1. Sabrina Meddah [1,2,*], 2. Abdelhakim Idir [1,4], 3. Sid Ahmed Tadjer [3], 4. Mohamed Zinelabidine Doghmane [1,5], 5. Madjid Kidouche[1]

Applied Automation Laboratory, F.H.C., University of Boumerdes, 35000 Boumerdes, Algeria (1),
Institute of Electrical and Electronic Engineering (IGEE), University of Boumerdes, 35000 Boumerdes, Algeria (2),
Laboratory for the electrification of industrial enterprises, University of Boumerdes, 35000 Boumerdes, Algeria (3),
Electrical Engineering Department, *Mohamed Boudiaf University of M'sila - Algeria* (4),
Reservoirs Evaluation Department, OPE, Exploration Division, SONATRACH, Hassi Messaoud 30500, Algeria (5)
ORCID: 1. 0000-0002-3510-5932; 2. 0009-0004-3896-5702, 3. 0000-0002-9715-8992

doi:10.15199/48.2024.03.35

PID Control Design of Strongly Coupled Axial-Torsional Vibrations in Rotary Drilling Systems

***Abstract**. Drilling operations can encounter considerable challenges posed by strong, coupled vibrations that exert a complex influence on rotary drilling system performance. These vibrations are classified into three distinct types based on their propagation direction: axial, lateral, and torsional. Previous research efforts have predominantly focused on examining each vibration type in isolation. However, the effectiveness and resilience of developed controllers are profoundly affected by the often overlooked coupling effects arising from other types of vibrations. In this study, we propose the implementation of a Proportional-Integral-Derivative (PID) controller for the coupled Axial-Torsional vibration system. The research presented herein is dedicated to investigate the performance of the controller under strongly coupled vibrations. To address the dynamic vibrations encountered during drilling, it is imperative to understand the intricate behavior of the drill bit in response to these vibrations before designing controllers to mitigate their impact. Numerous models have been proposed in the existing literature to elucidate the behavior of the drill string under axial-torsional vibrations. The objective of this research is to develop a comprehensive model of the drilling system and investigate the robustness of the PID controller to mitigate the adverse effects of coupled Axial-Torsional vibrations. By effectively analysing the obtained results, this study has contributed to the optimization and improvement of drilling operations under sever coupled vibrations.*

***Streszczenie.** Operacje wiercenia mogą napotkać znaczne wyzwania stawiane przez silne, sprzężone wibracje, które wywierają złożony wpływ na wydajność obrotowego systemu wiercącego. Te wibracje są klasyfikowane na trzy różne typy na podstawie kierunku propagacji: osiowe, boczne i skrętne. Dotychczasowe wysiłki badawcze skupiały się głównie na badaniu każdego rodzaju wibracji oddzielnie. Jednak skuteczność i odporność opracowanych regulatorów są głęboko dotknięte często pomijanymi efektami sprzężenia wynikającymi z innych rodzajów wibracji. W niniejszym badaniu proponujemy zastosowanie kontrolera Proporcjonalno-Całkowo-Różniczkowego (PID) dla sprzężonego systemu wibracji osiowo-skrętnych. Przedstawione w tym badaniu badania są poświęcone zbadaniu wydajności kontrolera w warunkach silnie sprzężonych wibracji. Aby skutecznie radzić sobie z dynamicznymi wibracjami napotykanymi podczas wiercenia, niezwykle istotne jest zrozumienie złożonego zachowania wiertła w odpowiedzi na te wibracje przed zaprojektowaniem kontrolerów w celu złagodzenia ich wpływu. W istniejącej literaturze zaproponowano wiele modeli w celu wyjaśnienia zachowania struny wiertniczej pod wpływem wibracji osiowo-skrętnych. Celem tego badania jest opracowanie wszechstronnego modelu systemu wiercenia i zbadanie odporności kontrolera PID na mitigację niekorzystnych skutków sprzężonych wibracji osiowo-skrętnych. Poprzez skuteczną analizę uzyskanych wyników, to badanie przyczyniło się do optymalizacji i poprawy operacji wiercenia w warunkach silnie sprzężonych wibracji. (**Projektowanie sterowania PID silnie sprzężonych drgań osiowo-skrętnych w obrotowych systemach wiertniczych**)*

Keywords: Axial-Torsional Coupled Vibrations; Decoupled Non-Linear Control; PID Controller; Rotary Drilling System.
Słowa kluczowe: Sprzężone drgania osiowo-torsyjne; Rozłączona nieliniowa regulacja; Regulatory PID; System wiercenia obrotowego.

Introduction

In the modern era, oil and gas extraction from underground reservoirs relies on drilling operations, with rotary drilling systems playing a key role. These systems uses a Top Drive mechanism to drive rotation and connect to the drill bit via a drill string [1]. The drilling setup consists of two main parts: the surface drilling rig (including the derrick and equipment for handling rods and controlling drill bit speed) and the subsurface equipment, which is the drill string assembly [2]. The assembly is divided into the upper section, composed of drill pipes, and the lower section, consisting of thicker pipes called the Bottom Hole Assembly (BHA). A thorough understanding of the design and function of these components is crucial for successful and efficient drilling operations [3].

Vibrations during petroleum well drilling have been extensively studied by researchers. Axial vibrations are observed first, propagating along the drilling axis and affecting the rate of penetration. In severe cases, like bit-bounce, intermittent contact between the drill bit and formations occurs. Lateral vibrations occur at the BHA level without axial propagation but can induce whirling and excite axial vibrations [4]. Torsional vibrations result from drill string elasticity and torque-on-bit interaction, potentially causing stick-slip phenomena. These vibrations can lead to various failures in drilling equipment, reduce the rate of penetration (ROP), and increase costs.

Previous researches have mainly addressed vibrations in rotary drilling systems and proposed controllers for their reduction [5]. Yet, the impact of axial vibrations on stick-slip severity and the coupled effect of axial-torsional vibrations on controller robustness have been overlooked. Few researchers have explored this interaction's influence on control performance [6]. A thorough analysis of vibration interactions and their effect on drilling performance is essential for effective control strategy development.

Axial, lateral, and torsional vibrations play crucial roles in drilling operations, highlighting the need for investigating their interactions to develop effective controllers for vibration reduction. Axial vibrations can result in severe issues like bit-bounce, while lateral vibrations may induce whirling and excite axial vibrations [7]. Torsional vibrations can lead to stick-slip, causing failures and decreased drilling efficiency. While prior research proposed controllers, it often neglected axial vibration's impact on stick-slip and the coupling of axial-torsional vibrations on controller robustness. Thorough analysis of these interactions is essential for devising robust control strategies [8].

While the PID controller has proven successful in various fields, its application to the specific context of axial-torsional vibrations in drilling operations has not been carefully studied [9]. This study introduces a novel PID controller designed for the coupled axial-torsional vibration system, a previously unexplored area. The potential of this empirically validated controller to address the challenges posed by axial-torsional vibrations offers a promising direction for future research and optimization strategies in drilling operations [9]. The primary focus of this research is to enhance the controller's robustness with the objective of minimizing coupled vibrations in rotary drilling systems.

Rotary Drilling Systems under Coupled Vibrations
System Description

The rotary drilling system plays a vital role in the petroleum industry by rotating the drilling tool to create boreholes from the surface to target reservoirs. The top drive connects with the bit through a series of drill pipes, forming the drill string (Fig.1). This system comprises two main components: the surface-based drilling rig, including a tower, lifting equipment, and handling floor, and the subsurface drill string, consisting of upper rods and a lower BHA containing the drilling tool, drill collar, and stabilizers. Vibrations during drilling operations, such as axial, lateral, and torsional vibrations, can lead to equipment damage and malfunctions [10, 11]. Axial vibrations affect the ROP, causing "bit-bounce", while lateral vibrations at the BHA level induce "whirling". Torsional vibrations, resulting from drill string elasticity and TOB interaction, can lead to the damaging "stick-slip" phenomenon, reducing drilling efficiency and increasing costs due to poor Borehole quality and sidetracking [12].

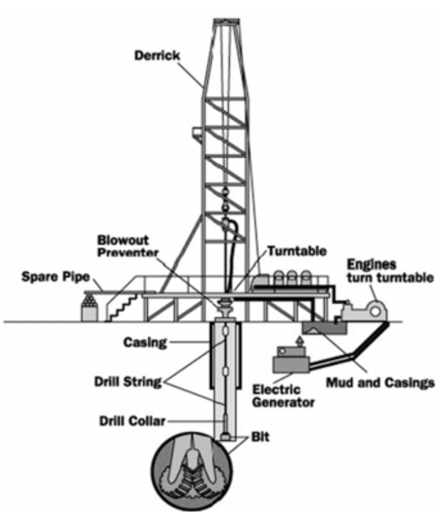


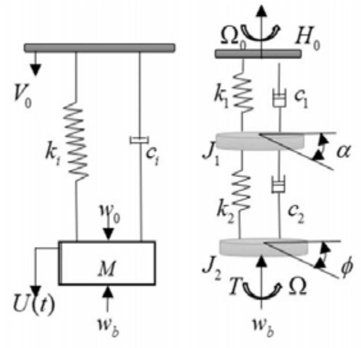
Fig.1. Rotary drilling system descriptive scheme.

Coupled Axial-Torsional Vibrations' modes

The coupling between axial and torsional vibrations, influenced by the transfer of the WOB through the drill string to the drill bit, is a common occurrence in drilling operations. This coupling can lead to the stick-slip, which in turn can result in whirling as axial vibrations propagate upwards. Moreover, the arrival of a reflected axial vibration wave to contact point within the drill string can trigger the transition from stick to slip phase, creating a repetitive cycle [13]. Recognizing the significance of these interactions, this study focuses on the coupled axial-torsional vibration mode due to its frequent occurrence and significant impact on the ROP and equipment integrity.

3 DOF Lamped model under Axial-Torsional Vibrations

In this section, a 3-degree-of-freedom (3-DOF) mass-damping model has been developed to study coupled axial-torsional vibrations. The model, depicted in Figure 2, encompasses both axial and torsional dynamics, as shown in (a) and (b) respectively [14].



(a) (b)

Fig. 2. Analogic model of drill string: (a) axil mechanical model, (b) torsional mechanical model.

The coupled axial-torsional vibration system involves a drill string experiencing axial displacement (*U*), torsional angle (*ϕ*), and bending angle (*θ*) [15, 16]. Equations of motion are derived from movement equations, resulting in the dynamic description of the system presented in equation (1). The parameters are elucidated in Table (1). Additionally, the mathematical expressions for the WOB and the TOB are provided in equations (2) and (3) respectively.

(1) $$\begin{cases} J_1\ddot{\alpha} + c_1(\dot{\alpha} - \Omega_0) + k_1(\alpha - \Omega_0 t) + c_2(\dot{\alpha} - \dot{\phi}) + k_2(\alpha - \phi) = 0 \\ J_2\ddot{\phi} + c_2(\dot{\phi} - \dot{\alpha}) + k_2(\phi - \alpha) = -T_b \\ M\ddot{U} + c_i\dot{U} + k_i(U - V_0 t) = w_0 - w_b \end{cases}$$

(2) $$w_b = \begin{pmatrix} k_c\left(\frac{w_0}{k_c} + U - V_0 t - s_0 \sin(n_b\phi)\right) & if\left(\frac{w_0}{k_c} + U - V_0 t\right) \geq s_0 \sin(n_b\phi) \\ 0 \quad if\left(\frac{w_0}{k_c} + U - V_0 t\right) < s_0 \sin(n_b\phi) & \end{pmatrix}$$

(3) $$T_b = w_b\left(\frac{2}{3} r_b \mu\left(\dot{\phi}\right) + \xi\sqrt{r_b\delta_c}\right)$$

Table 1. List of variables and constant used in the system model

Symbol	Description
V_0	Constant axial velocity
Ω_0	Constant torsional velocity
J_1	The moment of inertia of the drill string
J_2	The sum of the moment of inertia of the drill string and the moment of inertia of the drill bit.
c_i	The axial damping
c_1 , c_2	The torsional damping
k_i	The axial stiffness
k_1 , k_2	The torsional stiffness
M	The mass of drill string including BHA
w_0	The nominal drilling pressure
w_b	Force applied on the drill bit
T_b	Torque on bit

The formalisation of $\mu\left(\dot{\phi}\right)$ is expressed in equation (4).

(4) $$\mu\left(\dot{\phi}\right) = \frac{2}{\pi}\tanh^{-1}\left(\varepsilon\dot{\phi}\right)\left(\frac{f_1 - f_0}{1 + \delta_c\left|\dot{\phi}\right|} + f_0\right)$$

(5) $$\delta_c = \frac{2\pi R_p}{\Omega_0}$$

(6) $$R_p = c_1 w_0 \sqrt{\Omega_0} + c_2$$

The description of the constant parameters and their values are summarized in Table 2.

Table 2. Constant parameters values used for model simulation

Parameter	Description	Value
k_c	The contact stiffness with the formation	50000 N/m
s_0	Initial cutting depth	0.001 m
r_b	Drill radius	0.108 m
ε	Parameter in Eq (4)	60
f_0	Parameter in Eq (4)	1
f_1	Parameter in Eq (4)	1.6
C_1	constant	1.35×10^{-8}
C_2	constant	1.9×10^{-4}

The variable changes have been introduced in order to convert equation (1) into standard state space representation as follows: $\alpha = x_1$, $\dot{\alpha} = x_2$, $\dot{\phi} = x_3$, $\alpha - \phi = x_4$, $U = x_5$, and $\dot{U} = x_6$. The state space representation is given by equation (7).

(7) $$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\dfrac{k_1}{J_1}x_1 - \dfrac{c_1+c_2}{J_1}x_2 + \dfrac{c_1}{J_1}x_3 - \dfrac{k_2}{J_1}x_4 + \dfrac{k_1}{J_1}\Omega_0 t + \dfrac{c_1}{J_1}\Omega_0 \\ \dot{x}_3 = \dfrac{c_2}{J_2}x_2 - \dfrac{c_2}{J_2}x_3 + \dfrac{k_2}{J_2}x_4 - \dfrac{T_b}{J_2} \\ \dot{x}_4 = x_2 - x_3 \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = -\dfrac{k_i}{M}x_5 - \dfrac{c_i}{M}x_6 - \dfrac{k_i}{M}V_0 t - \dfrac{w_0 - w_b}{M} \end{cases}$$

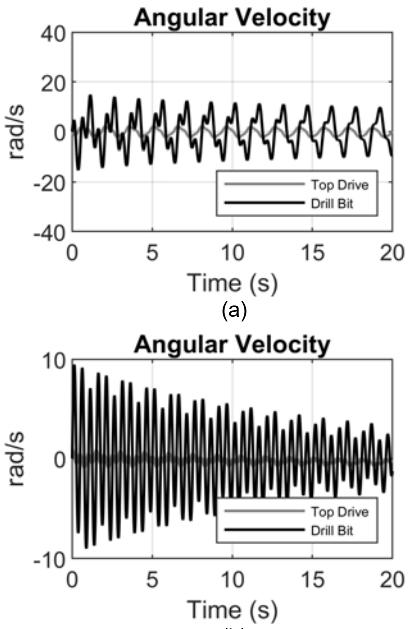
The nonlinear terms in equation (7) are: T_b that affects the angular velocity of the drill bit, $V_0 t$ and w_b that affects the axial speed of the drill bit. It should be noted from equations (2-4) that the Tob is expressed as nonlinear function of Wob, thus the dynamic of torsional vibrations depends strongly on the axial dynamic. From other side, the Wob is expressed as nonlinear function of rotation velocity of the drill bit, which means that the axial vibrations dynamic is also dependant on the torsional dynamic.

3 DOF model responses

This section focuses on analyzing the open-loop responses of the proposed model. Simulations conducted in the Matlab environment replicate real-life scenarios in petroleum drilling. The aim is to evaluate the model's reliability and emphasize the influence of coupled axial-torsional vibrations and their coupling in different scenarios.

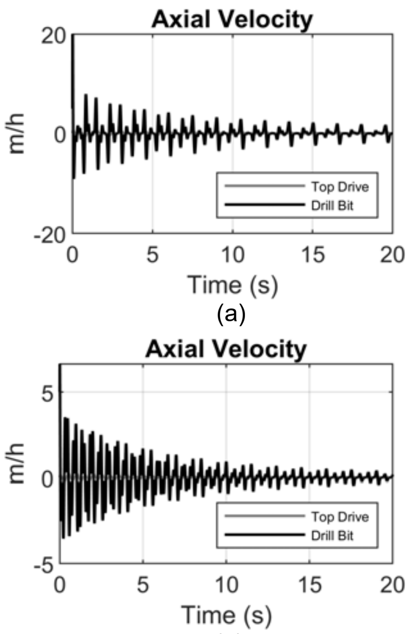
Scenario 1.1: Parametric variation of Ω_0

This scenario explores the effect of varying the constant torsional velocity on the system's oscillatory behavior. The study manipulates the constant torsional velocity within a range from 1 rad/s to 2.9 rad/s, while maintaining a fixed input speed of 0.1 m/s. The results, illustrated in Figure 3, reveal a direct link between reduced constant torsional velocity and heightened oscillatory behavior. This increase in oscillation suggests a more severe stick-slip condition, where lower torsional velocities bring the drill bit's angular velocity close to the critical threshold of complete sticking. On the contrary, significantly higher velocities surpassing the threshold minimize stick-slip severity. A similar trend is evident in the axial dynamic, as depicted in Figure 4, with higher constant torsional velocities leading to noticeable axial vibration intensification. These findings emphasize the crucial role of torsional dynamics and underscore the significance of analyzing the system's dynamic behavior.



(b)

Fig.3. Angular velocity for: (a) Ω_0 =2.9 rad/s, (b) Ω_0 =1 rad/s



(b)

Fig.4. Axial velocity for: (a) Ω_0 =2.9 rad/s (b) Ω_0 =1 rad/s

Scenario 1.2: Parametric variation of V_0

This scenario investigates the impact of varying the constant axial velocity (*V₀*) within a range of 0.05 m/s to 0.15 m/s, while keeping the constant torsional velocity fixed at 2 rad/s. Comparing Figure 5(a) and Figure 5(b), it's observed that the open-loop response of angular velocity remains unaffected by the changes in axial velocity constant. However, Figure 6 reveals a clear trend: an increase in constant axial velocity corresponds to a decrease in oscillation amplitude. Notably, this trend is linked to the occurrence and severity of the bit bounce phenomenon, primarily influenced by axial velocity constant values near the threshold. The severity is directly proportional to the difference between the constant and the threshold. These findings suggest that the influence of axial vibrations on torsional vibrations is less pronounced compared to the reverse effect. Therefore, while axial dynamics are relevant, the coupling between the two modes is predominantly one-sided.

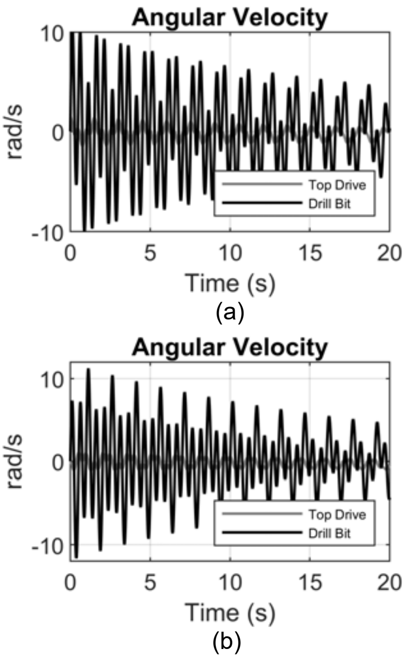


Fig.5. Angular velocity for: (a) V_0 =0.05 m/s (b) V_0 =0.15 m/s

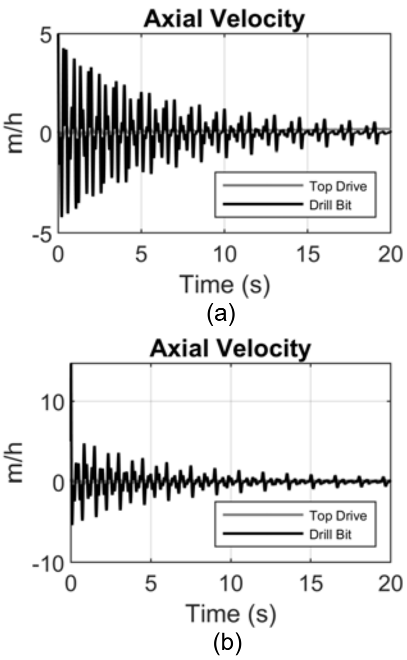


Fig.6. Axial velocity for: (a) V_0 =0.05 m/s (b) v_0 = 0.15 m/s

PID Control Design

The design of a PID controller for the studied system necessitates the judicious selection of suitable values for the proportional (P), integral (I), and derivative (D) gains, aiming to attain the desired control performance and robustness. The fundamental purpose of the PID controller entails minimizing the error between the actual output *(y(t))* of the system and the specified desired setpoint *(r(t))*. Where the outputs of the studied system is $y(t) = C \cdot X(t)$, $X(t)$ is 6×1 composed of states in equation (7). This objective is achieved through the dynamic adjustment of the control signal [17]. The instantaneous error signal *(e(t))* at any given time *(t)* is mathematically expressed as follows:

(8) $$e(t) = r(t) - y(t)$$

The proportional component of the PID controller exhibits a direct proportionality to the instantaneous error signal (e(t)), and its expression is defined as follows:

(9) $$P(t) = K_p.e(t)$$

A higher value of K_p will lead to a more significant correction in the control signal for a given error, but it may also cause the system to become more sensitive to unstructured disturbances that may occur due overloaded torque on bit. The integral component of the PID controller involves the accumulation of past error values over time, thereby contributing to the eradication of steady-state errors. Its mathematical representation is as follows:

(10) $$I(t) = K_i \int_0^t e(\tau) d\tau$$

The integral component reacts to the cumulative sum of past errors, enabling the system to address any long-term discrepancies between the setpoint and the actual output. The derivative component of the PID controller operates on the rate of change of the error signal, thereby facilitating the anticipation of future errors or even lower value of angular velocity that appear full stick-phase. Its mathematical formulation is as follows:

(11) $$D(t) = K_d \, de(t)/dt$$

The derivative component provides damping to the system, reducing overshoot and oscillations. Thence, the output control signal *u(t)* can be obtained by (12).

(12) $$u(t) = P(t) + I(t) + D(t)$$

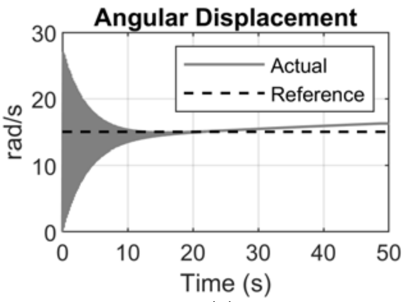
The PID controller's gains are crucial for determining the controller's performance. Selecting the optimal values typically involves a tuning process that aims to strike a balance between fast response, stability, and minimal overshoot [18]. The selection of a suitable tuning approach hinges on the system's intricacy and the specific control objectives under consideration. In this study, we have used trial and error methodology to obtain the optimal parameters of the controller [18]. Many simulation scenarios have been conducted for this purpose, in the next subsection we show the most significant results for the purposed of mitigating the coupled torsional-axial vibrations in optimal time [19].

Results and Discussion

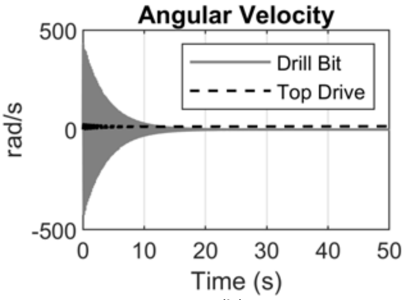
Results for Torsional Vibrations

Scenario 2.1: Constant input responses

In this scenario, the angular velocity reference is fixed at 15 rad/s. Subsequently, we derive the impulse response for both the angular displacement (Figure 7a) and the angular velocity of the drill bit (Figure 7b). The purpose of this scenario is to assess the severity of stick-slip vibrations by examining the behavior of the drill bit's angular responses. Both velocities closely track the desired reference velocity, although with slight overshoot. Notably, the system is swiftly stabilized, leading to a substantial reduction in stick-slip vibrations within a short duration (t=15s). This controller's performance is satisfactory and outperforms conventional manual techniques in petroleum drilling, resulting in a decrease in the severity of stick-slip vibration shocks [20]. This stabilization period of 15 seconds systematically minimizes oscillation amplitudes, reducing potential damage risks to both subsurface equipment and the borehole.



(a)



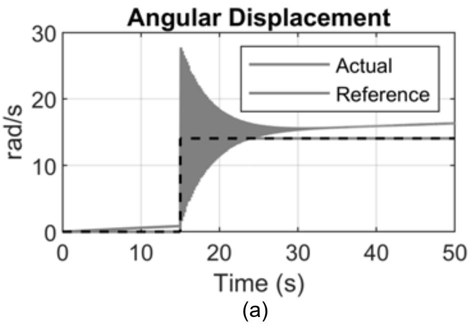
(b)

Fig.7. The constant input response of PID controller: a) control input, b) Angular velocity

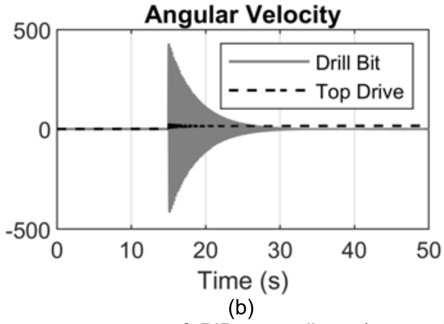
Further evaluation of the controller's effectiveness is carried out in subsequent subsections.

Scenario 2.2: Step input responses

In this scenario, the angular velocity reference experiences a step change from 0 to 15 rad/s at t=15s, simulating a manual adjustment commonly performed by experienced drillers to modify the top drive's angular velocity. This setup aims to replicate real-world drilling practices and investigate the system's response to such changes, particularly their impact on stick-slip vibrations. Figure 8(a) illustrates how abrupt shifts in input reference can potentially trigger stick-slip vibrations, providing valuable insights into the drilling system's dynamic behavior. This observation holds significance for practical drilling operations. Furthermore, Figure 8(b) displays the angular velocities of both the top drive and the drill bit.



(a)



(b)

Fig.8. The step response of PID controller: a) control input, b) Angular velocity

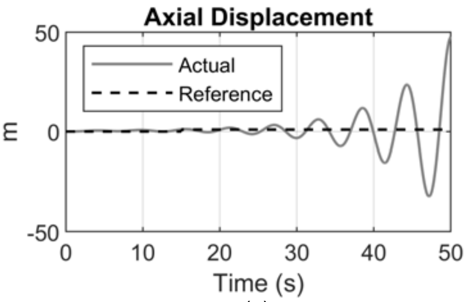
The performance achieved in this scenario is comparable to that of previous scenario. The PID controller has demonstrated high effectiveness in achieving the desired performance. Its dynamic response holds potential for reducing oscillations and diminishing the severity of torsional vibrations. However, while the controller exhibits significant improvement in mitigating vibrations, especially in the context of torsional vibrations, further investigation is needed to ascertain its capability for axial vibrations.

Results for Axial Vibrations

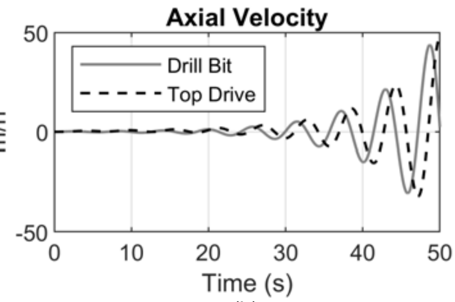
In this section, we assess the performance of the designed PID controller in mitigating both axial and torsional vibrations. The impact of torsional dynamics on axial vibrations is more significant, as seen in the open loop response scenario. However, the central focus of this section is to investigate whether eliminating stick-slip vibrations can halt axial vibrations without decoupling.

Scenario 3.1: Constant input response

Figure 9 displays the axial displacement and velocity of the closed-loop system. Precisely, Figure 9(b) presents the axial velocities of both the top drive and the drill bit when a PID controller operates under a constant input.



(a)



(b)

Fig.9. Responses of PID controller: a) control input, b) Axial velocity

Both angular velocities exhibit a rising trend over time, indicating increasing axial vibrations with drilling depth. Even though the designed controller effectively mitigates torsional vibrations, axial vibrations persist, revealing that the controller's influence on the two types of vibrations isn't simultaneous due to their strong interaction. This one-sided interaction hampers the controller's ability to concurrently manage both vibrations. Thus, the controller didn’t achieve simultaneous control due to the strong torsional-to-axial dynamic interaction. To validate this, another simulation scenario is performed in the following subsection.

Scenario 3.2: Step input response

In this scenario, a step response varying from 0 to 1 m/s at t=15s is proposed. Figure 9(b) shows the axial velocities of the drill bit (red) and the top drive (blue), revealing an unexpected rise in axial vibrations despite the controller's intervention and scenario change.

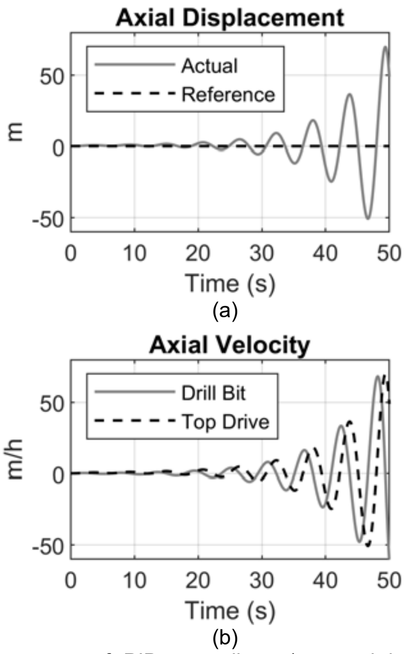


Fig.10. Responses of PID controller: a) control input, b) Axial velocity

The controller's reduction of stick-slip vibrations appears to have influenced axial vibrations' severity during the 15-second period. This demonstrates that eliminating stick-slip vibrations does not automatically lead to axial vibrations' mitigation, as seen in the oscillations during the controller's tuning. Thus, decoupling torsional and axial vibrations and designing separate controllers for each phenomenon becomes necessary. This decoupling approach could effectively address stick-slip and bit-bounce while minimizing interaction effects.

Conclusion

This study addresses the issue of coupled vibrations during drilling operations and their impact on controller performance. It focuses on coupled axial-torsional vibrations and highlights the need to consider their coupling effects in controller design. The proposed PID controller was effective in mitigating stick-slip vibrations but less for axial vibrations due to strong coupling. The controller's reliability is limited by the interaction between stick-slip and bit-bounce. Vibrations observed during stick-slip mitigation stimulate axial vibrations, making the controller unsuitable for such interactions. To address this, decoupling of axial-torsional vibrations is recommended, followed by designing individual controllers for each vibration while ensuring system stability. This research underscores the importance of managing coupled vibrations and suggests that decoupling can enhance system performance and stability.

Authors: *Ms. Sabrina Meddah, Dr. Sid Ahmed Tadjer, Dr. Mohamed Zinelabidine Doghmane, Prof. Madjid Kidouche, Boumerdes University 35000 Algeria, E-mail: s.meddah@univ-boumerdes.dz(*Corresponding author); s.tadjer@univ-boumerdes.dz;m.doghmene@univ-boumerdes.dz; mkidouche@univ-boumerdes.dz; Pr.* Abdelhakim Idir*, University of M'sila, 28000, E-mail: abdelhakim.idir@univ-msila.dz.*

REFERENCES

[1] Doghmane M. Z., Bacetti A., Kidouche M., "Stick-Slip Vibrations Control Strategy Design for Smart Rotary Drilling Systems, " ICAIRES 2020. Lecture Notes in Networks and Systems, vol 174. Springer(2021), Cham.

[2] Kessai I., Benammar S., Doghmane M. Z., "Dynamic failure analysis and lifetime estimation of Tool-string in rotary drilling system under Torsional-Axial coupled vibrations," Engineering Failure Analysis, (2022), vol. 134, no. 202,

[3] Mendil C., Kidouche Madjid., Doghmane M. Z., "Hybrid sliding PID controller for torsional vibrations mitigation in rotary drilling systems," Indonesian Journal of Electrical Engineering and Computer Science, (2021), vol. 22, no. 1, pp. 146-158,

[4] Riane R., Kidouche M., Doghmane M. Z., Illoul R., "Modeling of torsional vibrations dynamic in drill-string by using PI-observer," Lecture Notes in Electrical Engineering, vol. 682. Springer, Singapore (2021).

[5] Kuang-Chen L., Friend J. Yeo L., "The axial–torsional vibration of pretwisted beams," Journal of Sound and Vibration, (2009), vol. 321, no. 1–2, pp.115-136,

[6] Mendil C., Kidouche M., Doghmane, M. Z., "Hybrid Backstepping Sliding Mode Controller for Stick–slip Vibrations Mitigation in Rotary Drilling Systems," IETE Journal of Research, (2021), vol.69, no.6, pp.3477-3487.

[7] Mendil C., Kidouche M., Doghmane M.Z., "Modeling of Hydrocarbons Rotary Drilling Systems Under Torsional Vibrations: A Survey". ICAIRES 2020. Lecture Notes in Networks and Systems, vol 174. Springer, Cham.

[8] Idir A., Canale L., Bensafia Y., Khettab, K. "Design and Robust Performance Analysis of Low-Order Approximation of Fractional PID Controller Based on an IABC Algorithm for an Automatic Voltage Regulator System", *Energies*, (2022), vol. *15,* 8973.

[9] Idir A., Canale L., Tadjer S. A., Chekired F., "High Order Approximation of Fractional PID Controller based on Grey Wolf Optimization for DC Motor,". *2022 IEEE International Conference on Environment and Electrical Engineering and 2022 IEEE Industrial and Commercial Power Systems Europe (EEEIC / I&CPS Europe)*, Jun 2022, Prague, Czech Republic. pp.1–6.

[10] Kessai I., Doghmane M.Z., Benammar S., "Weight-On-Bit Variations Analyses in the Drillstring of Rotary Drilling Systems Under Stick-Slip Vibrations," Lecture Notes in Networks and Systems, vol 361. Springer, Cham.

[11] Kamel J. M., Yigit A., "Modeling and Analysis of Axial and Torsional Vibrations of Drillstrings with Drag Bits," International Petroleum Technology Conference, Doha, Qatar, January 2014.

[12] Riane R., Doghmane, M. Z., Kidouche M., Djezzar S., "Observer-Based H∞ Controller Design for High Frequency Stick-Slip Vibrations Mitigation in Drill-String of Rotary Drilling Systems," Vibration, (2022), vol.5, pp. 264-289.

[13] Hosseinzadeh A., and Bakhtiari-Nejad F., "A New Dynamic Model of Coupled Axial–Torsional Vibration of a Drill String for Investigation on the Length Increment Effect on Stick–Slip Instability," ASME. J. Vib. Acoust., (2017), vol. 139, no. 6, pp. 061016.

[14] Kim B. J., Palazzolo A., and Gharib M., "Drillstring Simulator: A Novel Software Model for Stick–Slip And Bit-Bounce Vibrations," In ASME International Mechanical Engineering Congress and Exposition, (2021), vol. 85611, pp. V07AT07A048, American Society of Mechanical Engineers.

[15] Sergey A. V., Alexander M. G., Alexey S. K., et al., "Influence of Torsional Motion on the Axial Vibrations of a Drilling Tool," J. Comput. Nonlinear Dynam., (2007), vol. 2, no.1, pp. 58-64.

[16] Yu T., Xinye L., Huabiao Z., "Nonlinear analysis of axial-torsional vibration of drill string based on a 3 DOF model," Advances in Mechanical Engineering, (2022), vol.14, no.6.

[17] Mohamed Helaimi M., Taleb R., Gabbar H. A., Othman M., " Frequency control of microgrid system based renewable generation using fractional PID controller," Indonesian Journal of Electrical Engineering and Computer Science (IJEECS), (2019), vol. 19, no. 2, pp. 745-755.

[18] Idir, A., Bensafia, Y., Khettab, K., and Canale, L., Performance improvement of aircraft pitch angle control using a new reduced order fractionalized PID controller, *Asian J Control* 25 (2023), 2588–2603.

[19] Doghmane M. Z., "Optimal Decentralized Control Design with Overlapping Structure," Magister Thesis, University M'hamed Bougara of Boumerdes, Algeria, 2011.

[20] Doghmane M. Z., Kidouche M., and Ahriche A., "Decentralized Overlapping Control Design with Application to Rotary Drilling System," IETE Journal of Research, (2023), vol.69, no.4, pp.2070-2079.