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doi:10.15199/48.2022.07.06

Hinf-FEEDBACK / FEEDFRWORD linear control applied to the 3DOF DELTA parallel robot

***Abstract** in this paper we propose an implementation of a Hinf-FEEDBACK / FEEDFRWORD multivariate linear controller on a 3DOF DELTA parallel robot. First, we linearized the system around an action point by implementing the "LINMODE" function in an MATLAB environment, then we compared the result of the proposed controller with that of the Hinf controller as well as that of the classic PID controller. We have found that the Hinf controller is robust compared to the PID controller and the proposed controller is more robust compared to the Hinf controller and the classic PID controller, and it follows the trajectory very well with good precision. And so we applied these controls to a system checked out from SOLIDWORKS.*

***Streszczenie** W niniejszym artykule zaproponowano implementację wielowariantowego regulatora liniowego Hinf-FEEDBACK / FEEDFRWORD na robocie równoległym DELTA 3DOF. W pierwszej kolejności dokonano linearyzacji układu wokół punktu akcji poprzez implementację funkcji "LINMODE" w środowisku MATLAB, a następnie porównano wynik działania proponowanego regulatora z wynikiem działania regulatora Hinf oraz klasycznego regulatora PID. Stwierdziliśmy, że kontroler Hinf jest odporny w porównaniu do kontrolera PID, a proponowany kontroler jest bardziej odporny w porównaniu do kontrolera Hinf i klasycznego kontrolera PID, i podąża za trajektorią bardzo dobrze z dobrą precyzją. I tak zastosowaliśmy te regulatory do układu sprawdzonego z SOLIDWORKS. (**Sterowanie liniowe Hinf-FEEDBACK / FEEDFRWORD zastosowane do robota równoległego 3DOF DELTA**)*

Keywords: Delta parallel robot, Dynamic model, robust Hinf-FEEDBACK / FEEDFRWORD, RMSE
Słowa kluczowe: Robot równoległy Delta, model dynamiczny, solidny Hinf-FEEDBACK / FEEDFRWORD, RMSE.

Introduction

Robots appeared in the early eighties conceived by the scientist Reymand Clavel in Switzerland (EPFC) [1]. These robots came into the field of industry from the start of their creation, and because of their speed and precision in the work, these systems have been extended to various fields, in particular (medical, pharmaceutical, military...) [2]. Parallel robots are known for their complex dynamics, which prompted many researchers to work on them. Due to the difficulty of controlling this type of automatic systems, researchers have led to the application and development of many linear and non-linear controllers on these systems.

Due to the expansion in many fields, the fact that impelled it to perform work requiring precision and heavy weight [3], which requires the controller to be robust and able to control the system in more difficult situations. Many researchers were incited to work on the development of control devices with the aim of reducing the signs of control and improving the performance and capacity of the controller in the most difficult situations. Formerly, the classic consoles fulfilled the requirements due to the work that the robot was doing, but as the needs increased, the robot were required to increase the performance and endure the work in the most difficult situations. The classic controllers became powerless, consequently, powerful controllers appeared, including Hinf controllers. Hinf controller is a robust [4] [5] [6] controller whose strength lies in the ability to control a group of systems adjacent to the system that the controller is computed on, including the real system. when the dynamics of the system deviates from the system that was calculated on it, then we find that the performance of the robot start to deteriorate, therefore, we attempted to solve this problem by introducing a FEED unit in order to improve its performance.

The Hinf controller provides a robust system yet the results obtained in previously published articles in this context, we revealed poor performances and does not meet the requirements of a robust controller. To improve the performance of the robot, we added the FEEDFORWARD console [7], which allowed us to obtain good results compared to Hind and PID controllers.

The article includes an introduction in section one. The dynamic model of 3-Dof Delta parallel robotic manipulators is explained in section two. In section three, control problem is given for the 3DOF DELTA parallel robotic manipulator model. Hinf controller design is provided for linear control of the Hinf scheme. The local linearization of robotic dynamics is obtained by the expansion of the Taylor series around its local equilibrium. In section 4, simulation tests are performed to assess the accuracy and robustness of the monitoring of the proposed non-linear control method. Finally, we have performed a robust test with the PID control and Hinf control in this part in order to compare its obtained results with our proposed idea, and therefore the robust control strength will appear. The study is concluded with the final results of the simulation tests.

Dynamic model of DPR

The inverse and direct geometric model (DGM/IGM) of the 3-DOF rigid-link DELTA robot and the architectural parameters of the robot extracted from SOLIDWORKS is given in [8]. Under the assumption that the masses of the links are distributor at the 3-links end [9]. So the inverse dynamic model is as follows.

(1) $$\tau = M(\vartheta)\ddot{\vartheta} + C(\vartheta,\dot{\vartheta})\dot{\vartheta} + G(\vartheta)$$

$M(\vartheta)$ is the inertia matrix dim (3×3), $C(\vartheta,\dot{\vartheta})$ represent the Coriolis torques / forces, centrifugal forces dim (3×1), $G(\vartheta)$ is the gravitational forces vector dim (3×1), τ is vector torque $\tau=[\tau_1\tau_2\tau_3]$, ϑ is the joint vector $\tau=[\vartheta_1\vartheta_2\vartheta_3]$ and $\dot{\vartheta},\ddot{\vartheta}$ is the speed and acceleration joint vector respectively.

With:

(2) $$M(\vartheta) = Ib + (mn + 3mab + 3mb)J^{'}J$$

(3) $\cdot C(\vartheta,\dot{\vartheta}) = J^{'}(mn+3mab+3mb)I\frac{dJ}{dt}\dot{\vartheta}$

(4) $G(\vartheta) = J'(mn+3mab)g - L(mb/2+mc+mab/2) gcos(\vartheta))$

were: "g" is the gravity acceleration, I represent the 3x3 identity matrix, J is the Jacobin matrix that represents the relation between the articular and operational speed $\dot{x} = J\dot{\vartheta}$

The direct dynamic model is the one which expresses the joint accelerations as a function of the positions, speeds and torques of the joints. It is then represented by the following relation:

(5) $\ddot{\vartheta} = M(\vartheta)^{-1}(\tau - C(\vartheta,\dot{\vartheta}) - G(\vartheta))$

Controller design

PID control design

We will convert the robot system to a linear model as shown in Equation 11. And we will control its joints by the PID controller [16] [8].

(6) $\tau = m_{max}\ddot{\vartheta} + c_{max}\dot{\vartheta} + g_{max}$

Where m_{max}, c_{max}, g_{max} – the maximum value of the element M_{jj}, C_{jj}, G_{jj} Respectively

The controller law is shown as follows

(7) $\tau = K_p(\vartheta_d - \vartheta) + K_d(\dot{\vartheta}_d - \dot{\vartheta})$

The calculation of the three constants (K_p, K_I, and K_d) will be based on Khalil Domber method

$K_{pjj} = 3m_{max\,jj}W^2 \quad K_{djj} = m_{max\,jj}W$

$K_{Ijj} = m_{max\,jj}W^3$

Hinf FEEDFRWORD control design

Description of the Hinf FEEDFRWORD

The Hinf robust controller philosophy is based on calculating the control K(s) for system by reducing the Hinf-norm of the close loop transformation matrix that is defined by the equation shown below [11].

(8) $\|\eta\|_\infty = \sup\sigma_{max}(\eta(jw))$

Wwere $\eta = \begin{bmatrix} W_1S \\ W_2KS \\ W_3T \end{bmatrix}$

$S = (I+GK)^{-1}$ and $T = I + S$

When I is the identity matrix, S is the sensitivity function matrix, T is the complementary sensitivity function matrix and the W_1, W_2, W_3 represent the weights penalize the errors signal, the control signal and the output signal respectively [12] as shown in Figure (1). The three weights should be stable, where W1 represents the desired performance, W2 represents the filter of control signal (limit control effort), and W3 represents the size of the acceptable disturbances. W2 and W3 can also be taken as the gains.

The augmented plane is based to create a new output, and these outputs may be either real or imaginary, in this case we have increased the error signal, control signal and output signal (fig 1 and 2). Where the transfer function between the new output and r input (reference signal r) is W_1S, W_2KS, W_3T respectively [13]

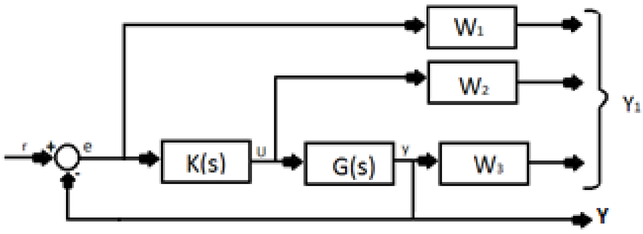

Fig.1. The augmented plane

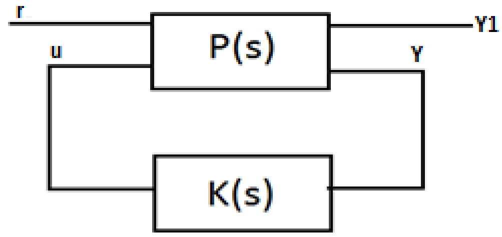

Fig.2. Closed-loop system

To calculate the robust Hinf feedback control K(s), we must solve the Riccati equation [14]. The Matlab robust control toolbox contains a lot of functions among them robust "hinfsyn" We will use that to compute the Hinf robust feedback control K(s). Finally, we will add the Feedforward to the control law to improve the tracking performance of the robot, as shown in the following equation.

$u = K(s)e + F(S)e$

where $S = ((C\times(B\times k_0 - A)^{-1})^{-1}$

$(B\times k_0 - A)^{-1}$ represent the close loop state matrix. The design of Hinf-F Feedforward is representing in following figure.

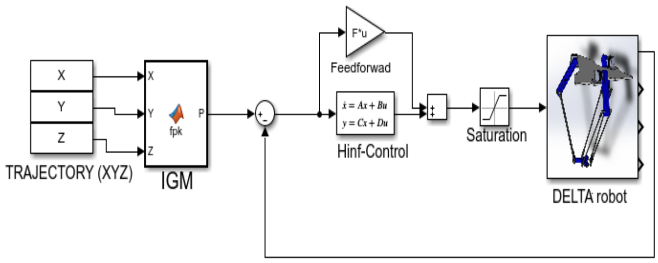

Fig.3. Hinf-FEEDFRWORD controller

Linearization

To linearize the dynamics of the robot, we will give the nonlinear direct model. So, we put the state vector in form $X = [X_1X_2X_3X_4X_5X_6] = [\vartheta_1\vartheta_2\vartheta_3\dot{\vartheta}_1\dot{\vartheta}_2\dot{\vartheta}_3]$ and the control inputs vector is $u = [u_1u_2u_3] = [\tau_1\tau_2\tau_3]$ then, we can write this nonlinear direct model as follows[15]

(9) $\dot{X} = f(x) + g_a(x)u_1 + g_b(x)u_2 + g_c(x)u_3$

with:
$f(x), g_a(x), g_b(x), g_c(x)$ nonlinear functions

The linearization procedure of the robotic model takes place around the operating point which is the current value of the robot state vector x and the value of the command input vector u. The linearization procedure gives the following differential equation (10), and the state space equation system (11).

(10) $\dot{X} = [\nabla xf + \nabla xg_a + \nabla xg_b + \nabla xg_c]x + g_a u_1 + g_b u_2 + g_c u_3$

where

(11) $$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

where:
$A = [\nabla xf + \nabla xg_a + \nabla xg_b + \nabla xg_c]$, $B = [g_a g_b g_c]$

We will use this function to calculate the linear model. Thus, the linear model for the DELTA Robot is given by the following equation.

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -54.6 & -0.44 & -0.44 & 0 & 0 & 0 \\ -0.44 & -54.6 & -0.44 & 0 & 0 & 0 \\ -0.44 & -0.44 & -54.6 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 5824 & 9.98 & 9.98 \\ 9.98 & 5824 & 9.98 \\ 9.98 & 9.98 & 5824 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

HINF DESIGN

In order to calculate the Hinf controller for the delta robot, we wrote the following code on Matlab [17]

```
P = augw (G, W1, W2, W3);
sys=minreal(P);
[K, CL, GAM, INFO] =hinfsyn(sys,3,3);
 K=minreal(K);
[Ak, Bk, Ck, Dk] =ssdata(K);
```

At first, we augment the system by the function “augw”, and the function “minreal” is utilized to eliminate uncontrollable or unobservable state in state-space models. Finally, we will place the controller as a state space form [18].
Where

(12) $$W_1 = \frac{(s+6000)}{(20s+6)} I$$

(13) $$W_2 = 0.5e^{-2} I$$

(14) $$W_3 = \frac{(s+1e^{-6})}{(1e^{-4}s+1e^{4})} I$$

The selection of the W1 weight is according to the required dynamics, depending on the static error e, bandwidth w_B, and peak magnitude M of S, a typical performance weight is the following:

(15) $$W_1 = \frac{\frac{s}{M} + w_B}{S + w_B s}$$

The choice of W3 is based on the uncertainty in the model. It is known that the higher is the frequency, the greater is the uncertainty. Thus, W_3 should be large at the high frequency to get a small “T” as shown in Fig. 4

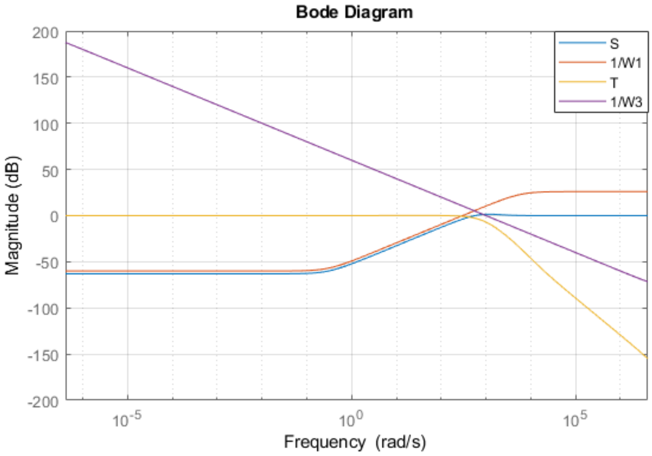

Fig.4. Bode plot of S , $\frac{1}{W_1}$ and T, $\frac{1}{W_3}$ for Hinf control

This weight is also depicted in Figure 6. It can be shown that this weight gives a good fit of the system G, except for frequencies that hovering around w=290 rad/s (Bandwidths), the singular values of the closed –loop system (CL) compared for the GAM= 0.7087=-3 dB is given in the following figure 5.

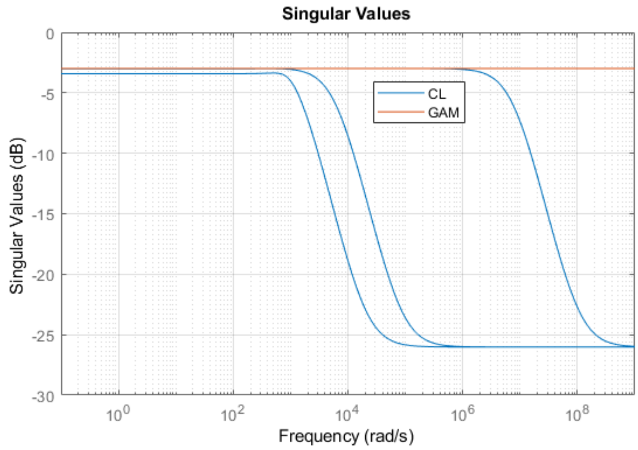

Fig.5. singular values of the Closed-Loop for Hinf control

FEEDFRWORD DESIGN

In this section, we will calculate the FEEDFRWORD controller by extracting the poles of the closed loop system and then extracting the robust poles P and after we calculate the gain k_0. The calculation of the FEEDFRWORD controller is as follows [19].

$$P = [-59 \pm 58.2i \quad -25.6 \pm 44.2i \quad -16 \quad -11]$$

$$k_0 = \begin{bmatrix} 0.87 & 1.38 & 7.51 & 0.14 & -0.49 & 0.43 \\ 45.85 & 21.92 & -13.87 & 2.43 & 1.24 & -1.40 \\ -41.33 & 28.14 & 38.70 & -3.76 & 0.21 & 2.53 \end{bmatrix}$$

(16) $$\text{So } F = (C \times (B \times k_0 - A)^{-1} B)^{-1}$$

where $$F = \begin{bmatrix} 1.86 & 1.25 & 7.36 \\ 45.71 & 22.90 & -13.98 \\ -41.46 & 28.01 & 39.69 \end{bmatrix}$$

SIMULATION AND RESULTS

In this part, we will test the controllers on an annular trajectory as shown in figure 6, we will follow the QUINTINQ method in calculate the trajectory, the controllers will be applied to the system extracted from SOLIDWORKS. The shape of the trajectory is illustrated in the following equation.

$$Z = -0.25 + e^{-2} t \,,\, Y = R \times \sin(wt) \,,\, X = R \times \cos(wt)$$

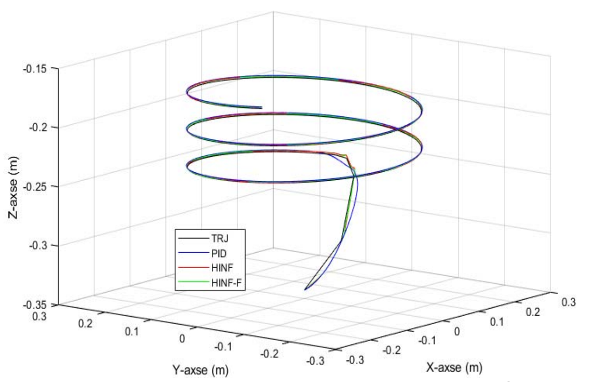

Fig.6.operational trajectory tracking under the proposed Controller

The following figure represents the errors in joint space

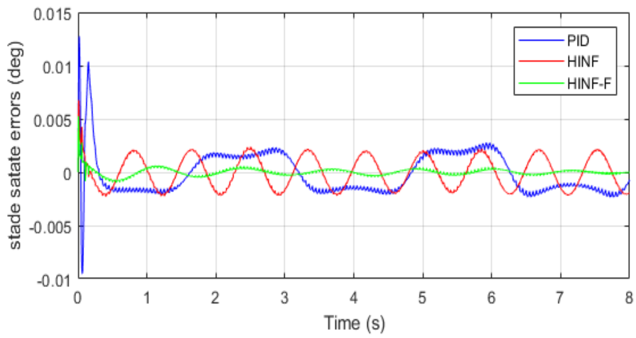

Fig.7.The stade state error for joint 1 (ϑ_1)

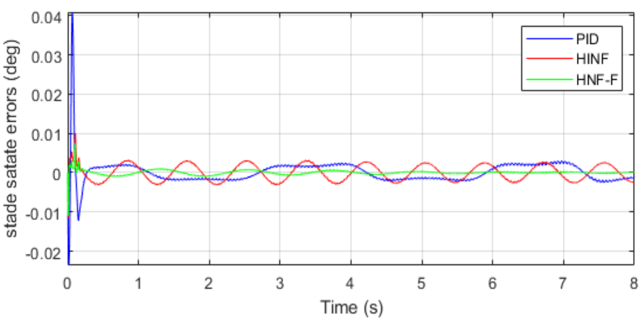

Fig.8.The stade state error for joint 2 (ϑ_2)

From the result shown in the figures above, we notice that the controllers work well on the system in the joint space, but the HNFF controller gave good results while comparing with the HNF and the latter is better at turn in relation to the PID controller.

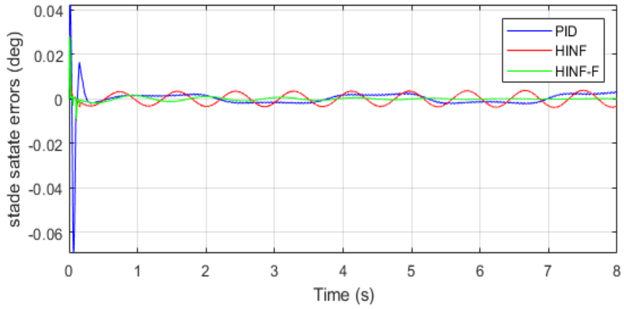

Fig.9.The Stade state error for joint 3 (ϑ_3)

The following figure represents the errors in operating space

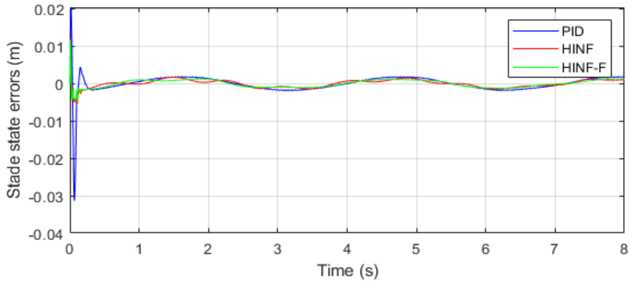

Fig.10.The stade state error for X-axes

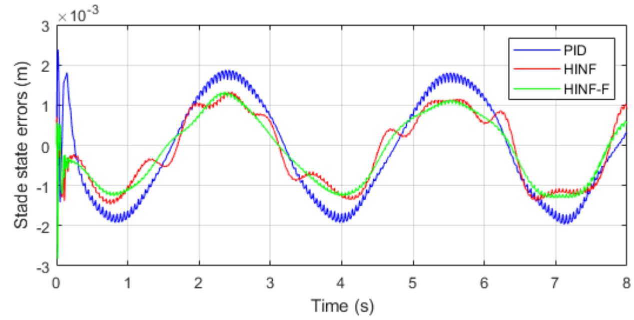

Fig.11.Thestade state error for Y-axes

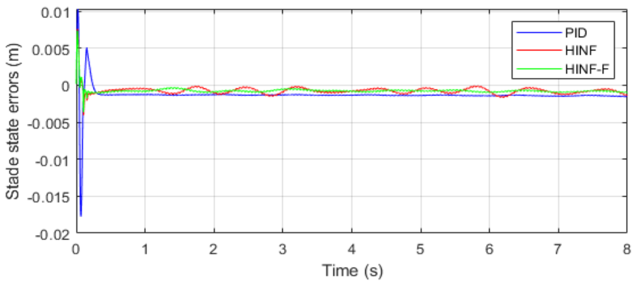

Fig.12.Thestade state error for Z-axes

The following figure represent The control signals for three controls PID and Hinf and Hinf FEEDFRWORD (Hinf-F) control

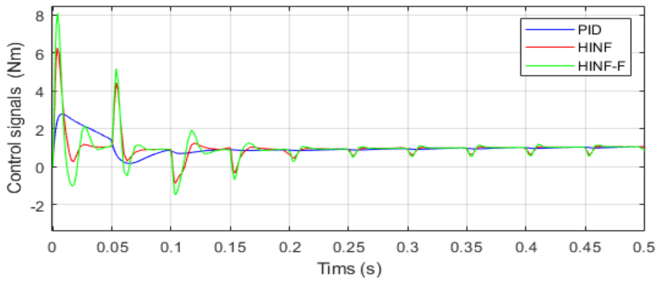

Fig.13.The control signals for joint 1 (ϑ_1)

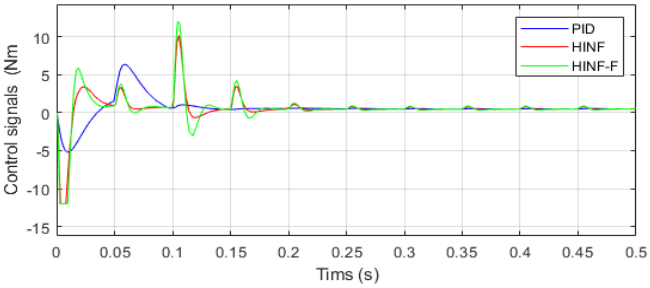

Fig.14.The control signals for joint 2 (ϑ_2)

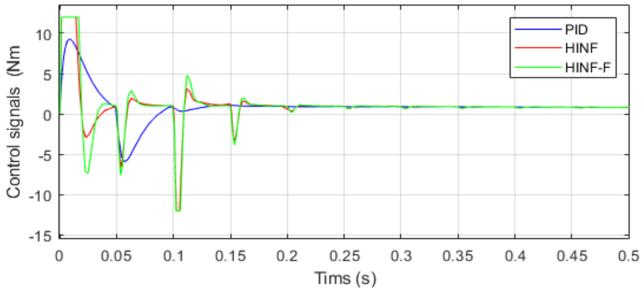

Fig.15.The control signals for joint 3 (ϑ_3)

In our experiment we used a trajectory in the form of an annular where its beginning was at point (0, 0, -2.5) and its end was at point (0.1331, -0.1515, -0.1584). The time of the tests took 8 seconds. During these tests, we noticed that all three controllers performed well on the system in the operating space and the results were close as the tracking error was no more than 0.002 degrees for all three controllers, but most importantly is that the Hinf controller was better than the other two controllers in the common space in terms of performance, where the trace error was no more than 0.0001 degrees, but it exceeded that value for the other two controllers. For the control signal, we have defined it between 12 and 12 N / m by the “saturation” function as shown in figure 3 below.

Robust test

The strength of robust control is not limited to performance and rejection; but the problem inquires whether the control maintains the stability of the system when we make variations on this model or not. To find out, in this experiment we put a 2 kg weight on a moving platform. In the results presented below, we calculated the corrector on a nominal model; We have changed some parameters in the model extracted from SOLIDWORKS. The corrector will be calculated on the nominal model, where we change the parameters as follows: (mn =0.187Kg To 1Kg). However, from the result obtained and summarized in all figures we can clearly see that the control performs well on the nonlinear model; We note that the setpoint and output are combined for servo regulation and control of positions and speeds, excellent performance, and very short response time, in the regulation mode [10].

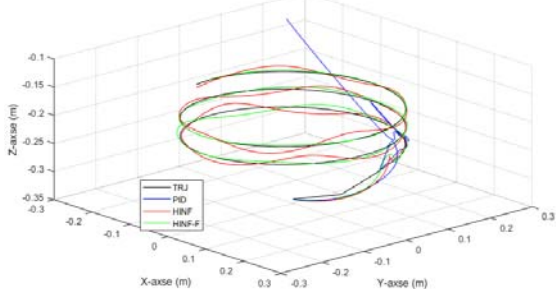

Fig.16. operational trajectory tracking

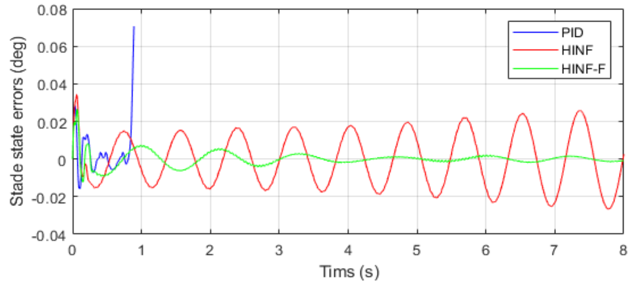

Fig.17.The stade state error for joint 1 (ϑ_1)

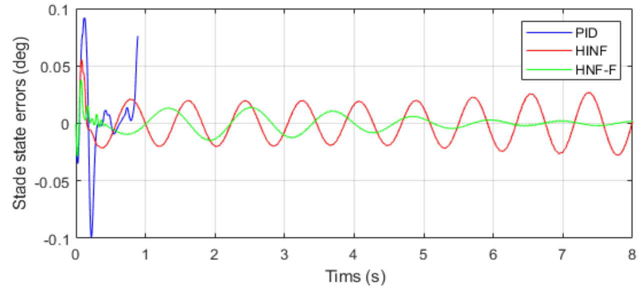

Fig.18.The stade state error for joint 2 (ϑ_2)

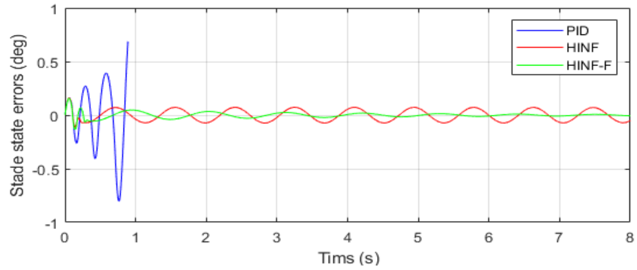

Fig.19.The stade state error for joint 3 (ϑ_3)

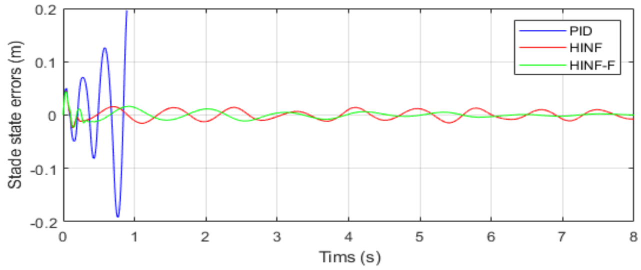

Fig.20.The stade state error for X-axes

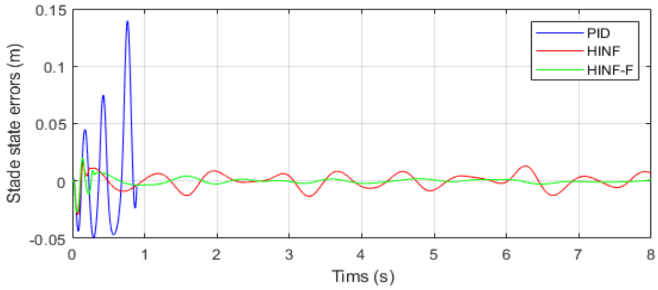

Fig.21.The stade state error for Y-axes

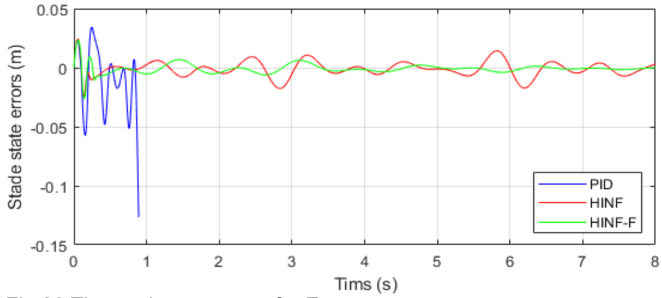

Fig.22.The stade state error for Z-axes

For more clarity we have used the root error mean square (REMS) as shown in Figure 25 and 26, as an indicator of separation in performance [20].

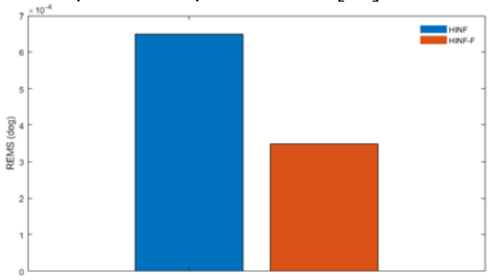

Fig.23.diagram representation of the RMSE Bare in joint space

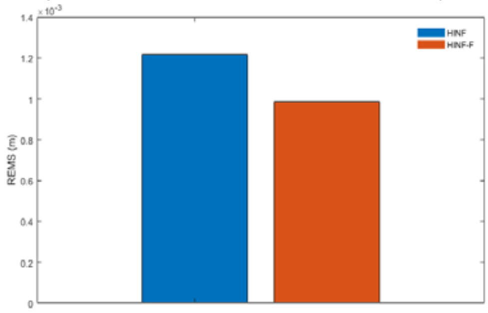

Fig.24.diagram representation of the RMSE Bare in operating space

In this part, we considered the difference between the three controllers in terms of Robustness, where the difference was very clear between them. We remarked that the Hinf controller is more robust in relation to the classic controllers, as the PID controller lost control of the system permanently while the Hinf controller controlled the system but with a clear lack of performance as the controller HINFF took control of the system and gave good results compared with HINF as the tracking signals did not exceed 2kg. HNFF improves RMSE by approximately 54% compared to HNF in joint space and by 36% in work space.

CONCLUSION

A robust Hinf-FEEDBACK / FEEDFRWORD control method has been developed for DELTA robotic. The corrector calculation is based on the local linearization of the robot dynamics around an operating point. The matrices of the Jacobean dynamic robot model are used to calculate an equivalent linearized manipulator model thanks to the expansion of the Taylor series. To compensate for the modelling error which is introduced by the approximate linearization of the Hinf control law required by the increase in the system, a gain of the controller is calculated by the solution of an algebraic equation Riccati. Thus, the strength of this command lies in the creation of variations on the model and the corrector will stabilize the modified system. It is common for robots to perform tasks such as weightlifting, but, during the modelling, we cannot take into account all the additional weights. So we have a robust control that can stabilize the system, and we took the additional weights as an internal disturbance of this model, and we saw that the corrector previously calculated stabilizes the system up to an additional 2kg weight.

Acknowledgement

Our special thanks to the examiners for their constructive comments. This study is supported by The Scientific Technological Research Council of Algeria. thanks Ministry of Higher Education and Scientific Research of Algeria for its continuous assistance in scientific research.

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