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Concerning the Problem of the Calculation of Magnetic Field Force Action on an Energized Ferromagnetic Conductor

Abstract. *The calculation of the force effect of a uniform magnetic field on an energized magnetic conductor is performed. It is shown that the calculation of the force of this field on the magnetic substance of the conductor for the known models of magnetization (the model of equivalent magnetizing currents and the model of equivalent magnetic charges) gives an ambiguous result. Thus, this ambiguity leaves A. Einstein's question on the determination of the force on the current in a magnetic substance topical.*

Streszczenie. *Wykonano obliczenia oddziaływania jednorodnego pola magnetycznego na przewodnik magnetyczny pod napięciem. Pokazano, że obliczenie siły tego oddziaływania na substancję magnetyczną przewodnika dla znanych modeli namagnesowania (model zastępczych prądów magnesujących i model zastępczych ładunków magnetycznych) daje niejednoznaczny wynik. Zatem pytanie A. Einsteina dotyczące wyznaczania siły działającej na prąd w substancji magnetycznej pozostaje wciąż aktualne.* (***Dyskusja problemu obliczania oddziaływania pola magnetycznego na orzewód ferromagntyczny)***

Key words: magnetic field, force action, magnetization model, equivalent magnetizing current, equivalent magnetic charge.
Słowa kluczowe:. pole magnetyczne, oddziaływanie, model magnesowania, równoważny prąd magnesowania, równoważny ładunek magnetyczny

Introduction

At present, several different approaches are known for calculating the constant magnetic field force effect on bodies whose matter has magnetic properties [1−6]. In particular, an approach is applied, which is associated with the integration over the volume of the body of the force density (specific force) determined from one of the models of the magnetized state of matter: a model of equivalent molecular (magnetizing) currents distributed with volume density $\bar{j}_m = rot\overline{M}$, and a model of equivalent magnetic charges distributed with volume density $\rho_m = -\mu_0 div\overline{M}$ (here $\overline{M}$ – the vector of the magnetization of the substance, μ_0 is the magnetic constant) [5, 7, 8].

It is known that the two mentioned methods of modeling the magnetized state of a substance provide the same result (integral equivalence) when determining the total force on the side of the magnetic field on a body in the volume of which there are no electric conduction currents (macroscopic currents) [9, 10].

The purpose of the paper

The purpose of the paper is, using a simple example, to show that the integral equivalence for the indicated methods of modeling the magnetized state of a substance is not preserved if conduction electric currents flow in the volume of the body on which the force action of a constant magnetic field is determined.

Problem statement

Consider a straight energized conductor made of a material with magnetic properties, which is placed in a uniform magnetic field with induction $\overline{B}_0$, directed orthogonal to the axis of the conductor (Fig. 1).

In the general case of an arbitrary external magnetic field, obviously, the total force on a magnetic conductor with a current in an external magnetic field (force $\overline{F}_\Sigma$) can be represented as the sum of the following forces: $\overline{F}_1$ – force on the electric current in the conductor and $\overline{F}_2$ – force on the magnetized substance of the conductor.

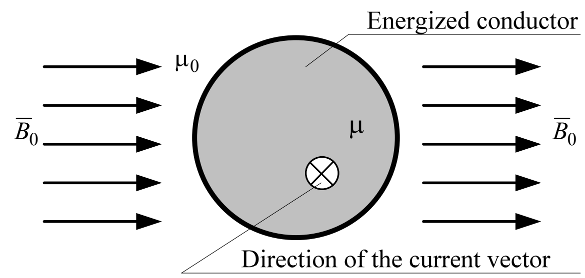

Fig. 1. An energized conductor made of a material with magnetic properties in a uniform magnetic field

In their turn, forces $\overline{F}_1$ and $\overline{F}_2$ can also be represented as definite sums of forces according to interactions between material objects of the system. Namely:

(1) $$\overline{F}_1 = \overline{F}_{10} + \overline{F}_{1M} + \overline{F}_{1i} ,$$

where $\overline{F}_{10}$ – force on the current from the external magnetic field; $\overline{F}_{1M}$ – force on the current, on the side of the magnetic field caused by the magnetization of the substance of the conductor; $\overline{F}_{1i}$ – force on the current on the side of the magnetic field created by the current itself;

(2) $$\overline{F}_2 = \overline{F}_{20} + \overline{F}_{2M} + \overline{F}_{2i} ,$$

where $\overline{F}_{20}$ – force on the magnetic substance of the conductor on the side of an external magnetic field; $\overline{F}_{2M}$ – force on the magnetic substance of the conductor on the side of the magnetic field created by this magnetized substance; $\overline{F}_{2i}$ – force on the magnetic substance of the conductor on the side of the magnetic field of the current.

Note that, according to the law on equality of action and reaction for components $\overline{F}_{1M}$ and $\overline{F}_{2i}$ in (1) and (2) equality $\overline{F}_{1M} = -\overline{F}_{2i}$ is true; and components $\overline{F}_{1i}$ and $\overline{F}_{2M}$ in (1) and (2) are numerically zero, since each of them represents the total force in a closed system.

Based on this, instead of (1) and (2), we can write sums

(3) $$\overline{F}_1 = \overline{F}_{10} + \overline{F}_{1M}$$

and

(4) $$\overline{F}_2 = \overline{F}_{20} + \overline{F}_{2i} ,$$

and for the total force $\overline{F}_{\Sigma}$, the following ratio can be written

(5) $$\overline{F}_{\Sigma} = \overline{F}_{10} + \overline{F}_{20} .$$

Thus, according to (5), in an arbitrary external magnetic field, the total force on an energized magnetic conductor is the sum of the force of the external magnetic field on the current in the conductor and the force on its magnetic substance.

In the case of a uniform external magnetic field, we will take into account that this field, as is known, does not create a force on any magnetic body in it. That is, we will take into account that in this case $\overline{F}_{20} = 0$, and the total force on the energized magnetic conductor is determined only by the action of an external field on the current in the conductor: $\overline{F}_{\Sigma} = \overline{F}_{10}$. As for the force on the magnetized substance of the conductor, then, according to (4), it is determined only by component $\overline{F}_{2i}$ (in this case $\overline{F}_{20} = 0$).

The calculation of this components of the force for two models of the magnetized state of matter (the model of equivalent magnetizing currents and the model of equivalent magnetic charges) in order to compare the obtained results is the task that is solved in this paper.

About the components of the force on the magnetic substance of the conductor on the side of the magnetic field of the current

In order to determine the considered force $\overline{F}_{2i}$, we expand it as follows

(6) $$\overline{F}_{2i} = \overline{F}_{2i0} + \overline{F}_{2ii} ,$$

where $\overline{F}_{2i0}$ and $\overline{F}_{2ii}$ – components of the force of the magnetic field of the current on the magnetized substance, the magnetization of which is caused, respectively, by the external magnetic field and the magnetic field of the current.

Due to the symmetry of the magnetization of the substance of the conductor with a current in the magnetic field of the current, it can be stated that for points of the conductor that are symmetrical about its axis, the components of force $\overline{F}_{2ii}$ will mutually balance so that the total force along these components in the entire conductor will be zero

(7) $$\overline{F}_{2ii} = 0 .$$

Then for the total force only on the magnetic substance of the conductor, taking into account (7), from (6) we obtain equality

(8) $$\overline{F}_{2} = \overline{F}_{2i0} .$$

With this approach, it becomes possible to strictly determine the total force on the magnetic substance of the conductor, based on the physical meaning of component $\overline{F}_{2ii}$, according to which it is necessary to preliminary find the magnetization of the conductor substance in a uniform magnetic field.

For this, we will take into account that in a uniform magnetic field, which is orthogonal to the axis of the conductor, it also creates a uniform field inside the cylindrical conductor with induction

(9) $$\overline{B} = \frac{2\mu}{\mu_0 + \mu} \overline{B}_0 ,$$

where μ – magnetic permeability of the conductor substance.

This allows writing the following relation for the magnetic conductor magnetization vector $\overline{M}_0$

(10) $$\overline{M}_0 = \frac{\mu - \mu_0}{\mu_0 \mu} \overline{B} ,$$

That is, according to (9) and (10), the following expression can be written for the magnetization of the conductor

(11) $$\overline{M}_0 = 2 \frac{\mu - \mu_0}{\mu_0 (\mu + \mu_0)} \overline{B}_0 .$$

Calculation of the force on the magnetized substance of the conductor according to the model of equivalent magnetic charges

According to the model of equivalent magnetic charges, the force effect of the magnetic field of the current in the conductor (denote its intensity as $\overline{H}_i$) on the substance, with magnetization $\overline{M}_0$ can be defined as the integral over the volume V of the conductor (as the volume of integration, we take the volume of a unit length of the conductor, which gives, with the indicated integration, the specific value of the force) of the following form

$$\overline{f}_{2i0} = -\mu_0 \int_V \overline{H}_i div \overline{M}_0 dv ,$$

which in the case under consideration (taking into account the plane-parallelism of the problem) can be written as an integral over the section of the cylinder, whence we find

(12) $$\overline{f}_{2i0} = -\mu_0 \int_{S_0} \overline{H}_i \, div \overline{M}_0 \, ds ,$$

where S_0 – cross-sectional area of a cylindrical conductor (perpendicular to the axis).

Since, according to (11), $\overline{M}_0 = const$ at all points of the conductor section, except for points on its surface (here the magnetization vector undergoes a discontinuity), then, according to the results of the previous section, instead of integral (12), we can write an integral of the form

(13) $$\overline{f}_{2i0} = -\mu_0 \int_{L_0} \overline{H}_i (\overline{n} \cdot \overline{M}_0) dl ,$$

where L_0 – circular contour limiting the cross-section of a cylindrical conductor (on the surface of the conductor); dl – the differential element of this circuit.

Note that when writing the integral in (13), we also take into account the continuity of vector $\overline{H}_i$ at points on the surface of the conductor (after all, vector $\overline{H}_i$ is the vector of the magnetic field of the current in the conductor, which is tangent to any circle inside the conductor and to the circle of the conductor section).

We find the initially vertical component of specific force (volume force density) $\overline{f}_{2i0}$, which we denote f_y. For which, obviously, it is necessary to determine the integral

(14) $$f_y = \mu_0 \int_{L_0} H_{iy} (\overline{n} \cdot \overline{M}) dl ,$$

where H_{iy} – vector $\overline{H}_i$ projection onto the vertical axis (Fig. 2).

For this purpose, we will take into account that (see Fig. 2)

(15) $$\overline{n} \cdot \overline{M} = M_0 \cos\alpha ,$$

(16) $$H_{iy} = H_i \cos\alpha ,$$

where (according to Ampere's circuital law)

(17) $$H_i = \frac{i}{2\pi R} .$$

Then, substituting (15) and (16) into (14), we obtain

(18) $$f_y = \mu_0 \frac{iM_0}{2\pi R}\int\limits_{L_0} \cos^2 \alpha dl \, .$$

However, since the differential element of the length of a circle is related to its radius by ratio $dl = Rd\alpha$ (here $d\alpha$ is the differential element of the angle with the vertex in the center of the circle, resting on an arc of length dl), the integration in (18) gives the following chain of equalities (taking into account the symmetry)

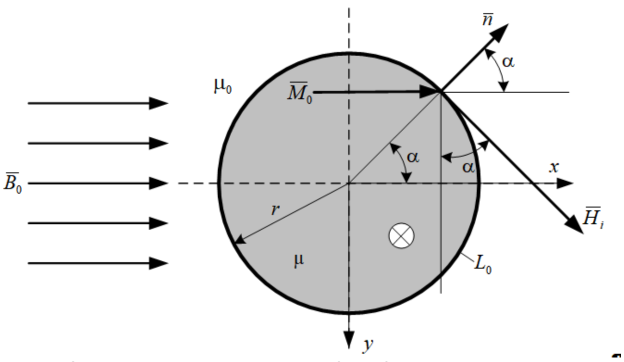

Fig. 2. Concerning the calculation of the force on a magnetic substance according to the model of equivalent magnetic charges

$$f_y = \frac{\mu_0}{\pi} iM_0 \int\limits_0^{\pi} \cos^2 \alpha d\alpha = \frac{\mu_0}{\pi} iM_0 (\frac{1}{4}\sin 2\alpha + \frac{\alpha}{2})\Big|_0^{\pi} = \frac{1}{2}\mu_0 iM_0 \, ,$$

or, taking (11) into account,

(19) $$f_y = iB_0 \frac{\mu - \mu_0}{\mu + \mu_0} \, .$$

As for the horizontal component of force $\overline{f}_{2i0}$ (denote it as f_x), for it, similarly to the previous one, we have (Fig. 2)

$$f_x = \mu_0 \int\limits_L H_{ix} (\overline{n} \cdot \overline{M}_0) dl \, ,$$

where $H_{ix} = H_i \sin \alpha$ – projection of vector $\overline{H}_i$ on axis x .

Further, taking into account equality (15), we have

$$f_x = \mu_0 H_i M_0 \int\limits_L \sin \alpha \cos \alpha dl \, ,$$

or, taking into account equality $dl = Rd\alpha$, we have

$$f_x = \mu_0 H_i M_0 R \int\limits_0^{\pi} \sin 2\alpha d\alpha = -\mu_0 H_i M_0 R \cos 2\alpha \Big|_0^{\pi} = 0 \, .$$

Thus, force $\overline{f}_{2i0}$ has only a component directed along the y-axis (perpendicular to the vectors of the current density and induction of the external field).

Calculation of the force on a magnetized substance according to the model of equivalent magnetizing currents

According to the model of molecular currents (equivalent magnetizing currents), a magnetized substance with magnetization $\overline{M}_0$ can be replaced by distribution of currents with volume density $rot\overline{M}_0$ in the space of the conductor with magnetic permeability μ_0 .

In this case, the magnetic field of the current in the conductor with induction $\mu_0 \overline{H}_i$, acts on equivalent currents in the conductor (in the volume of unit length), creating a force density (specific force)

(20) $$f_{2i0} = \mu_0 \int\limits_{S_0} rot\overline{M}_0 \times \overline{H}_i ds \, ,$$

(here the plane-parallel nature of the solved problem is taken into account).

As in the previous case, for all points of the conductor section (except for points on the surface of the conductor) vector $\overline{M}_0$ is constant. Then, instead of the integral in (20), using the concept of a surface rotor [11], we can write an integral of the form

(21) $$\overline{f}_{2i0} = -\int\limits_{L_y} \left[\overline{n} \times \overline{M}_0\right] \times \overline{H}_i dl \, .$$

Taking into account the geometry of the solved problem (Fig. 3), for the modulus of vector $\left[\overline{n} \times \overline{M}_0\right]$ equal to

$$\left|\overline{n}\right| \cdot \left|\overline{M}_0\right| \sin(\overline{n}, \overline{M}_0) \, ,$$

we have

$$\left|\overline{n}\right| = 1 \, ,$$

$$\left|\overline{M}_0\right| = M_0 \, , \qquad \sin(\overline{n}, \overline{M}_0) = \sin \alpha \, ,$$

which gives equality

$$\left|\left[\overline{n} \times \overline{M}_0\right]\right| = M_0 \sin \alpha \, .$$

In this case vector $\left[\overline{n} \times \overline{M}_0\right]$ is directed along the conductor axis (perpendicular to the plane of the figure, Fig. 3). I.e., vector $\left[\overline{n} \times \overline{M}_0\right]$ is orthogonal to vector $\overline{H}_i$. Therefore, the vector product $\left[\overline{n} \times \overline{M}_0\right] \times \overline{H}_i$ modulus is simply the product of the corresponding vectors

(22) $$\left[\overline{n} \times \overline{M}_0\right] \times \overline{H}_i = \left(\left[\overline{n} \times \overline{M}_0\right]\right) \left|\overline{H}_i\right| = H_i M_0 \sin \alpha \, .$$

Vector $\left[\overline{n} \times \overline{M}_0\right] \times \overline{H}_i$ is orthogonal both to vector $\left[\overline{n} \times \overline{M}_0\right]$ and vector $\overline{H}_i$, that is, along the radius line, as shown in Fig. 3.

In this case, in vector $\left[\overline{n} \times \overline{M}_0\right] \times \overline{H}_i$ we will be interested in the initially vertical component, which, taking into account (22), in the case under consideration, will be

(23) $$\left|\left[\overline{n} \times \overline{M}_0\right] \times \overline{H}_i\right| \sin \alpha = M_0 H_i \sin \alpha \, .$$

According to (23), the calculation of the component of force $\overline{f}_{2i0}$ along the y-axes from (21) gives the following chain of equalities

$$f_y = -\mu_0 M_0 H_i R \int\limits_{L_y} \sin^2 \alpha d\alpha = -2\mu_0 M_0 H_i R \left(\frac{\alpha}{2} - \frac{1}{4}\sin 2\alpha\right)\Bigg|_0^{\pi} =$$
$$= -\pi \mu_0 M_0 H_i R \, ,$$

which, taking into account (11) and (17), gives

(24) $$f_y = -iB_0 \frac{\mu - \mu_0}{\mu + \mu_0} \, .$$

As for the horizontal component of force $\overline{f}_{2i0}$ according to the model of equivalent magnetizing currents, for it, taking into account (22) and the above stated about the direction of vector $\left[\overline{n} \times \overline{M}_0\right] \times \overline{H}_i$, it is possible to write down integral

$$f_x = -\mu_0 H_i M_0 R \int\limits_{L_0} \sin \alpha \cos \alpha d\alpha \, ,$$

which is similar to the integral for f_x according to the model of equivalent magnetic charges. This gives for f_x in this case the identical zero too.

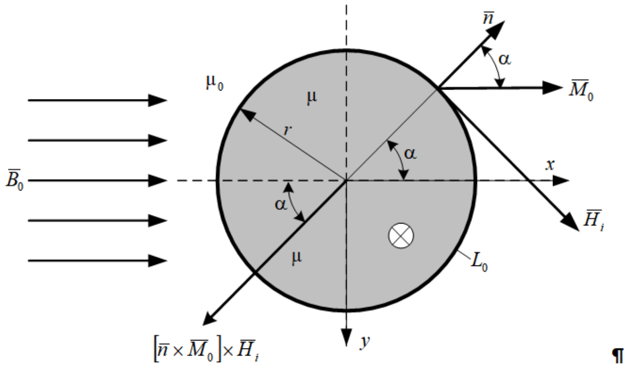

Fig. 3. Concerning the calculation of the force on a magnetic substance according to the model of equivalent magnetizing currents

Discussion of the obtained results

According to the above, both the model of equivalent magnetizing currents and the model of equivalent magnetic charges allow us to obtain a nonzero result for the force on the magnetic substance of the conductor in the considered case. Moreover, according to (19) and (24) for both considered models we have the same modulus of force density (specific force) on the magnetic substance, but the opposite direction of these forces. This means that the presence of a current in the magnetic substance of the conductor does not retain integral equivalence in determining the total force on a body made of a magnetic substance, which, as noted in the Introduction, takes place in the absence of electric currents in a magnetic substance.

This also means that, since, according to the posed problem, both considered models should give the same result for the total force on the energized conductor, then calculation the force on the current in the conductor using these both models will also give different results (so that the sum of the forces on the current and the magnetic substance be the same).

The fact that certain formulas are in contradiction is also mentioned in [12] and it leaves the problem of the physical causes of electromagnetic forces in magnets unsolved [12].

As a consequence, the question of the method for determining the force on the current in a magnetic substance, which was once posed by A. Einstein [13], seems to remain relevant.

Conclusions

1. It is shown that for the considered example, the integral equivalence between the models of the magnetized state (the model of equivalent magnetizing currents and the model of equivalent magnetic charges) is not preserved when determining the total force on the magnetic substance of a body through which an electric current flows.

2. The problem of calculation the magnetic field force effect on an electric current in a body made of a magnetized substance requires an additional solution.

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REFERENCES

[1] Mozhen Zh., Mechanics of electromagnetic continuous media. M.: Mir, 1991. 560 p. (Rus)
[2] Zagirnyak M.V., Determination of the ponderomotive magnetic force when calculating the field by the conformal transformation method, *Tekhnichna electrodynamika*, (2020), nr 3, 9-14.
[3] Bossavit A., On forces in magnetized matter, *IEEE Transactions on Magnetics*, 50 (2014), nr 2, 229-232.
[4] Spalek D., Generalization of Maxwell stress tensor method for magnetically anisotropic regions, *IEEE Transactions on Magnetics*, 55 (2019), nr 12, 1-6.
[5] Henneberger G., Sattler Ph. K., Shen D., Nature of the equivalent magnetizing current for the force calculation, *IEEE Transactions on Magnetics*, 28 (1992), nr 2, 1068-1071.
[6] Kassid F. Ezzahra, Azelmad Essaadia, Bousshine Lahbib, Berdai Abdelmajid,Magnetic force computation for a magnetic separator based on permanent magnet, *Przeglad electrotechniczny,* 97 (2021), nr 10, 28-32.
[7] Choi Hong Soon, Park Il Han, Lee Se Hee, Concept of virtual air gap and its applications for force calculation, *IEEE Transactions on Magnetics*, 42 (2006), nr. 4, 663-666.
[8] Bobbio Scipione, Delfino Federico, Girdinio Paola, Molfino Paolo, Equivalent sources methods for the numerical evaluation of magnetic force with extension to nonlinear materials. *IEEE Transactions on magnetics*, 36 (2000), nr. 4, 663-666.
[9] Tamm I. E., Fundamentals of the theory of electricity: Textbook for HEI. 11-th ed., rev. M.: FIZMATLIT, 2003. 616 p.
[10] Müller W., Comparison of different methods of force calculation, *IEEE Transactions on magnetics*, 26 (1987), nr 2, 1058-1061.
[11] Korn G., Mathematical handbook for scientists and engineers. Definitions, theorems and formulas for reference and review. M.: Book on Demand, 2014. 832 p.
[12] Tsybulevskii F. I., On the calculation of the forces acting on energized magnetics in a magnetic field, *Russian Electromechanics.* (1991), No. 1, 5-11. (Rus)
[13] Einstein A., On ponderomotive forces acting on ferromagnetic energized conductors in magnetic field: Einstein A. Collection of research papers. In 4 v. V. 3. M.: Nauka, 1966. P. 240-241. (Rus)