**1. Mykhaylo ZAGIRNYAK, 2. Viktor LYASHENKO, 3. Elena KOBILSKAYA**

Kremenchuk Mykhailo Ostrohradskyi National University,
ORCID: 1. 0000-0003-4700-0967; 2. 0000-0002-4538-631X;3; 0000-0002-4210-1534

doi:10.15199/48.2022.11.18

# Modeling of a wire antenna electromagnetic field

***Abstract**. A mathematical model of the electromagnetic field of a wire antenna in a steady (stationary) mode of radiation is built, which is presented in the form of a system of Maxwell's equations. Analytical representations are found for the components of the electric field strength of the wire antenna, which allowed calculating the value of the current along the antenna. The method used to calculate the electromagnetic field strength of a wire antenna can also be used when calculating the electromagnetic field with more complex boundary conditions. Numerical calculations of the current distribution along the antenna given in the paper are close to the experimental values.*

***Streszczenie.** Zbudowano matematyczny model pola elektromagnetycznego anteny drutowej, który przedstawiono w postaci układu równań Maxwella. Dla składowych natężenia pola elektrycznego anteny drutowej znaleziono analityczne odwzorowania, co pozwoliło na obliczenie wartości prądu wzdłuż anteny. Metoda wykorzystywana do obliczania natężenia pola elektromagnetycznego może być również wykorzystywana do rozwiązywania problemów o bardziej złożonych warunkach brzegowych. Podane w pracy obliczenia numeryczne rozkładu prądu wzdłuż anteny są zgodne z danymi eksperymentalnymi. (**Modelowanie pola elektromagnetycznego anteny**)*

**Keywords:** wire antenna, mathematical model, current distribution, electric field strength.
**Słowa kluczowe:** antena drutowa, model matematyczny, rozkład prądu, natężenie pola elektrycznego.

## Introduction

In modern radio engineering systems, there are trends towards the improvement of wire antennas, which in turn necessitates the modeling of the electromagnetic field of antennas and a detailed research of their characteristics. Linear type antennas are characterized by the fact that their cross-sectional dimensions are small compared to the wavelength. Usually, such antennas are made from a piece of wire or several wires or in the form of a rod. A vertical wire antenna in the form of an asymmetric vibrator is considered in this paper. An asymmetrical vibrator is a vibrator located above a conductive surface, which is connected at one end to the feeder conductor, and the other is connected to a conductive surface, for example, the ground or the body of the object. Restrictions on physical parameters determine the range of use of the antenna [1-5]. The following requirements are put forward to wire antennas: simplicity of design and operation; symmetrical directional diagram in the plane of the electric field strength vector; circular directional diagram in the plane of the magnetic field strength vector; the presence of adjustment elements; specified level of radiated power (for transmitting antennas); high efficiency coefficient, steady mode of operation in case of changing weather conditions (rain, snow); low level of cross-polarization radiation; convenient mounting on a support, providing protection against lightning and static charges.

It should be noted that in most papers, the wire antenna is researched taking into account the Pocklington integral equation, for the solution of which the method of moments is usually used [6].

Thus, in [6], the current distribution on the surface of the antenna grid is calculated analytically by solving the Pocklington integral equation. The Galerkin method is used for the numerical solution of the Pocklington integral equation. An algorithm for solving Pocklington's integral equation is proposed, numerical calculations are performed and the results of analytical and numerical solutions are compared.

In [7], a dipole antenna made of aluminum and iron that operates at the frequency of a TV channel (450–950 $MHz$) is considered. The study of the dipole antenna was carried out by observing the bandwidth, return loss and radiation.

In [8, 9], the method of moments is used to calculate the current along the antenna. In [8], a dyadic Green's function was constructed to obtain the solution of the wave equation, with the help of which the numerical solution of the problem of calculating the electromagnetic field of wire antennas, regardless of their shape and position, was obtained.

In [9], dipole antennas are modeled using the integral equations of Gallen and Pocklington. The Pocklington equation is solved in the MATLAB software environment.

Numerical analysis of a dipole antenna is presented in [10]. The method of moments and the method of finite elements are used to obtain the values of the antenna characteristics. The basic characteristics of antenna are analyzed on the basis of numerical calculations.

In [11], a description of the method and results of calculating the parameters of the wire antenna based on the application of the finite element method is given.

The disadvantage of the numerical methods for calculating the parameters of the wire antenna proposed in [6-11] is that they allow solving partial problems, which makes it impossible to analyze the phenomenon as a whole. To research the electric field of an arbitrary wire antenna, it is necessary to solve many partial problems and then generalize them. However, the analysis can be carried out more effectively using an analytical solution.

In [12], using the Fourier method, the authors managed to obtain an analytical solution for determining the azimuthal component of the magnetic field strength of a wire antenna in the form of an infinite series. This paper is a continuation of [12]. Using the solution of the magnetic field strength problem obtained in [12], we obtain the characteristics of the electromagnetic field of the wire antenna, in particular the electric field strength.

## Purpose of the paper

The purpose of the paper is to develop a mathematical model of the electromagnetic field of a wire antenna, finding the analytical solution for the electric field strength and calculating the current of a wire antenna.

## Physical model

A wire antenna represented by an asymmetric vibrator located above the ground at a distance $d$ is considered. $L_A$ is the length of the antenna housing (Fig. 1). Given the axial symmetry of the antenna, the following assumptions are valid: surface electric currents together with magnetic equivalent currents are represented as an infinite function to be calculated; the surface of the earth is taken as a surface with reflective properties, which determines the symmetry of the scalar field of electric and magnetic field strengths relative to this surface.

The magnetic field of a wire antenna (Fig. 1) is a transverse magnetic field, or, in other words, the magnetic field has only one curvilinear component in the $\varphi$-direction [2], and the electric field has two orthogonal rectilinear components of the cylindrical coordinate system (Fig. 1.).

The influence of the conductivity of the grounding layer near the antenna is not taken into account, since the scattering effect can be neglected for distances close to the antenna.

From a practical point of view, the case of a thin antenna is of particular interest, for the radius of which the relations $a<<l$, $a<<\lambda$ are valid ($a$ is the radius of the antenna, $\lambda$ is the length of the electromagnetic wave in a vacuum, $l$ is the length of the antenna).

The mathematical model of the electromagnetic field of the wire antenna makes it possible to find an analytical solution to the given problem, makes it possible to study and predict the behavior of the system (Fig. 1) depending on the change of parameters and initial conditions of the problem.

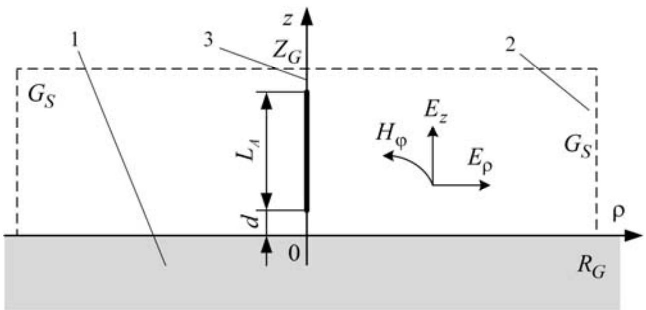



Fig.1. Scheme of an asymmetric vibrator in the form of a wire antenna.

1 – surface of the earth, 2 – external border, 3 – axis of symmetry.

**Mathematical model**

In the general case, the electromagnetic field of a wire antenna in the stationary mode of radiation is described by the following system of equations (Maxwell's equation) [13]:

$$rot\overline{H} = j\omega\varepsilon\overline{E}, \quad (1)$$

$$rot\overline{E} = -j\omega\mu\overline{H}, \quad (2)$$

$$div\overline{H} = 0, \quad (3)$$

$$div\overline{E} = 0, \quad (4)$$

where $\overline{H}$ and $\overline{E}$ are magnetic and electric field strength vectors, respectively; $\varepsilon$ and $\mu$ are, respectively, the electric permittivity and magnetic permeability of electromagnetic field; $\omega$ is the angular frequency; $j$ is an imaginary unit ($j^2$=1).

To build a mathematical model of the electromagnetic field of a wire antenna, we add to equations (1) – (4) the boundary conditions resulting from the consideration of the symmetry of the field strength $H\varphi$ with respect to the surface of the earth and that on the end surface (at the height $z$=$Z_G$) and the side surface ($\rho$=$R_G$) of the antenna, the tangential component of the tension vector is zero (the vector is orthogonal to these surfaces)

$$\left.\frac{\partial H_\varphi}{dz}\right|_{z=0} = 0, \quad (5)$$

$$\left.\frac{1}{j\omega\varepsilon}\frac{\partial H_\varphi}{\partial z}\right|_{z=Z_G} = 0, \quad (6)$$

$$\left.\frac{1}{j\omega\varepsilon}\left(\frac{\partial H_\varphi}{\partial \rho} + \frac{H_\varphi}{\rho}\right)\right|_{\rho=R_G} = 0. \quad (7)$$

The boundary condition on the axis of symmetry $z$ ($\rho$=0) is defined as the boundary condition for the minimum possible approach to the axis ("isolated axis"). If the axis is "isolated" by a cylindrical surface $\rho$=$a$, then the boundary condition can be written in the form:

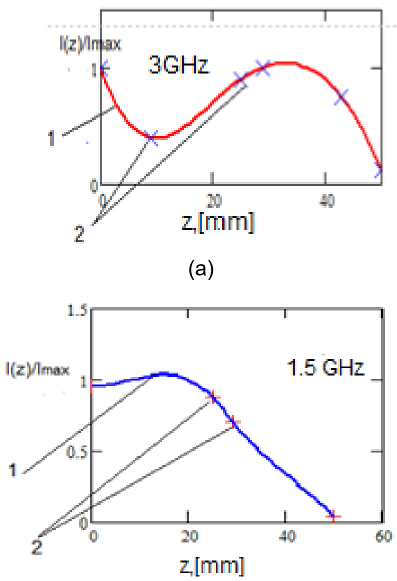
$$\left.-\frac{1}{j\omega\varepsilon}\frac{\partial H_\varphi}{\partial z}\right|_{\rho=a} = \left.E_\rho(z)\right|_{\rho=a}, \quad (8)$$

where $E_\rho(z)$ is the distribution function of the normal component of the $\overline{E}$ vector on the antenna surface. Considering the analytical representation for the magnetic field strength [12] obtained from (1) – (8), we find the components of the electric field strength vectors, namely the normal component $E_\rho$ of the electric field strength vector and the tangential component $E_z$ of the electric field strength vector. From (1) in cylindrical coordinates we have:

$$E_z = \frac{1}{j\omega\varepsilon}\frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho H_\varphi\right], \quad E_\rho = -\frac{1}{j\omega\varepsilon}\frac{\partial H_\varphi}{\partial z}. \quad (9)$$

The magnetic field strength on the surface of the vibrator determines the current in the antenna, and the normal component of the electric field strength vector on the side surface determines the linear charge density along the vibrator

$$I(z) = 2a\pi H_\varphi(a,z), \quad Q(z) = 2\pi a\varepsilon E_\rho(a,z). \quad (10)$$




(a)

(b)

Fig.2. Normalized current distribution along the antenna at a frequency value of (*a*) 3 $GHz$ and (*b*) 1.5 $GHz$, respectively.

1–calculated value of normalized current, 2–value of normalized current obtained experimentally.

Fig. 2 shows a comparison of the calculated value of the normalized current with the current obtained experimentally [14].

Calculations were made for the following parameter values and geometric dimensions $L_A$=0.005$m$, $d$=0.005$m$, $a$=0.0015$m$, $R_G$=3$L_A$, $Z_G$=3$L_A$. The control values of the current obtained experimentally (points 2 in Fig. 2) are close to the values obtained by calculation (curve 1, Fig. 2).

Tables 1 and 2 present a comparison of the calculated and experimental values of the normalized current at different frequencies [14].

Table 1. Value of normalized current, $f$=3 $GHz$

| Control point,[mm] | The calculated value of the normalized current, $I_n^r$ | Values of the normalized current obtained experimentally, $I_n$ | $\delta I_n$ |
|---|---|---|---|
| z=9 | 0.4 | 0.43 | 6% |
| z=25 | 0.9 | 0.94 | 4% |
| z=43 | 0.75 | 0.8 | 6% |

Table 2. Value of normalized current, $f$=1.5 $GHz$

| Control point, [mm] | The calculated value of the normalized current, $I_n^r$ | Values of the normalized current obtained experimentally, $I_n$ | $\delta I_n$ |
|---|---|---|---|
| z=25 | 0.87 | 0.9 | 3% |
| z=29 | 0.7 | 0.75 | 6% |
| z=43 | 0.25 | 0.27 | 7% |

Calculations in the computer algebra system Mathcad are carried out.

From the expressions (10), it obviously follows that for given values of the current (hence the charge), the magnetic field strength and the normal component of the electric field strength vector on the surface of the vibrator grow inversely proportional to its radius.

## Conclusions

A mathematical model of the electromagnetic field of a wire antenna has been built.

The peculiarity of the presented mathematical model is that when the problem is formulated, the boundary condition on the $z$ axis is obtained if the axis is "isolated" by a cylindrical surface. This makes it possible to obtain an analytical solution using the Fourier method regarding the azimuthal component of the magnetic field strength of a wire antenna in the form of an infinite sum, and then using (1) – (8) to calculate the components of the electric field strength vectors.

The approach used in modeling the electromagnetic field of a wire antenna, due to its generality, is the basis for determining the electromagnetic field of a wire antenna even under more complex boundary conditions.

Analytical representations for the normal and tangential components of the electric field strength have been obtained. This made it possible to calculate the current along the antenna, as well as the linear charge density. A comparison of the calculated values of the normalized current with the experimental values has been presented. The calculated values of the normalized current are in good agreement with the experimental values of the normalized current (the error did not exceed 7%).

***Authors**: prof. dr eng. sc. Mykhaylo Zagirnyak, Kremenchuk Mykhailo Ostrohradskyi National University, 20 Pershotravneva str., 39600 Kremenchuk, E-mail: mzagirn@gmail.com; prof. dr eng. sc. Victor Lyashenko, E-mail: viklyash2903@gmail.com; dr eng. sc. Elena Kobilskaya, kobilskaya1983@gmail.com.*

REFERENCES

[1] Sheikholeslami Kandelousi M., Electric Field. London, IntechOpen, 2018. p. 322.
[2] Sahraei A., Hadi A., Reza Y., A simple cavity method which increases bandwidth of cylindrical dipole antennas to almost 100%, *IET Microwaves, Antennas& Propagation*, (2022), 526–536.
[3] Amani N., Jafargholi A., Pazoki R., Abroadband VHF/UHF-loaded dipole antenna inthevicinity of ahum an body, *IEEE Trans.Antenn. Propag,* 65(10)(2017), 5577–5582.
[4] Stumpf M.,Time-Domain Electromagnetic Reciprocity in Antenna Modeling, IEEE Press – Wiley, 2019, 256 p.
[5] Yazdani R., A compacttriple-band dipole array antenna for selected sub1GHz, 5Gand WiFi access point applications, *IETMicrow. Antennas Propag*, 15(2021), 1866–1876.
[6] Parhizgar Naser, Calculating surface current distribution in antenna array in the presence of mutual coupling by analytical solving of Pocklington's integral equation, *Archives of Electrical Engineering,* 67(2018), 65–79.
[7] Rahmatia S., Fransiska D. W. E., Pratama N. I. H., Wulandari P., Samijayani O. N., Designing dipole antenna for TV application and rectangular microstrip antenna working at 3 GHz for radar application, *2017 5th International Conference on Cyber and IT Service Management (CITSM)*, 2017, 1–6.
[8] Eszes A., Maros D., Determine the Current Distribution for an Arbitrary Shaped Thin-Wire Antenna by Solving Its Dyadic Green's Function. *IEEE International Conference and Workshop in Óbuda on Electrical and Power Engineering (IEEE CANDO-EPE 2019)*, (2019), 000011–000016.
[9] Qiaoqiao Li, Modeling and Simulation of Current Distribution of Wire Antenna Based on MOM. *7th International Conference on Education, Management, Computer and Society (EMCS 2017)*, 2 (2017), 1057 – 1062.
[10] Karlo Queiroz da, Gleida Tayanna Conde de, Gabriel Silva Pinto, Andrey Viana Pires. Numerical Analysis of Broadband Dipole-Loop Graphene Antenna for Applications in Terahertz Communications. *Antennas and Wave Propagation*, (2018), 3 – 18.
[11] Marinović, Tomislav, Maaskant, Rob, Mittra, Raj, Vandenbosch Guy., Comparison of CBFM-Enhanced Iterative Methods for MoM-Based Finite Antenna Array Analysis. *IEEE Transactions on Antennas and Propagation,* (2021), *3538 – 3548.*
[12] Zagirnyak M., Branspiz Y., Kuczmann M., Calculation of magnetic component of wire antenna electromagnetic field. *Przegląd Elektrotechniczny*, 89(2013), nr. 2b, 21–24.
[13] Lyashenko V., Kobilskaya E., Zaika A., Demyanchenko O., Hryhorova T., Mathematical Model of heat Transfer in an Electric Machine. *AIP Conference Proceedings(AMiTaNS'18),* 2025(2018), 080006-1–080006-7.
[14] Kuczmann M., Feeding models of wire antennas. Przeglądu, *Przegląd Elektrotechniczny*, 3 (2011), 107–110