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Sea Clutter Modelling using Compound Gaussian with Nakagami Texture plus Thermal Noise

***Abstract**. Sea clutter modelling is crucial issue for constant false alarm rate (CFAR) radar detection. Several works had shown that sea clutter has a non-Gaussian nature. several heavy-tailed distributions have been proposed to model the sea clutter such as compound K (Ck), compound inverse Gaussian (CIG) and compound inverted exponentiated Rayleigh distribution (CIER). Recently, a new compound model has introduced by using Nakagami-distributed texture (CGNG) to model sea clutter with high-resolution at medium/high grazing angles. In this manuscript, we propose to extend the CGNG distribution to cover the presence of additive thermal noise and show the modelling performance using high resolution sea clutter data.*

***Streszczenie.** Modelowanie zakłóceń morskich ma kluczowe znaczenie dla wykrywania radarów ze stałym współczynnikiem fałszywych alarmów (CFAR). Kilka prac wykazało, że bałagan morski ma charakter niegaussowski. do modelowania bałaganu morskiego zaproponowano kilka rozkładów o grubych ogonach, takich jak związek K (Ck), złożony odwrotny rozkład Gaussa (CIG) i złożony odwrócony wykładniczy rozkład Rayleigha (CIER). Niedawno wprowadzono nowy model złożony, wykorzystujący teksturę rozproszoną Nakagami (CGNG) do modelowania bałaganu morskiego z wysoką rozdzielczością przy średnich/wysokich kątach wypasu. W tym manuskrypcie proponujemy rozszerzenie rozkładu CGNG, aby uwzględnić obecność addytywnego szumu termicznego i pokazać wydajność modelowania przy użyciu danych dotyczących zakłóceń morskich o wysokiej rozdzielczości. (**Modelowanie bałaganu morskiego przy użyciu złożonego rozkładu Gaussa z teksturą Nakagami i szumem termicznym**)*

Keywords: Modeling, High-resoulution, Sea clutter, CGNG distribution.
Słowa kluczowe: Modelowanie, wysoka rozdzielczość, bałagan morski, dystrybucja CGNG.

Introduction

Radar clutter modelling is crucial issue in radar detection, almost of constant false alarm rate (CFAR) detectors are developed based on the statistical model of the clutter [1-5]. In conventional radar with low-resolution, the clutter was supposed to be Gaussian distributed and optimal detectors are developed under this assumption. For modern radars working with high-resolution, the assumption of Gaussian clutter is not valid and the clutter has non-Gaussian nature. Compound Gaussian models are largely used to model high-resolution sea clutter, they are defined as mixture of two components; speckle and texture. In [6], the speckle component is interpreted as the consequence of the gravity waves with an amplitude Rayleigh distributed and the texture is the result of the capillary waves. In [7], the compound K distribution is introduced to model the clutter of sea surface. The texture component of the CK distribution obeys Gamma law. After that, the additive thermal noise is considered present in the CK distribution in [3]. Several statistical analyses of high-resolution sea clutter are carried out using real data such as in [8] and [9]. Furthermore, in [10] the CIG is proposed and validated to model sea clutter by experimental study using the database IPIX (Intelligent PIxel Processing). Moreover, the CIER distribution is developed in [11], this distribution is based on the inverted exponentiated Rayleigh texture.

Recently, in [12], the authors introduced the compound Gaussian with Nakagami texture to model sea clutter at medium/high grazing angles. In this manuscript, we propose to extend the CGNG distribution to cover the presence of additive thermal noise in order to enhance the modelling performance of high-resolution sea clutter.

Compound Gaussian Models

Compound Gaussian models are heavy-tailed non-Gaussian models, defined as a mixture of two components; the texture which represents the average local level of the clutter and the speckle component which obeys Rayleigh distribution. The total probability density function (PDF) is given as

$$p(z)=\int_0^{\infty} p(z/y)p(y)dy \tag{1}$$

where $p(x/y)$ and $p(y)$ are the speckle and the texture respectively.

The CGNG model is proposed in [12] by using Nakagami distribution as texture component, the Nakagami PDF is given as [12]

$$p(y)=\frac{2\Gamma^{2v}(v+0.5)y^{2v-1}}{\Gamma^{2v+1}(v)b^{2v}}exp\left(-\left[\frac{y\Gamma(v+0.5)}{b\Gamma(v)}\right]^2\right) \tag{2}$$

where v represent the shape and b the scale parameters. b is the average power of the clutter.

Substituting the Nakagami texture (2) in (1), we obtain the CGNG PDF as [12]

$$p(z)=\frac{4\Gamma^{2v}(v+0.5)}{\Gamma^{2v+1}(v)}\frac{z}{b}\int_0^{+\infty} y^{2v-2}\times exp\left(-\left[\frac{y^2\Gamma^2(v+0.5)}{\Gamma^2(v)}+\frac{z^2}{by}\right]\right)dy \tag{3}$$

The complimentary cumulative density function (CCDF) of CGNG is given as function of normalized threshold T as [12]

$$CCDF(T)=\frac{2\Gamma^{2v}(v+0.5)}{\Gamma^{2v+1}(v)}\int_0^{+\infty} y^{2v-1}\times exp\left(-\left[\frac{y^2\Gamma^2(v+0.5)}{\Gamma^2(v)}+\frac{T^2}{by}\right]\right)dy \tag{4}$$

CGNG plus-noise

In the presence of additive thermal noise, the compound Gaussian models are modified by increasing the average power of the speckle component as [3]

$$p(z/y)=\frac{z}{\sigma^2+2y^2/\pi}exp\left(-\frac{z^2}{2\sigma^2+4y^2/\pi}\right) \tag{5}$$

where $2\sigma^2$ is the power of the thermal noise.

Substituting the speckle (5) and the texture (2) in (1), we obtain the PDF of CGNG plus-noise as

$$(6)\qquad p(z)=\frac{4\Gamma^{2v}(v+0.5)z}{\Gamma^{2v+1}(v)b^{2v}}\int_0^\infty\frac{y^{2v-1}}{\sigma^2+2y^2/\pi}\times$$
$$exp\left(-\frac{z^2}{2\sigma^2+4y^2/\pi}-\left[\frac{y\Gamma(v+0.5)}{b\Gamma(v)}\right]^2\right)dy$$

The CCDF of CGNG plus-noise is given as

$$(7)\qquad CCDF(T)=\int_0^\infty exp\left(-\frac{T^2}{2\sigma^2+4y^2/\pi}\right)\times$$
$$\frac{2\Gamma^{2v}(v+0.5)y^{2v-1}}{\Gamma^{2v+1}(v)b^{2v}}exp\left(-\left[\frac{y\Gamma(v+0.5)}{b\Gamma(v)}\right]^2\right)dy$$

In order to show the modeling ability of the CGNG plus-noise, it will compared with the well known CK, CIG and CIER distributions in the presence of thermal noise.

The compound K distribution has been widely used to model non-Gaussian sea clutter, the texture of the CK is gamma distributed as [3]

$$(8)\qquad p(y)=\frac{2c^{2\gamma}y^{2\gamma-1}}{\Gamma(\gamma)}exp(-c^2y^2)$$

where γ and c are the shape and the scale parameters respectively.

The PDF of compound K distribution in presence of additive thermal noise is given as [3]

$$(9)\qquad p(z)=\int_0^\infty\frac{z}{\sigma^2+2y^2/\pi}exp\left(-\frac{z^2}{2\sigma^2+4y^2/\pi}\right)\times$$
$$\frac{2c^{2\gamma}y^{2\gamma-1}}{\Gamma(\gamma)}exp(-c^2y^2)dy$$

The CCDF of compound K distribution is given by

$$(10)\qquad CCDF(T)=\int_0^\infty exp\left(-\frac{T^2}{2\sigma^2+4y^2/\pi}\right)\frac{2c^{2\gamma}y^{2\gamma-1}}{\Gamma(\gamma)}\times$$
$$exp(-c^2y^2)dy$$

The CIG distribution is proposed in [10], this model is characterized by inverse Gaussian texture, the CIG PDF is given as [10]

$$(11)\qquad p(z)=\int_0^\infty\frac{z}{\sigma^2+4y^2/\pi}exp\left(-\frac{z^2}{2\sigma^2+4y^2/\pi}\right)\times$$
$$\frac{\lambda^{1/2}}{\sqrt{2\pi}y^{3/2}}exp\left(-\lambda\frac{(y-\mu)^2}{2\mu^2y}\right)dy$$

where λ and μ are the shapes parameters and the scale parameter respectively.

The CCDF of CIG is

$$(12)\qquad CCDF(T)=\int_0^\infty exp\left(-\frac{T^2}{p_n+4y^2/\pi}\right)\times$$

The CIER model is base on the inverted exponentiated Rayleigh texture, the PDF of the CIER distribution is given as [11]

$$(13)\qquad p(z)=\int_0^\infty\frac{2\alpha\beta zy^{-3}}{\sigma^2+2y^2/\pi}exp\left(-\frac{z^2}{2\sigma^2+4y^2/\pi}-\frac{\beta}{y^2}\right)\times$$
$$\left(1-exp\left(-\frac{\beta}{y^2}\right)\right)^{\alpha-1}dy$$

where α is the shape parameter and β is the scale parameter.

The corresponding CCDF of the CIER distribution is given as [11]

$$(14)\qquad CCDF(T)=\int_0^\infty 2\alpha\beta y^{-3}exp\left(-\frac{T^2}{2\sigma^2+4y^2/\pi}-\frac{\beta}{y^2}\right)\times$$
$$\left(1-exp\left(-\frac{\beta}{y^2}\right)\right)^{\alpha-1}dy$$

Modelling Performance analysis

The performance of the CGNG plus-noise is performed by several comparisons using the IPIX real database [8, 10, 11], where of the real PDF and CCDF will be compared with the theoretical models. The mean square criterion (MSE) is used to measure the modelling performance. The curve fitting method [13] is used to estimate the parameters of the different distributions.

First, in Figs. 1 and 2, the CGNG plus-noise is compared with the CGNG free-noise. The curves of Fig. 1 are obtained from the 5th cell range, resolution 3m and polarization HH, the Fig. 2 is obtained from the 7th cell range, resolution 30m and polarization VV. It is observable that the PDFs and CCDFs of the CGNG plus-noise offers the best fit to the real data, and confirm that the real data are effectively contain the thermal noise.

Now, the modelling performance is also assessed by comparing the CGNG with the existing distributions CK, CIG and CIER, the noise is considered present in all distributions. The Figs. 3 and 4 show the curves of the PDFs and the CCDFs of the CGNG, CK, CIG and CIER. The PDFs and CCDFs curves plotted in the Fig.3 are carried out using the data set of the 6th cell range, resolution 15m and polarization HH. The Fig. 4 is obtained by using the 20th cell range, resolution 3m and polarization VV. We observe that the CGNG gives a good fit to the real data with the lowest values of the MSE as shown in Table 1. The obtained results show the capability of the CGNG plus-noise to model sea clutter with high-resolution.

$$\frac{\lambda^{1/2}}{\sqrt{2\pi}y^{3/2}}exp\left(-\lambda\frac{(y-\mu)^2}{2\mu^2y}\right)dy$$

a)

b)

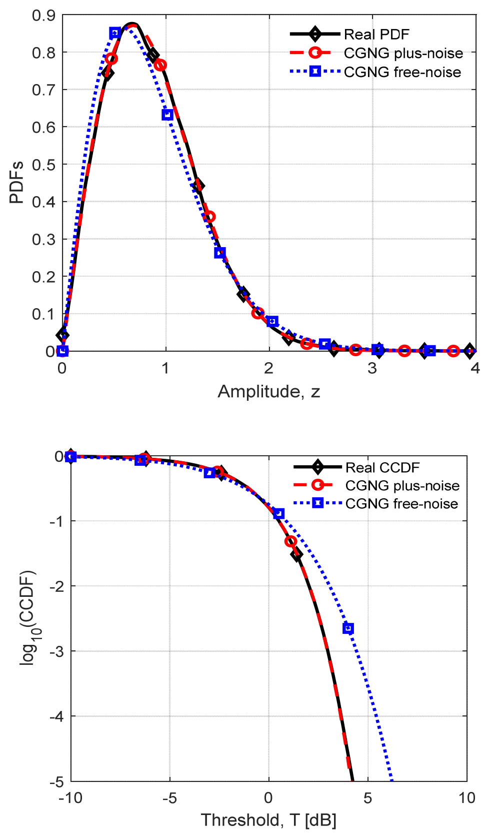
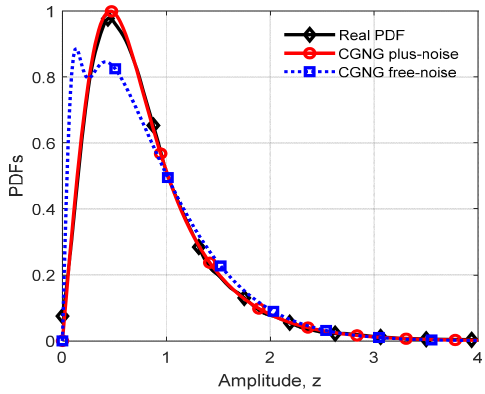


Fig.1. PDFs (a) and CCDFs (b) of CGNG plus-noise and CGNG free-noise using IPIX data of the 5th cell range, resolution 3m and polarization HH.

a)



b)

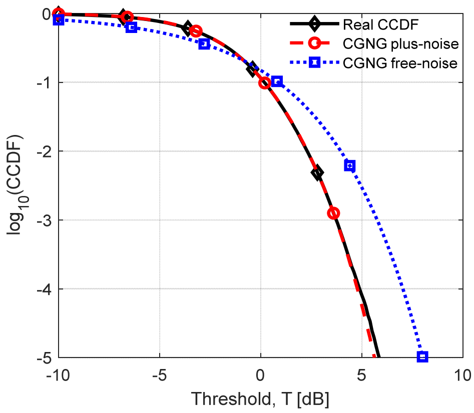
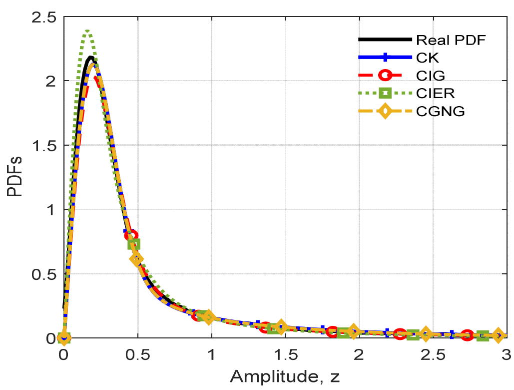


Fig.2. PDFs (a) and CCDFs (b) of CGNG plus-noise and CGNG free-noise using IPIX data of the 7^{th} cell range, resolution 30m and polarization VV.

a)



b)

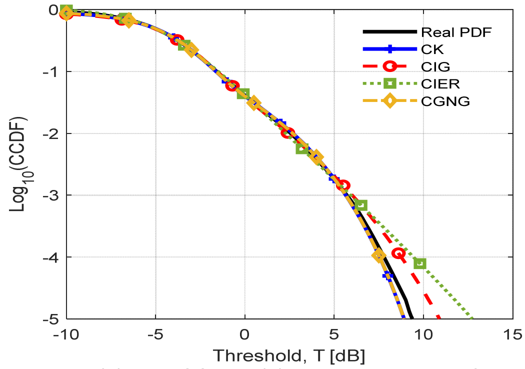
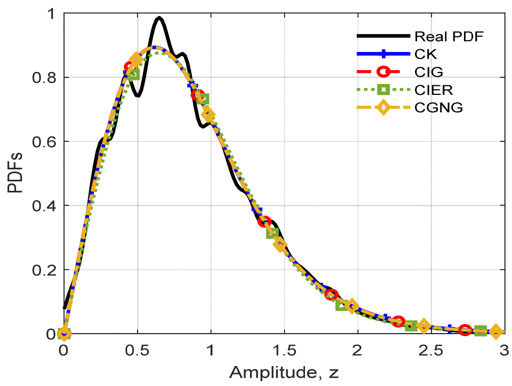


Fig.3. PDFs (a) and CCDFs (b) in the presence of noise using IPIX data of the 23^{th} cell range, resolution 15m and polarization HH.

a)



b)

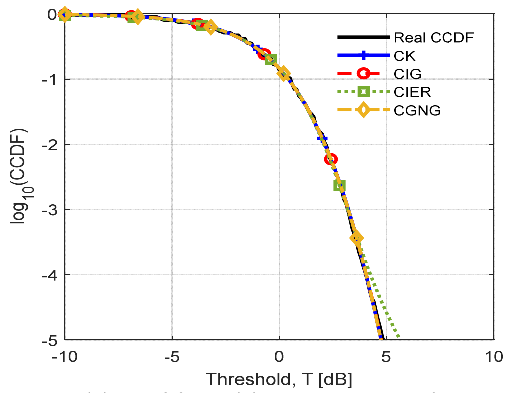


Fig.4. PDFs (a) and CCDFs (b) in the presence of noise using IPIX data of the 20^{th} cell range, resolution 3m and polarization VV

Table 1. MSEs values in the presence noise

IPIX data range	MSE	CK	CIG	CIER	CGNG
23^{th} cell range of resolution 15m and polarization HH	PDFs MSE	0.0007	0.0013	0.0020	0.0007
	CCDFs MSE	0.0414×10^{-3}	0.1306×10^{-3}	0.0856×10^{-3}	0.0414×10^{-3}
20^{th} cell range of resolution 3m and polarization VV	PDFs MSE	0.6613×10^{-3}	0.6658×10^{-3}	0.6670×10^{-3}	0.6613×10^{-3}
	CCDFs MSE	0.3939×10^{-4}	0.3962×10^{-4}	0.3959×10^{-4}	0.3939×10^{-4}

Conclusion

In this paper, the CGNG model is extended to cover the additive thermal noise. Exploiting IPIX database, the CGNG plus-noise is compared with CGNG free-noise and the results show that CGNG plus-noise is closer and fits the real PDF and CCDF. After that, CGNG plus-noise is also compared with the CK, CIG and CIER distributions in the presence of noise, The results show the ability of the CGNG plus-noise to model high-resolution sea clutter.

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