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Utilization of Rayleigh approximation for analysis of magnetoelastic effect in truss elements

***Abstract**. The paper presents the results of measurements of hysteresis loops in low magnetizing fields in steel truss elements under mechanical stress. This low magnetizing fields area is known as the Rayleigh region. Magnetic hysteresis loops under the influence of external forces in the range of up to 10.2 kN were measured. The investigation was performed for a truss made of ferromagnetic materials: 13CrMo4-5 and X30Cr13 steel. Based on the results, a modified Rayleigh approximation was proposed to include the effect of external force introducing stress to the material. The proposed modified Rayleigh approximation was verified. The high value of R^2 determination coefficient confirms the correctness of the proposed solution for the discussed case.*

***Streszczenie.** W artykule przedstawiono wyniki pomiarów pętli histerezy w słabych polach magnesujących w stalowych elementach kratownicy poddanych naprężeniom mechanicznym. Obszar słabych pól magnesujących nazywany jest obszarem Rayleigha. Wykonano pomiary pętli histerezy magnetycznej pod wpływem sił zewnętrznych w zakresie do 10,2 kN. Badania przeprowadzono dla kratownicy wykonanej z materiałów ferromagnetycznych: stali 13CrMo4-5 i stali X30Cr13. Na podstawie uzyskanych wyników, zaproponowano zmodyfikowaną postać aproksymacji Rayleigha w celu uwzględnienia wpływu siły zewnętrznej wprowadzającej naprężenia. Zaproponowana modyfikacja aproksymacji Rayleigha zostało zweryfikowana. Wysoka wartość współczynnika determinacji R^2 potwierdza poprawność zaproponowanego rozwiązania dla rozważanego przypadku. (**Wykorzystanie przybliżenia Rayleigha do analizy efektu magnetosprężystego w elementach kra**townicy)*

Keywords: Rayleigh approximation, magnetoelastic effect, steel truss
Słowa kluczowe: aproksymacji Rayleigha, efekt magnetosprężysty, kratownica stalowa.

Introduction

Steel structures have been widely used in the construction, energy industry and transportation for many years. For this reason, Non-Destructive Testing (NDT) methods for steel structural components are dynamically developed and increasingly used. Magnetoelastic effect can be utilized for this type of measurement. To determine the stress, a model of the influence of stress on magnetic properties is required. Elaborated physical models often use extensive equations and consume large amounts of computing power [1]. However, sometimes there is a need to quickly model hysteresis loop or magnetic parameters for specific construction case on the ground of relatively uncomplicated mathematical dependencies approximating the physical nature of phenomenon. Such approach can be used for preliminary verification of the measurement method, e.g., distribution of windings, selection of the most optimal configuration, and in constructions where measurement conditions change dynamically. This paper proposes the utilization of Rayleigh approximation for analysis of magnetoelastic effect in specific truss configuration used in typical steel constructions.

Magnetoelastic effect

Magnetoelastic effect involves changes of the magnetic parameters of a magnetic material under the influence of stresses σ from external forces F. This effect is most often observed as a change of the shape of the magnetic hysteresis loop. Utilizing the magnetoelastic effect, by observing changes in the magnetic parameters of structural steels, it is possible to determine the stresses appearing in the material under the influence of external forces [2, 3].

Rayleigh region

The Rayleigh region is set at the initial part of the magnetization curve of a ferromagnetic material. The magnetizing field is calculated from the equation:

$$H = \frac{IN}{l_e} \tag{1}$$

where: H – magnetizing field, I – current, N – number of windings, l_e – effective length of the magnetic flux path. Therefore, it can be clearly seen that performing measurements in low magnetizing fields of Rayleigh region can reduce the current or the number of windings, which simplifies installation for stress assessment in the construction.

In Rayleigh region the dependence between magnetic flux density B and magnetizing field H can be described using a second-degree polynomial [4]:

$$B(H) = \mu_0 \mu_i H + \mu_0 \nu_R H^2 \tag{2}$$

where: B – magnetic flux density, H – magnetizing field, μ_0 – magnetic permeability of free space, μ_i – initial relative magnetic permeability, ν_R – Rayleigh coefficient. Initial relative magnetic permeability μ_i characterizes linear reversible changes of magnetic flux density B under the influence of the magnetizing field H. These changes are the result of domain wall deflections. In turn, the Rayleigh coefficient ν_R describes the nonlinear irreversible changes of magnetic flux density B under the influence of the magnetizing field H that result from displacement of the domain walls.

The hysteresis loop in the Rayleigh region exhibits characteristic lenticular shape and can be approximated by the Rayleigh equations [5]:

$$\begin{aligned} B_{\swarrow}(H) &= \mu_0 \left[(\mu_i + \nu_R H_m) H + \frac{\nu_R}{2} (H_m^2 - H^2) \right] \\ B_{\nearrow}(H) &= \mu_0 \left[(\mu_i + \nu_R H_m) H - \frac{\nu_R}{2} (H_m^2 - H^2) \right] \end{aligned} \tag{3}$$

where: $B_{\swarrow}(H)$ – magnetic flux density for upper, decreasing branch of the hysteresis loop $B_{\nearrow}(H)$ – magnetic flux density for lower, increasing branch of the hysteresis loop, H_m – maximum magnetizing field.

Method of investigation

Special measurement system, adjusted to investigate the magnetoelastic properties of truss structures, was used to perform the measurements. The schematic block diagram of the system is presented in Fig. 1.

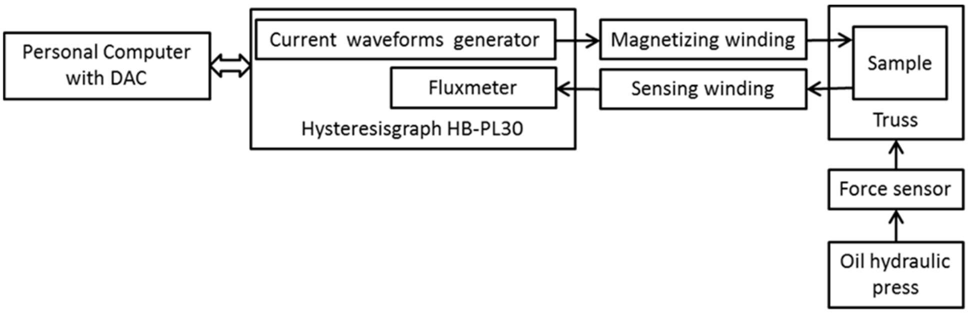

Fig.1. Schematic block diagram of the measurement stand

The mechanical setup includes the investigated truss and hydraulic press with sensor for measurement the applied force. For the measurement of magnetic characteristics, a PC-controlled hysteresisgraph system HB-PL30 was utilized. Magnetizing and sensing coils were wound on the investigated elements of the truss, which were connected to the hysteresisgraph magnetizing output and induced voltage input respectively [6].

Fig.2. Mechanical setup

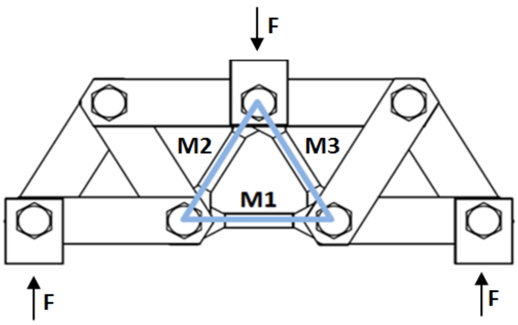

Fig. 3. Truss with the sample

The truss is designed so that the three middle elements, which are the measurement sample, are replaceable. These elements have a reduced cross-section area so when the force is applied, only they are destroyed, instead of the entire truss. The magnetizing and sensing coils are wound on the three elements of the truss, so that the magnetic circuit M is closed through all three elements. Element M1 is subjected to tension, while elements M2 and M3 are compressed [7].

The measurement samples were made of ferromagnetic materials used in mechanical structures: 13CrMo4-5 and X30Cr13 steels. These materials are used, among others, in the construction of bridge structures, power poles, high-pressure boilers, etc.

The mechanical setup is presented in Fig. 2, and the truss with the sample in Fig. 3.

Results

The investigation was performed for trusses made of X30Cr13 and 13CrMo4-5 steels [8]. The truss was placed in the press and an external force was applied in the range from 0 to 10.2 kN. The hysteresis loop was measured for each applied force. The magnetizing field range was selected so that the measurements were made in the Rayleigh region, where hysteresis loop can be approximated by the Rayleigh equations (3).

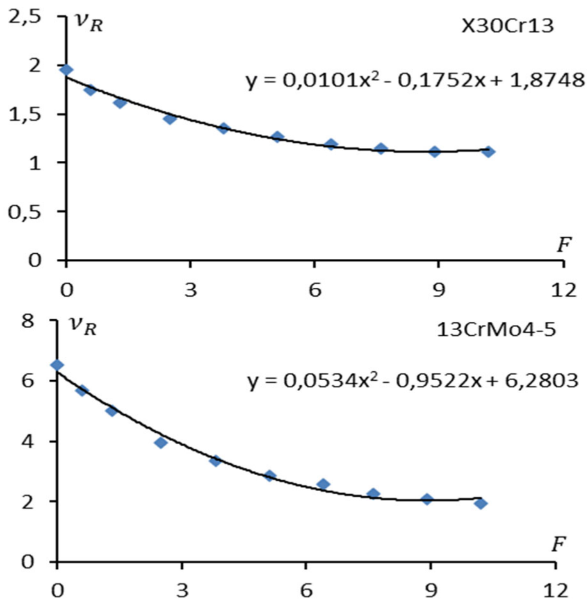

Fig. 4. The dependence of the Rayleigh coefficient ν_R on the applied force F

Then, based on the results of the measurements and the formula (3), the parameters μ_i and ν_R were determined for each value of the applied force. After analysing the values obtained, it was proposed that the parameter μ_i can be considered to have a constant value. However, parameter ν_R depends quadratically on the value of the applied force F and can be written with the equation:

$$\nu_R = aF^2 + bF + c \tag{4}$$

where: F is applied force, a, b, and c are coefficients of the polynomial.

The Table 1 shows determined values of initial relative magnetic permeability μ_i.

Tab. 1. Determined values of initial relative magnetic permeability μ_i

steel	initial relative magnetic permeability μ_i
X30Cr13	58
13CrMo4-5	232

Fig. 4 shows the dependence of the parameter ν_R on the applied force F and values of coefficients a, b, and c of the polynomial.

The Rayleigh equations for hysteresis loop will take the form:

$$B_{\swarrow}(H) = \mu_0\left[(\mu_i + (aF^2 + bF + c)H_m)H + \frac{(aF^2 + bF + c)}{2}(H_m^2 - H^2)\right]$$
$$B_{\nearrow}(H) = \mu_0\left[(\mu_i + (aF^2 + bF + c)H_m)H - \frac{(aF^2 + bF + c)}{2}(H_m^2 - H^2)\right] \tag{5}$$

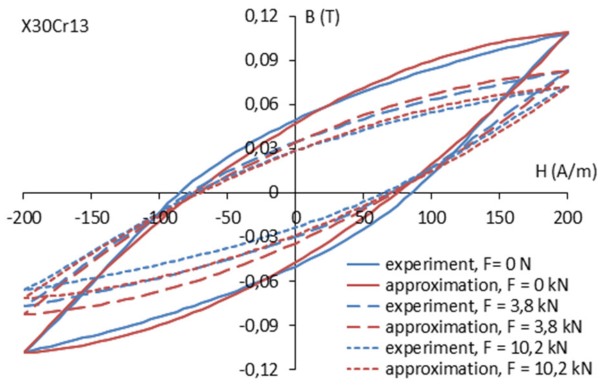

Fig. 5. Hysteresis loops of X30Cr13 steel for three different values of external force: blue line – experimental results, red line – modified Rayleigh approximation

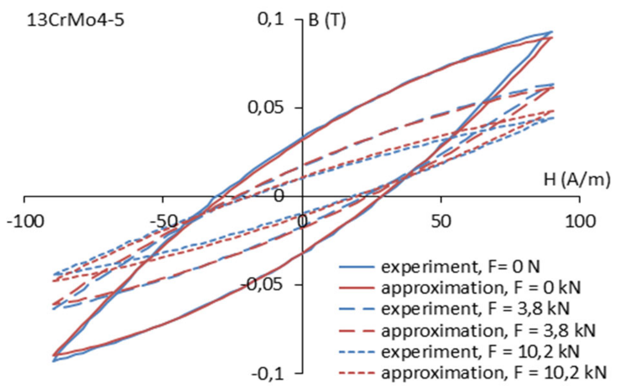

Fig. 6. Hysteresis loops of 13CrMo4-5 steel for three different values of external force: blue line – experimental results, red line – modified Rayleigh approximation

Next, verification was made to assess if the modified Rayleigh approximation would give the expected results. Fig. 5 presents hysteresis loops for three different values of external force for X30Cr13 steel. Fig. 6 presents hysteresis loops for three different values of external force for 13CrMo4-5 steel.

From the presented characteristics, it can be seen that a significant effect of stress from external forces on the magnetization characteristics was observed for X30Cr13 and 13CrMo4-5 steels.

A high value of R^2 determination coefficient indicates that the modification introduced in the original Rayleigh approximation accurately represents the effect of stress on the magnetic characteristics of structural elements of the investigated truss.

Tab. 2. Determined values of R^2 determination coefficient

F	R^2	
kN	X30Cr13	13CrMo4-5
0	0,99	0,99
3,8	0,99	0,99
10,2	0,99	0,99

Conclusions

The obtained results confirm the validity of the proposed concept. Modified Rayleigh approximation can be utilized to determine the hysteresis loop under stress in the specific truss configuration under investigation. Methodology based on modified Rayleigh approximation can be utilized in NDT for stress assessment in elements of the investigated steel truss providing the following advantages:

- utilization of low magnetizing fields reduces required number of windings or the amplitude of magnetizing current,
- calculation based on simple quadratic functions does not require high computational power and can be performed relatively quickly even with standard PC,
- proposed approximation provides high value of R^2 determination coefficient, exhibiting high compliance with the observed changes of magnetic properties under applied force.

The presented method, due to low computational complexity, can be utilized to test and optimize the method of measurement of stresses in truss structures such as bridges, power poles and building structures. High compliance with the measurement results also indicates that the methodology is applicable in monitoring the state of stress in such structures.

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