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Do fractional derivatives make physical sense?

Abstract. *This is my next (tenth) original or co-authored article in which I criticize the incorrect use of fractional integrals and derivatives in physics and especially in electrical engineering.*

Streszczenie. *Jest to kolejny (dziesiąty) mój autorski lub współautorski artykuł w którym krytykuję błędne wykorzystanie ułamkowych całek i pochodnych w fizyce a w szczególności w elektrotechnice. (**Czy pochodne ułamkowe mają sens fizyczny?**)*

Keywords: Fractional integrals and derivatives, Maxwell's laws, physical meaning.
Słowa kluczowe: Całki i pochodne ułamkowe, prawa Maxwella, znaczenie fizyczne

Introduction

In the Financial Times, a newspaper published in Great Britain, in the issue of MONDAY 19 JUNE 2017 on page 3, there was an article "China targets academic fraud after journal retractions" describing errors in the evaluation of scientific articles. There are more errors of this type. I have described them in many publications. The list of errors was presented during ISTET'21 in the publication [1] "Can science exist without free discussion?" ISTET'21 took place online in Szczecin in 2022. The article received two positive reviews and was published in 2023 in the well-known scientific journal COMPEL. One of the reviewers of this article highly appreciated my fight for scientific veracity. More than a year has passed since the publication of this article, but I have no indication that anyone has read it. I have not noticed any reaction from the Archive of Electrical Engineering, a body of the Polish Academy of Sciences, nor the Bulletin of the Polish Academy of Sciences or other bodies of the Polish Academy of Sciences. I also did not notice any reaction of POZNAN UNIVERSITY OF TECHNOLOGY ACADEMIC JOURNALS or from the Author of the article "Analysis of linear continuous-time systems by the use of the conformable fractional calculus and Caputo" in AEE VOL. 67(3), pp. 629–639 (2018). On pages 638-639 you can find my critical review of this work not disclosed in the table of contents of this issue of AEE. In 2017, the online publisher 978-1-5386-1528-7/17/$31.00 ©2017 IEE published the article "Fractional- order modeling of electric circuits: modern empiricism vs. Classical science". In my opinion, it is substantively incorrect but also constitutes a serious error in the cited literature. Four of the five authors of this article are professors at the Opole University of Technology. One of them is the chairman of the Committee on Electrical Engineering of the Polish Academy of Sciences. This five-person team of authors did not notice my reaction to the article by prof. P. Ostalczyk in Przegląd Elektrotechniczny 4/2017. At least a few of the five-person team attended my lecture at the Opole University of Technology and did not react to my speech. I delivered a paper at the University of Hanover at a meeting of two Institutes of this University, the Institute of Theoretical Electrical Engineering and the Institute of Analysis, the paper "Fractional Derivatives and the Laws of Electrical Engineering". No one denied my arguments. So far, I have not noticed any reaction to this speech. I gave a speech of similar content in Xian, China, in 2017. The conference organizers awarded it. This is an astonishing situation. No one denies my public speeches against the incorrect use of fractional derivatives in electrical engineering in Poland and abroad. Examples of this are my speeches in Ilmenau and Hanover and in Opole during the Committee of Electrical Engineering meeting of the Polish Academy of Sciences. My former PhD student, Professor Tomasz Chady, submitted papers on the application of fractional derivatives in NDT in 2019 in Sofia during ISTET'20 and at the QNDE. Testing conference in the USA. No one denied his theses. I recently talked to a former PhD student, now a professor, about the situation that had arisen. Unfortunately, he could not explain it. It is an astonishing situation. Physics, including electrical engineering, is quantum in nature, so I believe that fractional derivatives and integrals cannot be used in the description of electromagnetic phenomena. Fractional derivatives and integrals may be used but with the explicit understanding that this is a non-physical approximation since it is not possible to transform fractional integrals and derivatives into integer integrals and derivatives.

Mathematical methods used in electrical engineering

Electrical engineering is a relatively young science. It can be assumed that J. C. Maxwell formulated its foundations. J. C. Maxwell did not used fractional derivatives and fractional integrals. Prof. St. Fryze (1885-1964), one of founder polish electrical engineering, professor of the Lwów and Silesian Polytechnics also did not used fractional derivatives and fractional integrals. . As a student, I met Prof. St. Fryze in 1954, ten years before his death. Maxwell's equations are usually written in electromagnetism books like this:

(1) $$curl\boldsymbol{H} = \boldsymbol{J} + \varepsilon\frac{\partial \boldsymbol{E}}{\partial t}$$

(2) $$div\ \boldsymbol{B} = 0$$

(3) $$curl\ \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

(4) $$div\ \boldsymbol{D} = \rho$$

Translating them into a fractional form:

(5) $$curl_l^{\propto}\boldsymbol{H} = \boldsymbol{J} + \varepsilon\frac{\partial^{\propto}\boldsymbol{E}}{\partial t^{\propto}}$$

(6) $$div_r^{\propto}\ \boldsymbol{B} = 0$$

(7) $$curl_r^{\propto}\boldsymbol{E} = -\frac{d^{\gamma}\boldsymbol{B}}{dt^{\gamma}}$$

(8) $$curl_r^{\propto}\boldsymbol{D} = \rho$$

or in the form:

(9) $$curl_l^{\propto}\boldsymbol{B} = \mu\boldsymbol{J} + \mu\varepsilon\frac{\partial^{\propto}\boldsymbol{E}}{\partial t^{\propto}}$$

(10) $$div_r^{\propto}\ \boldsymbol{B} = 0$$

(11) $$curl_r^{\propto}\boldsymbol{E} = -\mu\frac{\partial^{\gamma}\boldsymbol{H}}{\partial t^{\gamma}}$$

(12) $$div_l^{\propto}\boldsymbol{D} = \rho$$

I believe, has no physical meaning. There are two reasons for my opinion. The first reason is the dimensional homogeneity of equations (5-12) and the second is the quantum nature of matter and energy. Let's look at the first

reason. First, let`s analyze the dimensions of the individual components of equation (5). Obviously, the dimension of the left-hand side of this equation $\boldsymbol{div}_l^{\alpha}\boldsymbol{H}$ changes with the change of the differentiation index α and the dimension of the component *J* is independent on this index but dimension of the component $\boldsymbol{\mu\varepsilon}\frac{\partial^{\alpha}\boldsymbol{E}}{\partial t^{\alpha}}$ is dependent. Such an equation does not satisfy the basic condition for writing equations describing physical phenomena. It cannot be that one side (in this case the left) has a variable dimension depending, for example, on the differentiation index *α* and the other side (in this case the right) has a constant plus dimension depending on the index *α*, because the dimension of the component ***J*** is constant, independent of the differentiation index *α*. Equation (5) is dimensionally homogeneous only when *α*=1. For other values of *α*≠1, equation (5) loses its validity. The second reason is the quantum nature of matter and energy. Energy, according to quantum theory, changes discretely by a quantum or its multiple. In the case of equation (6), the first reason disappears because the right side of this equation is equal to zero. The second reason remains valid. In the case of equation (7), the matter becomes even more complicated. The differentiation indices of both sides of the equation *α* and *γ* can have different values, which makes it even more difficult to maintain the dimensional homogeneity of this equation. As far as the quantum aspect is concerned, the reservation remains valid. Equation (8) is also complicated in terms of physical dimensionality. The dimension of the left side of this equation $\boldsymbol{div}_l^{\alpha}\boldsymbol{D}$ depends on the differentiation index *α*, and the right side is equal to the constant value. All equations from (5) to (8) do not satisfy the condition of the correctness of the notation of physical equations. Physical equations are written (when necessary) utilizing derivatives and integrals of integer order and are not expressed by fractional derivatives and integrals of non-integer order. Similar reservations apply to the notation of Maxwell's laws expressed by equations (9) - (12). Maxwell's equations, as well as other equations of physics, can be written in various equivalent forms. However, they should always be written in different equivalent forms provided that derivatives and integrals appearing in these laws are of integer order.

Summary

In the introduction I have described the lack of discussion on an important issue in theoretical electrical engineering. A little over a year has passed since the publication of this article. To this day I have not noticed any reaction to this article. I will see what happens with the current article. I believe that there is no easier and cheaper method of avoiding serious and costly errors in Science than free and open discussion. We can all make mistakes in our scientific work. There is nothing wrong with criticism of erroneous theories. This is not criticism of the authors but only criticism of erroneous theories as such. There is nothing more practical than a good theory. We are in the early stages of development of artificial intelligence. Artificial intelligence can contribute to the turbulent development of Science. Artificial intelligence used incorrectly in Science can cause serious damage. For this reason, free and open discussion is particularly important for further development of Science. I would like to draw attention to how the lack of free discussion on the development of nuclear energy caused serious material losses. In the eighties of the last century, preparatory work was carried out in Poland in the field of nuclear energy. The first Polish nuclear power plant was to be built in Żarnowiec. I participated in this project led by Prof. Z. Pawłowski regarding non-destructive testing at this power plant. Had a suitable nuclear power plant been built in Poland at the right time, Poland would have had a profit exceeding the country's annual budget. There are more such cases. I have serious doubts about the so-called magnetotherapy. Criticism of errors arising in the process of scientific research and formulation of new theories is not directed against the creators but only against the errors that arise. I am aware of the low effectiveness of scientific criticism. However, it is worth conducting it purely for scientific and economic reasons.

Reading the monograph of the Nobel Prize winner Prof. R. Penrose "The road to reality" [2] (Polish edition from 2010) I did not notice any fractional derivatives anywhere. Isn't that intriguing?

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REFERENCES

[1] Sikora, Ryszard “Can science exist without free discussion?” COMPEL - Vol. 42 No. 3, 2023 pp. 687-694

[2] Roger Penrose “The road to reality” Prószyński Media Sp. Z o.o. Warszawa 2010.