$(15) \quad u_{qe} = L_\sigma \frac{d}{dt} i_{qre} + \left(R_s + \frac{L_m^2}{L_r^2} R_r \right) i_{qs} + L_\sigma \omega_e i_{qs} - \frac{L_m R_r}{L_r^2} \psi_{qr} + \frac{L_m}{L_r} p \omega \psi_{dr}$$

The component connected with time-varying sliding line is given by

$$(16) \quad u_{qst} = L_\sigma D$$

and the switching action is realised by the element

$$(17) \quad u_{qsw} = \gamma_q \text{sgn}(s_q).$$

Finally, the control law for the q-axis, which ensures, that the derivative of the Lyapunov function remains negative semi-definite for any time t , is given by

$$(18) \quad u_q = u_{qe} + u_{qsw} + \begin{cases} L_\sigma D, & t \leq t_0 \\ 0, & t > t_0 \end{cases}.$$

C. Time-Varying Switching Line

The general form of the control law have already been derived. However, the parameters for the time-varying sliding line must be determined. Once again, the parameters for d- and q-axis are found using the same methods. Thus, only d-axis will be deeply investigated.

The use of the time-varying sliding line is supposed to ensure, that the plant's representative point always remains on the sliding surface. Hence, for $t = 0$,

$$(19) \quad s_d(0) = 0 = e_d(0) + A + B \cdot 0$$

which leads to

$$(20) \quad A = -e_d(0).$$

The relationship between two other parameters can be determined from equation (7) for $t = t_0$, since the equation

$$(21) \quad \lim_{t \rightarrow t_0^-} s_d(t) = \lim_{t \rightarrow t_0^+} s_d(t)$$

must be true, we know, that

$$(22) \quad e_d(t_0) = e_d(0) + A + B t_0,$$

which finally leads to

$$(23) \quad t_0 = -\frac{A}{B}.$$

To find the appropriate value of parameter B , we consider the absolute value of control voltage u_d . Since in every real device there is a voltage limit, the absolute value $|u_d|$ must always be less than u_{dmax} . It follows from (23), that such choice of the parameter B will lead to the shortest possible value of t_0 . We write

$$(24) \quad \left| L_\sigma \frac{d}{dt} i_{dre} + \left(R_s + \frac{L_m^2}{L_r^2} R_r \right) i_{ds} - L_\sigma \omega_e i_{qs} - \frac{L_m R_r}{L_r^2} \psi_{dr} - \frac{L_m}{L_r} p \omega \psi_{qr} + L_\sigma B + \gamma_d \text{sgn}(e_d) \right| \leq u_{dmax}.$$

Since we assume $i_{dre} = \text{const}$, this relation can be simplified

$$(25) \quad \left| \left(R_s + \frac{L_m^2}{L_r^2} R_r \right) i_{ds} - L_\sigma \omega_e i_{qs} - \frac{L_m R_r}{L_r^2} \psi_{dr} - \frac{L_m}{L_r} p \omega \psi_{qr} \right| \leq u_{dmax} - L_\sigma B - \gamma_d.$$

If all of the machine parameters are known, the value of B can be determined by solving this inequality. Extremum values should be considered for the signals ω , ω_e , i_d , i_q , etc.

One should remember, that the possible extremum values of these signals will be different depending on the initial state of the induction machine. This happens due to significant difference in time constants of the mechanical and the electrical equations of the induction motor model. For example, ω can be considered a constant value, since the motion of the sliding line will be occurring only for a short period of time in the beginning of the regulation process. However, this value will be different when the motor has been free-running and when it has not been rotating at the moment the control is enabled.

Another way to determine the time-varying sliding line parameters is to choose time t_0 arbitrarily, calculate B from (23) and verify, whether the inequality (25) is fulfilled.

For the other axis, steps from (19) to (25) must be repeated. Hence, we get two relationships

$$(26) \quad C = -e_q(0)$$

and

$$(27) \quad t_0 = -\frac{C}{D}.$$

If the time t_0 or parameter B was chosen as described previously, the value D can be simply calculated. However, to ensure, that the control signal will be limited for both axes, one should also consider the equation

$$(28) \quad \left[\left(R_s + \frac{L_m^2}{L_r^2} R_r \right) i_{qs} + L_\sigma \omega_e i_{ds} - \frac{L_m R_r}{L_r^2} \psi_{qr} + \frac{L_m}{L_r} p \omega \psi_{dr} \right] \leq u_{q_{max}} - L_\sigma D - \gamma_q.$$

When two equations are considered, the biggest value of t_0 must be chosen.

Since the limitation of voltage in an inverter regards usually the length of voltage vector u_{dq} not the coordinates separately, both $u_{d_{max}}$ and $u_{q_{max}}$ must be predefined to meet required conditions.

D. Plant's trajectory

There is one, very interesting conclusion, which follows from the presented considerations. Namely, knowing, that the average values of sign functions, given by $\frac{1}{t} \int_0^t \text{sgn}(s(\tau)) d\tau$, will only compensate for the inaccurate values of the system's parameters, when the control signals u_d and u_q , defined for $t \leq t_0$, are plugged into equations (2) and (3), we get

$$(29) \quad \frac{d}{dt} i_{ds} = B,$$

$$(30) \quad \frac{d}{dt} i_{qs} = D.$$

Integrating both sides of these equations we obtain

$$(31) \quad i_{ds} = Bt + i_{ds}(0),$$

$$(32) \quad i_{qs} = Dt + i_{qs}(0).$$

This results means, that when the time-variant sliding line is changing its position, the currents of both axes increase linearly. This also means, that for the control laws presented in this paper, the plant's trajectory in $\{e_d, e_q\}$ coordinate system will be following a straight line passing through point $(0, 0)$.

Simulation Example

The presented results have been tested using simulation created in PSIM. For this purpose, a three-phase squirrel-cage induction motor model was used. The parameters have been obtained through the identification of a real induction machine and they are given in the table below.

Parameters			
Symbol	Value	Symbol	Value
R_r	2.73Ω	U_n	230/400V Δ/Y
R_s	2.84Ω	I_n	8.4/4.85A Δ/Y
L_s	285.8mH	n_n	1420rpm
L_r	285.8mH	P_n	2.2kW
L_m	275.0mH	$\cos(\varphi_n)$	0.82
p	2	J	4840g · cm ²

To verify the control strategy presented in this paper, following steps have been taken. Firstly, the induction motor was started. At the start moment, values of all the signals were equal to zero. After exactly 0.95s from the beginning of the simulation, all the transistors have been disabled and the motor was left free-running. The experiment was conducted twice to show, that the systems operation is identical regardless the remaining value of magnetic flux. The control system was enabled again after 650ms during the first test and after 50ms during the second test.

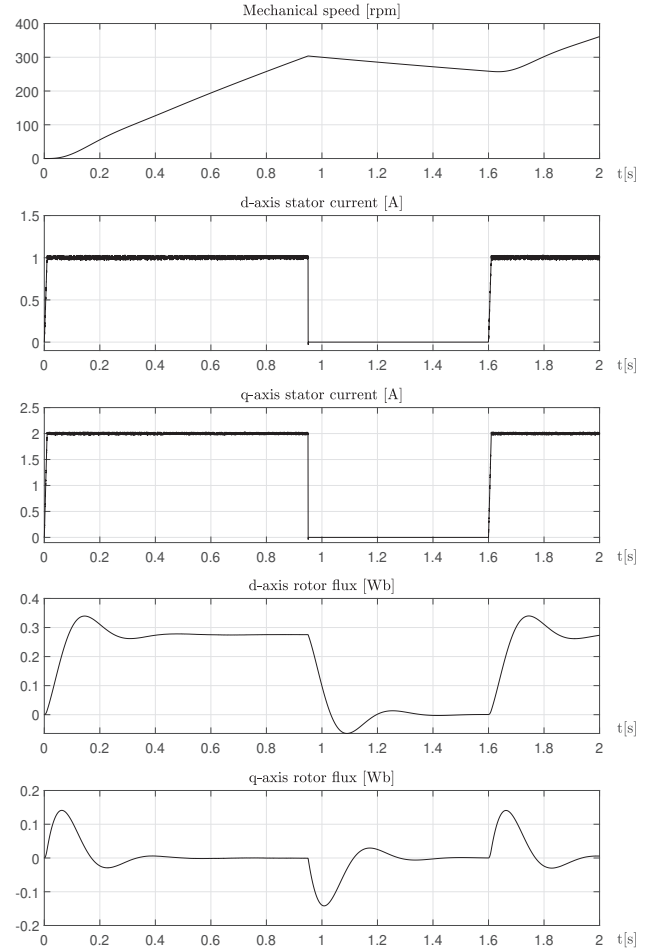


Fig. 2. Measured signals over time. Voltage was lost in $t = 0.95$ and it was restored in $t = 1.6$ s.

The measured and calculated signals are shown in figures 2 and 3 and the current plot was enlarged in figure 5. It can be seen from these graphs, that the regulation process is fast, effective and there is no overshoot. Moreover, figure 3 shows, that the system is robust to R_r parameter change and the inaccuracy in flux estimation is compensated by the switching component of the control law.

Since the rotor flux is not available for measurement it was calculated using equations (4) and (5). The exact values of the motor's parameters are not always obtainable. It is particularly hard to overcome the problem of possible parameters' changes, which occur due to temperature. There are many accurate solutions to the problem of rotor flux estimation, which are crucial especially, when sensorless control methods are used [11]. However, the proposed control strategy is robust to this parameter change, what was confirmed by decreasing the value of R_r by 15% and R_s by 5% in all the model equations used to calculate the value of equivalent control voltage.

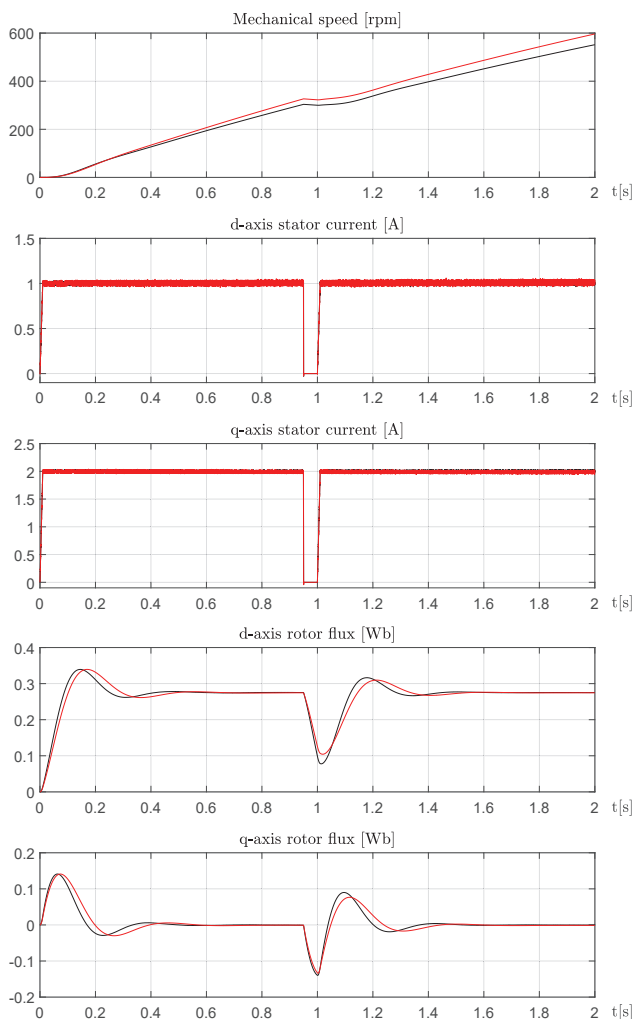


Fig. 3. Measured signals over time; plots marked with black colour were obtained for accurate value of R_r and R_s parameters and the red ones – for inaccurate. Voltage was lost in $t = 0.95$ and it was restored in $t = 1.0$ s.

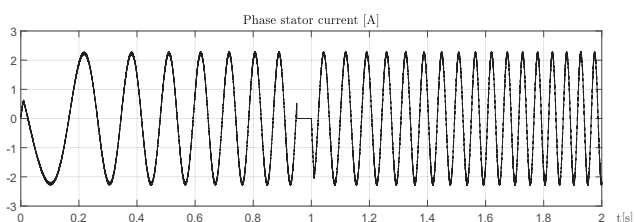


Fig. 4. Phase current of phase A.

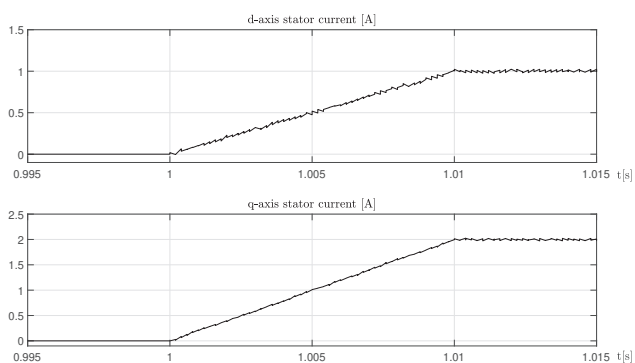


Fig. 5. Enlarged fragment of currents plot showing the moment of induction motor restart.

One may also see from the presented plots, that the control process was successfully restored regardless the value of

the remaining magnetic flux.

The time t_0 was arbitrarily chosen to be equal to 10ms. This value leaves a great margin of voltage and it can be verified with (25) and (28), that even for the worst case scenario during the motor restart, time t_0 could be decreased to the value lower than 1ms.

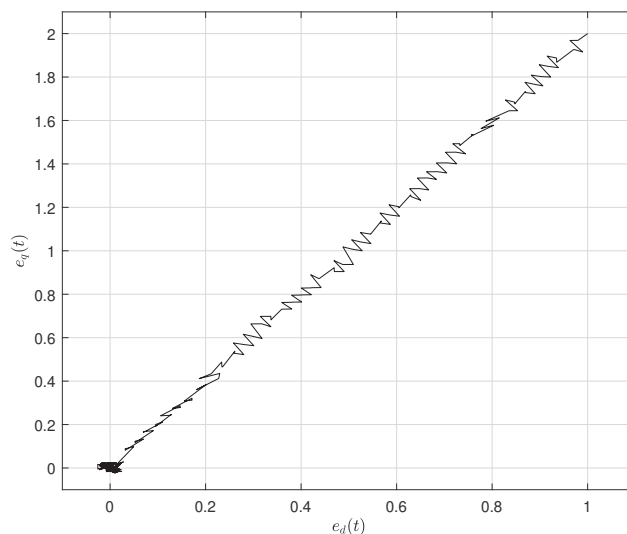


Fig. 6. Plant's trajectory in $\{e_d, e_q\}$ coordinate system.

Final Remarks

Presented simulation experiments show, that the considered control strategy enables precise induction motor current control. This method is excellent for controlling the processes, which require fast recovery from the fault condition or after an emergency

The research also show, that the properties of the SMC are particularly useful when the exact values of the plant's parameters are unknown.

Additional advantage of the presented control strategy is the lack of the uncontrollable transient states, which often occur due to control signal limitations or temporary lack of the switching action during the reaching phase of the sliding mode control algorithm.

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