Andrey Grishkevich

WSB University in Chorzów
ORCID. 0000-0002-2008-0672

doi:10.15199/48.2022.12.69

Combinatorial algorithm of enumeration the failure states of complex systems

***Abstract**. A method of enumeration failure states of complex electrical systems was developed as a combinatorial problem of forming the classes of cross-sections. An example of the presented algorithm work is given*

***Streszczenie.** Opracowano metodę wyliczania stanów awaryjnych złożonych układów elektrycznych jako kombinatorycznego problemu tworzenia klas zbiorów krytycznych. Podano przykład działania prezentowanego algorytmu. (**Kombinatoryczny algorytm wyliczania stanów awaryjnych złożonych systemów**).*

Keywords: electric power system, structural reliability of electric power systems, minimal-cut failure state.
Słowa kluczowe: system elektroenergetyczny, niezawodność strukturalna systemów elektroenergetycznych, stan minimalny uszkodzenia.

Introduction

While considering different states of the system's elements it is also necessary to consider different states for the whole system, particularly, the failure states. It is natural to suppose that each failure state contributes to the resulting indices of the system reliability. This originates typical combinatorial problems of the classification of failure states into classes on the basis of their contribution to the resulting indices of the system reliability and the problems of enumeration of all representatives of these classes.

The formulation of the problem

Let $L=\{E^l : l = 1, 2, \ldots, n\}$ be a set of system elements. Each element of the system E can be in one of three states E_α, $\alpha \in \{N, R, S\}$, where E_N is an operating state (state of normal work), E_S is a state between the fault and switching (before switching), E_R is a repair state (after switching). Transitions between states for one element are described by $E_N \to E_S \to E_R \to E_N \to \ldots$.

State of the system ω is determined by state of each element and can be described as a set $\omega=\omega(L)=\{\omega(E^l) : E^l\in L,\ l = 1, 2, \ldots, n\} = \{E^l_{\alpha_l} : E^l\in L,\ \alpha_l \in \{N, R, S\},\ l = 1, 2, \ldots, n\}$. Let us denote $\Omega=\{\omega\}$ the set of all system`s states; $Z_{En}=\varnothing$, $Z_{Er}=\{E\}$, $Z_{Es}\subseteq L(E\in Z_{Es})$ are zones of influence of elements E in different states (finding of the element E in state S is equal to finding of the set of elements Z_{Es} in state R; finding of the element E in the state E_R corresponds to the removal of this element from the system, i.e. consideration of the system $L\backslash E$ elements); $T_\omega=(\cup_{Er\in\omega}E) \cup (\cup_{Es\in\omega}Z_{Es})\subseteq L$ the set of failing elements in state ω; $R=\{r : r\subseteq L, |r|\leq 3\}$ is a set of assemblages of elements which synchronous finding in state R leads to system failure. The state ω is a system failure state, if exist $r\in R$, such as $r\subseteq T\omega$, or a operating state (state of successful work) otherwise. Thus $\Omega=\Omega_F\cup\Omega_W$ ($\Omega_F\cap\Omega_W = \varnothing$), where Ω_F is a set of failure states, Ω_W – set of operating states (states of successful work). A failure state is a minimal-cut failure state (MC-state) [1], if for all $E_\alpha\in\omega$ the translation of element E from state E_R into state E_N, or from state E_S into state E_R, returns the system into operating state (state of successful work). It is required to enumerate all the minimal-cut failure states (MC-state).

The classification of cross-section on the basis of minimal-cut failure states (MC-states)

Consideration of set $\Omega(J)=\{\omega : J\subseteq L,\ \forall E\in L\backslash J\Rightarrow En\in\omega\}$ of states of elements $J\subseteq L$. Lets each set of system elements be characterised by the set of states $\mathbf{\Omega}(J)=\Omega(J)\backslash\cup_{I\subset J(I\neq J)}\Omega(I)$ and for any $I\neq J$ $\mathbf{\Omega}(I)\cap\mathbf{\Omega}(J)=\varnothing$ is true. The set $J\subseteq L$ is a cross-section, if MC($\mathbf{\Omega}(J)$)$\neq\varnothing$. The set of all cross-sections will be denoted by $\Re$.

Let function $\psi : L\to L$ be a bijection. Determine

$$\psi(\omega) = \psi(\omega(L)) = \psi(\{E^1{}_{\alpha_1}, E^2{}_{\alpha_2},\ldots E^n{}_{\alpha_n}\}) = \\ = \{\psi(E^1{}_{\alpha_1}), \psi(E^2{}_{\alpha_2}),\ldots \psi(E^n{}_{\alpha_n})\} = \\ = \{\psi(E^1)_{\alpha_1}, \psi(E^2)_{\alpha_2},\ldots \psi(E^n)_{\alpha_n}\} = \omega(\psi(L)) = \eta$$

the transformation of state ω into state η. The cross-section $I,J\in\Re$ we will consider to be θ-equivalently ($I\equiv J(\theta)$), if there is a bijection ψ_{IJ} so as ψ_{IJ}(MC($\mathbf{\Omega}(I)$)) = MC($\mathbf{\Omega}(\psi_{IJ}(I))$) = MC($\mathbf{\Omega}(J)$).

Consideration of the factorset of cross-section set according to equivalently

$$\Re/\theta = \{[J]\theta\} = \{[J_i]\theta : i=1, 2, \ldots, 15\},$$

where $J_1=J_2=(I)$ are one-element cross-sections, $J_3=\ldots=J_6=(I, K)$ – two-element cross-sections, $J_7=\ldots=J_{15}=(I, K, O)$ – three-element cross-sections, gives us the classification of cross-sections on the basis of MC-states:

MC($\mathbf{\Omega}(J_1)$) = $\{I_R\}$,
MC($\mathbf{\Omega}(J_2)$) = $\{I_S\}$,
MC($\mathbf{\Omega}(J_3)$) = $\{I_RK_R\}$,
MC($\mathbf{\Omega}(J_4)$) = $\{I_SK_R\}$,
MC($\mathbf{\Omega}(J_5)$) = $\{I_SK_S\}$,
MC($\mathbf{\Omega}(J_6)$) = $\{I_SK_R, I_RK_S\}$,
MC($\mathbf{\Omega}(J_7)$) = $\{I_RK_RO_R\}$,
MC($\mathbf{\Omega}(J_8)$) = $\{I_SK_RO_R\}$,
MC($\mathbf{\Omega}(J_9)$) = $\{I_RK_SO_S\}$,
MC($\mathbf{\Omega}(J_{10})$) = $\{I_SK_SO_S\}$,
MC($\mathbf{\Omega}(J_{11})$) = $\{I_SK_RO_R, I_RK_SO_R\}$,
MC($\mathbf{\Omega}(J_{12})$) = $\{I_RK_SO_S, I_SK_RO_S\}$,
MC($\mathbf{\Omega}(J_{13})$) = $\{I_SK_RO_R, I_RK_SO_S\}$,
MC($\mathbf{\Omega}(J_{14})$) = $\{I_SK_RO_R, I_RK_SO_R, I_RK_RO_S\}$,
MC($\mathbf{\Omega}(J_{15})$) = $\{I_RK_SO_S, I_SK_RO_S, I_SK_SO_R\}$.

Thus, instead of enumeration of all MC-states we can enumerate the assemblages of system elements, belonging to the different classes of the equivalently. The advantages of such a method are usage of subset of a system element and decreasing of dimensions of the considered set, because one subset of system elements can be characterised by more than one MC-states. The presented classification specifies the classification [2], in which both passive and active failures are considered. Proposed algorithm of forming the classes of cross-sections.

Algorithm of forming the classes of cross-sections

Let us consider forming the classes of cross-sections on the basis of sets R i Z_{I_S} ($I \in L$). Suppose that $\{I, K\} = \{K, I\}$, $(I, K) \neq (K, I)$, $(I, \{K, O\}) = (I, \{O, K\})$, $(I, \{K, O\}) \neq (K, \{I, O\})$, $(I, \{K, O\}) \neq (O, \{I,K\})$.

$Z_x = \{I : I \in L, x \in Z_{I_S}\}$ for all $x \in L$.

In addition let us suppose that for all $r \in R$ set r does not contain identical elements.

$M_1 = \{I\ (I_R \in \Omega_F) : \{I\} \in R\}$;

$M_3 = \{\{I, K\}\ (I_RK_R \in \Omega_F) : \{I, K\} \in R, I \neq K, I \notin M_1, K \notin M_1\}$;

$M_7 = \{\{I, K, O\}\ (I_RK_RO_R \in \Omega_F) : \{I, K, O\} \in R, I \neq K, I \neq O, K \neq O, I \notin M_1, K \notin M_1, O \notin M_1, \{I, K\} \notin M_3, \{I, O\} \notin M_3, \{K, O\} \notin M_3\}$.

On giving a system by a graph one-, two- and three-element cuts of the graph could be taken instead of sets M_1, M_3, M_7.

$Q_2^1 = \{I : x \in M_1, I \in Z_x\}$;

$Q_2^2 = \{I : \{x, y\} \in M_3, I \in Z_x \cap Z_y\}$;

$Q_2^3 = \{I : \{x, y, z\} \in M_3, I \in Z_x \cap Z_y \cap Z_z\}$;

$M_2 = \{I\ (I_S \in \Omega_F) : I \in Q_2^1 \cup Q_2^2 \cup Q_2^3, I \notin M_1\}$;

$Q_4^2 = \{(I, K) : \{x, K\} \in M_3, I \in Z_x, I \neq K\}$;

$Q_4^3 = \{(I, K) : \{x, y, K\} \in M_7, I \in Z_x \cap Z_y, I \neq K\}$;

$Q_4 = \{(I, K)\ (I_SK_R \in \Omega_F) : (I, K) \in Q_4^2 \cup Q_4^3, I \notin M_1, I \notin M_2, \{I, K\} \notin M_3\}$;

$M_6 = \{\{I, K\}\ (I_SK_R, I_RK_S \in \Omega_F) : (I, K) \in Q_4$ and $(K, I) \in Q_4\}$;

$M_4 = \{(I, K)\ (I_SK_R \in \Omega_F) : (I, K) \in Q_4$ and $(K, I) \notin Q_4\}$;

$Q_5^2 = \{\{I, K\} : \{x, y\} \in M_3, I \in Z_x, K \in Z_y, I \neq K\}$;

$Q_5^3 = \{\{I, K\} : \{x, y, z\} \in M_7, I \in Z_x \cap Z_y, K \in Z_z, I \neq K\}$;

$M_5 = \{\{I, K\}\ (I_SK_S \in \Omega_F) : \{I, K\} \in Q_5^2 \cup Q_5^3, I \notin M_1, I \notin M_2, K \notin M_1, K \notin M_2, \{I, K\} \notin M_3, (I, K) \notin Q_4, (K, I) \notin Q_4\}$.

The sets M_2, M_4, M_5, M_6 forming with some other suppositions was considered in [3].

$Q_8 = \{(I, \{K, O\})\ (I_SK_RO_R \in \Omega_F) : \{x, K, O\} \in M_7, I \in Z_x, I \neq K, I \neq O, (I, K) \notin Q_4, I \notin M_2, I \notin M_1, \{I, K\} \notin M_3, (I, O) \notin Q_4, \{I, O\} \notin M_3, \{I, K, O\} \notin M_7\}$.

Let us clear up forming the set Q_8 (see class MC($\mathbf{\Omega}(J_8)$). It is formed in such a way, that $I_SK_RO_R$ is a minimal-cut failure state, $I_SK_RO_R \in \Omega_F$, and states I_SK_R $(O_R \to O_N)$, I_SO_R $(K_R \to K_N)$, $I_RK_RO_R$ $(I_S \to I_R) \in \Omega_W$ (see changes in Fig.1). In Fig.1. the circles corresponding to the failure states are filled in, and the circles corresponding to the MC-states are enlarged and signed. The circles corresponding to the states of successful work and against which checks are carried out are highlighted with a thick line and signed.

The states $I_RK_SO_R$, $I_RK_RO_S$, $I_RK_SO_S$ could be as the operating states so the failure states. That is why it requires additional checks (see bellow) for to form class $[J_8]\theta$ from set Q_8. State $I_SK_RO_R \in \Omega_F$, this belonging is determined by the way of forming the set Q_8: $\{x, K, O\} \in M_7$ $(x_RK_RO_R \in \Omega_F)$, $I \in Z_x$ $(I_SK_RO_R \in \Omega_F)$; besides by checking $I \neq K$, $I \neq O$ $(|\{I, K, O\}| = 3)$ is protected (provided). To check that $I_SK_RO_R$ is MC-state is quite complicated. Thus, checking $I_SK_R \in \Omega_W$ is done in several steps: either I_SK_R $(O_R \to O_N)$ is not minimal-cut failure state ($(I, K) \notin Q_4$), or I_S $(K_R \to K_N)$ … $(I \notin M_2)$, or I_R $(I_S \to I_R)$ … $(I \notin M_1)$, or I_RK_R $(I_S \to I_R)$ … ($\{I, K\} \notin M_3$), or I_R $(K_R \to K_N)$ … – has been already checked, or K_R $(I_R \to I_N)$ … – checked during forming the set M_7. Analogous checks are required for to determine $I_SO_R \in \Omega_W$ (additionally $(I, O) \notin Q_4$, $\{I, O\} \notin M_3$ are checked) i $I_RK_RO_R \in \Omega_W$ (additionally $\{I, K, O\} \notin M_7$ is checked). These checks are determined by the states, to which it could be transitioned from state $I_SK_RO_R$ (see Fig.1), and known to this step minimal-cut failure states (sets M_1, M_3, M_7, M_2, Q_4, M_5). Fig.2 illustrates two symmetrical cases. In practice, the checks are similar, only they are applied to some permutation (you need to change the order of the elements during the checks).

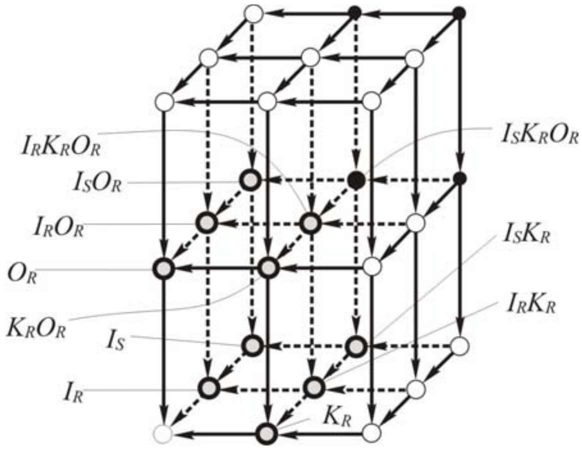

Fig.1. Forming the set Q_8

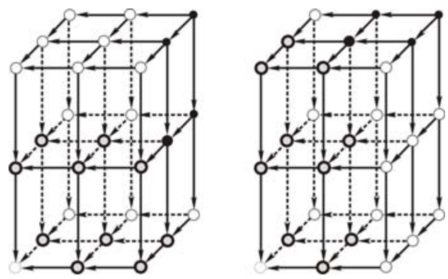

Fig.2. Symmetrical cases the set Q_8

$Q_9 = \{(I, \{K, O\})\ (I_RK_SO_S \in \Omega_F) : \{I, y, z\} \in M_7, K \in Z_y, O \in Z_z, I \neq K, I \neq O, K \neq O, K \notin M_1, K \notin M_2, O \notin M_1, O \notin M_2, \{I, K\} \notin M_3, \{I, O\} \notin M_3, \{K, O\} \notin M_3, (K, I) \notin Q_4, (O, I) \notin Q_4, (O, K) \notin Q_4, (K, O) \notin Q_4, \{I, K, O\} \notin M_7, \{K,O\} \notin M_5, (O,\{I,K\}) \notin Q_8, (K,\{I,O\}) \notin Q_8\}$;

$M_8 = \{(I, \{K, O\}) : (I, \{K, O\}) \in Q_8, (I, \{K, O\}) \notin Q_9, (K, \{I, O\}) \notin Q_8, (O, \{I, K\}) \notin Q_8\}$;

$M_9 = \{(I, \{K, O\}) : (I, \{K, O\}) \in Q_9, (I, \{K, O\}) \notin Q_8, (K, \{I, O\}) \notin Q_9, (O, \{I, K\}) \notin Q_9\}$;

$M_{10} = \{\{I, K, O\}\ (I_SK_SO_S \in \Omega_F) : \{x, y, z\} \in M_7, I \in Z_x, K \in Z_y, O \in Z_z, I \neq K, I \neq O, K \neq O, I \notin M_1, I \notin M_2, K \notin M_1, K \notin M_2, O \notin M_1, O \notin M_2, \{I, K\} \notin M_3, \{I, O\} \notin M_3, \{K, O\} \notin M_3, (I, K) \notin Q_4, (K, I) \notin Q_4, (I, O) \notin Q_4, (O, I) \notin Q_4, (K, O) \notin Q_4, (O, K) \notin Q_4, \{I, K\} \notin M_5, \{I, O\} \notin M_5, \{K, O\} \notin M_5, \{I, K, O\} \notin M_7, (I, \{K, O\}) \notin Q_8, (K, \{I, O\}) \notin Q_8, (O, \{I, K\}) \notin Q_8, (I, \{K, O\}) \notin Q_9, (K, \{I, O\}) \notin Q_9, (O, \{I, K\}) \notin Q_9\}$;

$M_{11} = \{(\{I, K\}, O) : (I, \{K, O\}) \in Q_8$ and $(K, \{I, O\}) \in Q_8$ and $(O, \{I, K\}) \notin Q_8\}$;

M_{12}={({I, K}, O) : (I, {K, O}) ∈ Q_9 and (K, {I, O}) ∈ Q_9 and (O, {I, K}) ∉ Q_9};

M_{13}= $Q_8 \cap Q_9$;

M_{14} = {{I, K, O} : (I, {K, O}) ∈ Q_8 and (K, {I, O}) ∈ Q_8 and (O, {I, K}) ∈ Q_8};

M_{15} = {{I, K, O} : (I, {K, O}) ∈ Q_9 and (K, {I, O}) ∈ Q_9 and (O, {I, K}) ∈ Q_9}.

On done suppositions in result we receive

$M_i = [J_i]\theta$, i = 1, 2, ..., 15.

Example

1. Initial given (sets R, Z_{Es}): R={1}; Z_{2s}={1} (simplified for the sake of brevity, actually Z_{2s}={1,2}).

L={1,2}, Ω={1_N2_N, 1_R2_N, 1_S2_N, 1_N2_R, 1_R2_R, 1_S2_R, 1_N2_S, 1_R2_S, 1_S2_S}; **Ω**(1)={1_N2_N, 1_R2_N, 1_S2_N}, T_{1r2n}=T_{1s2n}={1}, 1_R2_N∈Ω_F, T_{1n2n}=∅, 1_N2_N∈Ω_W, MC(**Ω**(1))={1_R}; **Ω**(2)={1_N2_N, 1_N2_R, 1_N2_S}, T_{1n2s}={1,2}, 1_N2_S∈Ω_F, T_{1n2r}={2}, 1_N2_R∈Ω_W, MC(**Ω**(2))={2_S}. The classes of cross-sections (sets Ji): J_1=(1), J_2=(2); [J_1]θ={J_1}={1}, [J_2]θ={J_2}={2}.

2. R={{1, 2}}; Z_{3s}={1,2}. J_3=(1,2), J_2=(3).

3. R= {{1, 2}}; Z_{3s}={1}. J_3=(1,2), J_4=(3, 2).

4. R={{1, 2}}; Z_{3s}={1}, Z_{4s}={2}. J_3=(1,2), [J_4]θ ={(3, 2), (4, 1)}, J_5=(3, 4).

5. R={{1, 2}, {1, 4}, {2, 3}}; Z_{3s}={1}, Z_{4s}={2}. [J_3]θ ={(1, 2), (1, 4), (2, 3)}, J_6=(3, 4).

6. R={{11, 12, 13}}; Z_{1s}={11,12,13}. J_2=(1), J_7=(11, 12, 13).

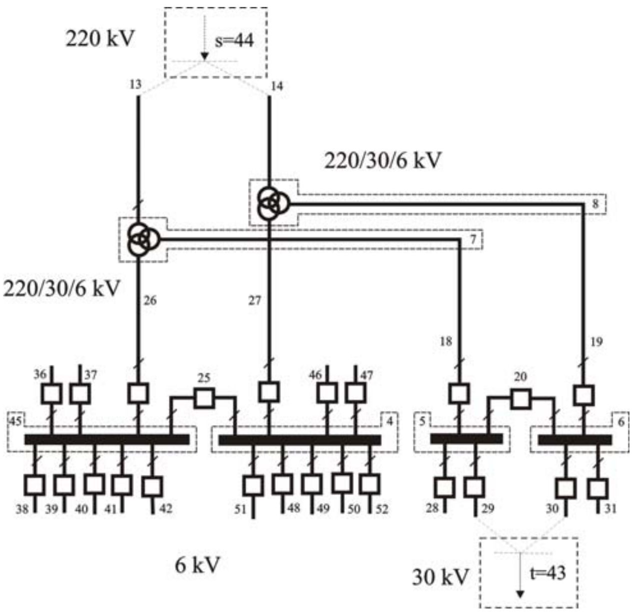

Fig.3. Fragment of electric system scheme

7. R={{1, 11, 12}}; Z_{2s}={11, 12}. J_4=(2, 1), J_7=(1, 11, 12).

8. R={{11, 12, 13}}; Z_{1s}={11}, Z_{2s}={12, 13}. J_4=(2, 11), J_5=(2, 1), J_7=(11, 12, 13), J_8=(1, 12, 13).

9. R={{1, 12, 13}, {1, 2, 14}}; Z_{2s}={12,13}, Z_{1s}={14}. J_6=(2, 1), [J_7]θ ={(1, 12, 13), (1, 2, 14)}.

10. R={{1, 2, 3}}; Z_{4s}={1}, Z_{5s}={2}, Z_{6s}={3}. J_7=(1, 2, 3), [J_8]θ ={(6, 1, 2), (5, 1, 3), (4, 2, 3)}, [J_9]θ ={(3, 4, 5), (2, 4, 6), (1, 5, 6)}, J_{10}=(4, 5, 6).

11. R={{1, 12, 13}, {11, 2, 13}}; Z_{1s}={11}, Z_{2s}={12}, Z_{3s}={13}. [J_7]θ ={(1, 12, 13), (11, 2, 13)}, [J_8]θ ={(3, 1, 12), (3, 11, 2)}, J_{11}=(2, 1, 13), J_{12}=(1, 2, 3).

12. R={{11, 2, 3}, {1, 12, 13}}; Z_{1s}={11}, Z_{2s}={12}, Z_{3s}={13}. [J_7]θ ={(11, 2, 3), (1, 12, 13)}, [J_8]θ ={(3, 1, 12), (2, 1, 13)}, J_{13}=(1, 2, 3).

13. R={{11, 2, 3}, {1, 12, 13}, {1, 2, 13}}; Z_{1s}={11}, Z_{2s}={12}, Z_{3s}={13}. [J_7]θ ={(11, 2, 3), (1, 12, 3), (1, 2, 13)}, J_{14}=(1, 2, 3).

14. R={{1, 12, 13}, {11, 12, 3}, {11, 2, 13}}; Z_{1s}={11}, Z_{2s}={12}, Z_{3s}={13}. [J_7]θ ={(1, 12, 13), (11, 12, 3), (11, 2, 13)}, [J_{11}]θ ={(3, 1, 12), (2, 1, 13), (2, 3, 11)}, J_{15}=(2, 1, 3).

15. Fig.3 shows the fragment of a part of electric system representing the circuit of the substation, the sets R = M_3 = {{14, 13}, {8, 7}, {19, 18}, {6, 18}, {19, 5}, {6, 5}, {30, 29}, {8, 13}, {19, 7}, {6, 7}, {30, 5}, {14, 7}, {8, 18}, {8, 5}, {6, 29}}; Z_5 = {18, 20, 28, 29}, Z_6 = {19, 20, 30, 31}, Z_7 = {13, 18, 26}, Z_8 = {14, 19, 27}, Z_{13} = {7, 18, 26}, Z_{14} = {8, 19, 27}, Z_{18} = {5, 7, 13, 20, 26, 28, 29}, Z_{19} = {6, 8, 14, 20, 27, 30, 31}, Z_{29} = {5, 18, 20, 28}, Z_{30} = {6, 19, 20, 31}.

List all minimal cross-sections (minimal-cut failure states), where number of elements in states R и S does not exceed quantity 2: M_2 = {{20}}, M_4 = {(26, 14), (27, 13), (26, 8), (27, 7), (26, 19), (27, 18), (28, 19), (31, 18), (13, 6), (26, 6), (28, 6), (14, 5), (27, 5), (31, 5), (28, 30), (31, 29), (30, 7), (31, 7), (28, 8), (29, 8)}, M_5 = {{27, 26}, {14, 28}, {14, 29}, {27, 28}, {27, 29}, {30, 13}, {30, 26}, {31, 13}, {31, 26}, {31, 28}}, M_6 = {{18, 14}, {19, 13}, {29, 19}, {30, 18}}. An example of the operation of a computer program for the formation of cross-sections is shown in Fig. 4.

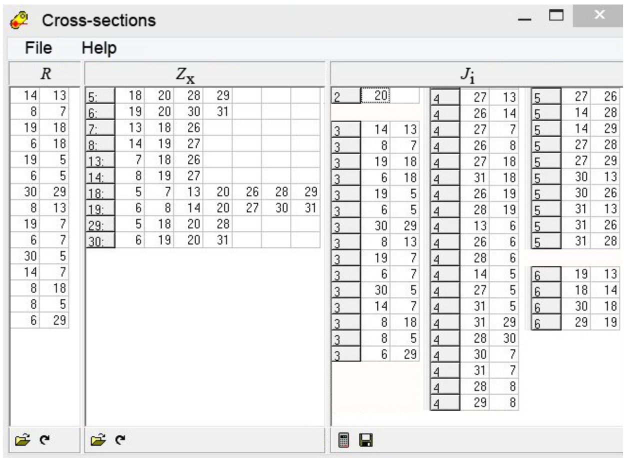

Fig.4. Computer program for the formation of minimal cross-sections (minimal-cut failure states) [J_i]θ : i = 2, 3, 4, 5, 6

Conclusions

The concept of a cross-section of a complex system is introduced and a classification of sections is obtained based on the contribution to the resulting reliability indicators.

Fifteen different classes of sections are identified in the set of one-, two- and three-element sections, and a combinatorial algorithm for the formation of these classes is developed.

The presented model can be used to form failure states of real electric power systems.

REFERENCES

[1] Endrenyi J., Reliability Modeling in Electric Power Systems, John Wiley, New York, 1978

[2] Billinton R., Allan R., Reliability Evaluation of Power Systems, Springer Science+Business Media: New York, 1996

[3] Grishkevich A., Simulation models for assessment of structural reliability indicators distribution for power supply systems, 2019 Applications of Electromagnetics in Modern Engineering and Medicine / ed. by Korzeniewska E., Janów Podlaski: Institute of Electrical and Electronics Engineers, 2019, s.52-55