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Rotating machinery single plane balancing non-contact method

Abstract. *The paper introduces a method for single plane balancing of rotating machinery using order analysis of data acquired by the proximity probes. It provides a detailed description of calculating the unbalance magnitude and location followed by the description of the algorithm responsible for distribution of discrete trim weights values in discrete locations in order to counter the unbalance. The method could be used when the bearings of the rotating machinery are not accessible for accelerometers installation, when great accuracy is needed or when there are only a few angular locations available for installation of few possible trim weights. Experiments conducted using the method helped to reduce the vibration caused by unbalance by the order of three to ten times (depending on the initial vibration) in every case which proves its effectiveness.*

Streszczenie*. Artykuł ten opisuje metodę wyważania jednopłaszczyznowego maszyn wirujących wykorzystującą analizę rzędów na podstawie danych z czujników zbliżenia. Artykuł przedstawia dokładny opis obliczania amplitudy i kąta niewyważenia oraz opis algorytmu odpowiedzialnego za rozmieszczenie dostępnych mas w dostępnych kątowych lokalizacjach w celu minimalizacji niewyważenia. Zaprezentowana metoda może zostać wykorzystana, gdy nie ma możliwości zainstalowania akcelerometrów na łożyskach maszyny wirującej, gdy potrzebna jest duża dokładność lub gdy konstrukcja maszyny wirującej pozwala jedynie na instalację określonej kombinacji mas w określonych miejscach. W przypadku wszystkich prób wyważania za pomocą przedstawionej metody udało się ograniczyć wibracje spowodowane niewyważeniem o trzy do dziesięciu razy (w zależności od początkowego poziomu wibracji), co dowodzi skuteczności metody.* ***(Metoda bezkontaktowego wyważania jednopłaszczyznowego maszyn wirujących)***

Keywords: rotating machinery, balancing, order analysis, polar plots, combinatorics.
Słowa kluczowe: maszyny wirujące, wyważanie, analiza rzędów, wykresy polarne, kombinatoryka.

Introduction

The unbalance is an unwanted property which unfortunately all the rotors (some more then others) posses. If not treated properly the unbalance will always cause an undesirable rotor vibration, especially when rotating at or close to the critical frequency [1]. Therefore, the balancing is the process of attenuating the inherent vibration caused by unbalance. It's done by adding or removing mass from the rotor in order to make the mass distribution even about the rotation centerline. The purpose of the method presented is to find how much mass and at what angular location. It's important to state here, that the vibration caused by the unbalance can only be attenuated, it cannot be entirely removed as no matter how precise the balancing, there will always be some unbalance left in the rotating system [2].

The usual approach when dealing with unbalance is to use a dedicated balance machine. These are specialized devices designed to adopt a wide variety of rotors of different size and shape. They typically consist of a variable frequency drive that controls the motor which drives a gearbox. The rotor being balanced needs to be disassembled from the rotating system it's a part of and then placed on the supports of the balance machine. Then the rotor is coupled to the drivetrain of the balance machine, usually by a belt. The balancing itself is performed by spinning the rotor and reading the data from accelerometers installed on the supports and the data from a sensor that detects a specific spot on the rotor. The spot detection is then used in the calculation of the rotor speed and angle [2]. Balance machines can help to greatly reduce the unbalance and they should be used whenever possible, however there are two considerable drawbacks associated with them – first, they are very expensive and second, the rotor needs to be disassembled before the balancing.

The method presented in this paper can give the same results as a balance machine in terms of reducing the unbalance level, but it is both inexpensive and not invasive. All it takes to conduct the balancing is a data acquisition system and two sensors that don't even need to be coupled to the rotor as they are the eddy current and inductive probes. This means that the balancing could be performed on any rotating machinery in the field as long as there is an access to the rotor.

Test stand

The test stand (Fig. 1) used to research and verify the presented balancing method consist of variable frequency drive (VFD), asynchronous electric motor capable of spinning up to 3000 rpm, rotor being balanced, 8 steel wheels (each weighing 1 kg) that could be installed on the shaft in order to simulate different rotors, inductive proximity sensor with PNP output (Panasonic GX-M12A-P-Z), eddy current proximity probe connected to a dedicated transducer with analog 0÷10VDC output (Bently Nevada 3300XL), programmable automation controller (PAC, National Instruments cRIO-9063) and a PC.

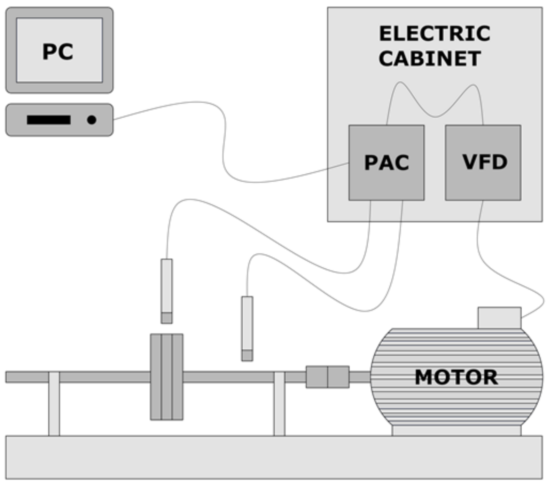

Fig. 1 – test stand configuration (3 steel wheels installed)

The PNP proximity sensor is mounted on an Ω shaped bracket directly over the shaft and detects a notch every revolution (Fig. 2). Signal from this probe is acquired by National Instruments cRIO-9375 digital input-output card which allows for the measurements to be taken every 7μs (around 140kS/s). Given that the maximum motor speed is 3000 rpm (50Hz), a decision has been made to limit the sampling rate to 20kS/s. Even when running the motor at 50 Hz, the system acquires 400 samples each revolution which allows to detect the notch with accuracy better than 1 degree. However, this is the worst case scenario -

when spinning the motor at lower rotational speed, the accuracy of the notch detection is getting higher.

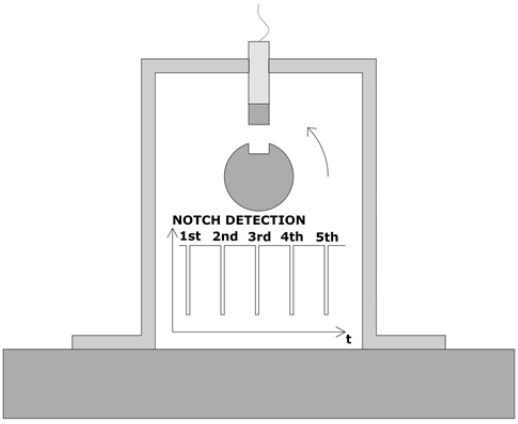

Fig. 2 – PNP inductive proximity sensor assembly

The analog eddy current proximity probe (latter referred to as analog proximity probe) is also mounted on an Ω shaped bracket (Fig. 3). It detects a change in proximity to one of the steel wheels mounted on the rotor. The sampling rate has been set at 20 kS/s, however the data acquisition system is capable of acquiring the data at higher rates (up to 100 kS/s for analog signals). If the maximum operating speed of the rotor is higher than 3000 rpm, then the sampling rate can be adjusted accordingly. The measuring range of this probe is 200÷3000 μm, however not the entire range is linear. Through a calibration using certified height gauge the linear range of the proximity probe has been established to be 400÷2800 μm. The clearance of the analog proximity probe to the steel wheel, when the rotor is not spinning, was set right in the middle of this linear range. In each steel wheel there are 16 evenly distributed angular locations where balance weights might be added in form of bolts and washers.

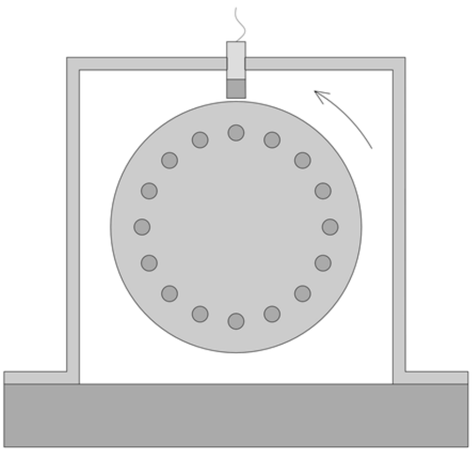

Fig. 3 – analog eddy current proximity probe assembly

Order analysis

Vibration measurements can be acquired either as an acceleration, velocity or displacement [3]. The method presented in this paper uses displacement (measured by the analog proximity probe) as a source of vibration information, however both velocity and acceleration could be calculated be means of differentiation. Magnitude and phase of the vibration at frequency of rotation are the indicators of an unbalance [4]. The phase is measured as relative to a known zero angle, in this case a notch on the shaft detected by the PNP proximity sensor. When analyzing the magnitude and phase response associated with the rotor weight distribution the total unbalance can be considered as a single heavy spot in the rotor. Important thing to remember is that before the critical speed the heavy spot will fly out and after the critical speed the point opposite to the heavy spot will fly out. At exactly the critical speed the heavy spot will lead the geometric center of the rotor by 90°. Another way of thinking about this is that there is a 180° phase shift when going through a resonance. The highest vibration magnitude will almost always be detected at critical speed so the phase measured at this speed needs to be adjusted by 90° to properly determine the angle of unbalance.

To perform the balancing it is required to run up to the maximum operating speed while acquiring the measurements from both proximity probes. As the speed changes so is the frequency at which the magnitude and phase associated with the unbalance shall be calculated. Because of the changing speed simple application of the Fast Fourier Transform [5] to the acquired measurements gives a meaningless data.

Fig. 4 shows an example of sine wave signal which frequency increases linearly from 1 Hz to 40 Hz. Even though the amplitude of the sine wave is not changing the frequency spectrum of the signal shows that it does.

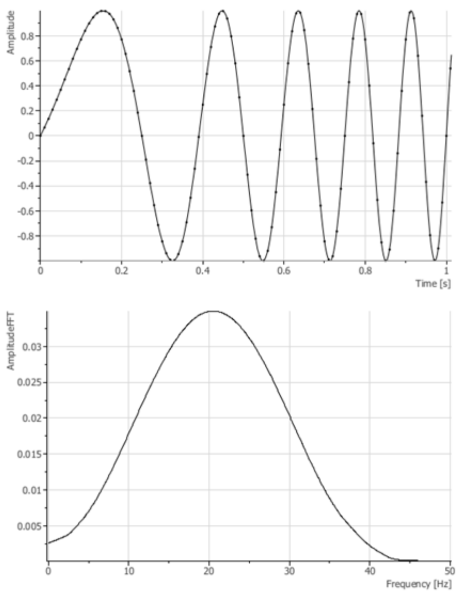

Fig. 4 – time domain signal and frequency response of a sine wave with linearly increasing frequency

To make the data useful for balancing purposes it needs to be represented in a different form. Instead of calculating components that are periodic in time (FFT applied to a time domain signal), components that are periodic in rotational angle shall be calculated. This technique is called order analysis [6]. Just as a signal of one Hertz frequency takes one second to finish a full cycle, a first order signal takes one full revolution to finish a cycle. It is therefore a perfect tool for balancing as the first order signal is the direct representation of the unbalance.

To perform the order analysis the first step is to resample the time domain signal in such a way that each sample isn't taken with equal time intervals but with equal angle steps. This is done by splitting the time domain signal into segments, each one revolution long. A rising edge of the signal acquired by the PNP proximity sensor marks a beginning of each segment and it is therefore the zero degree angle reference. By calculating the time intervals between each following rising edges the rotational speed can be calculated. An arbitrary number of evenly distributed angle locations is then chosen (for example 360, meaning that there should be a sample taken each one degree). Given the zero degree angle reference and the speed measurements of the current and previous revolutions, exact moments of signal being at the specific angles are calculated. Last step is to sample the time domain signal at the calculated moments. The output of the whole process is a so called even angle signal. By taking the FFT of this signal the order spectrum is obtain. Fig. 5 shows the signal from

Fig. 4 resampled into the even angle domain. Even though the frequency of the signal in the time domain is changing, the order response shows a single peak at first order of amplitude equal to the amplitude of the signal.

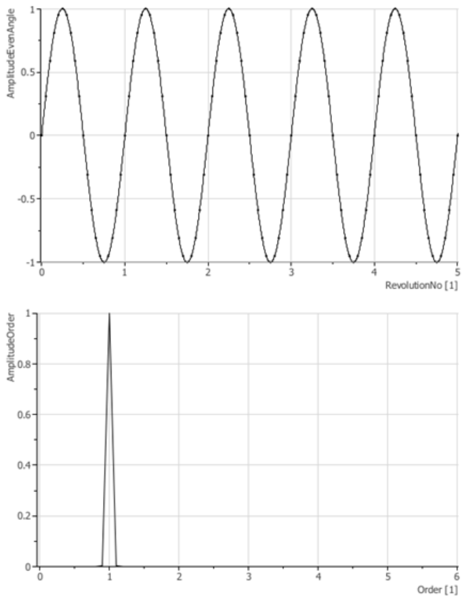

Fig. 5 – even angle signal and order response of a sine wave with frequency increasing linearly in the time domain

Phase lead versus phase lag

Given two periodic signals phase can be defined as a relative shift between them. If the signals are periodic in time domain, then the phase is measured in seconds, however if the signals are both even angle signals then the shift is measured in degrees (or radians, depending on the unit chosen). In the case of the presented method a more meaningful way of thinking about the phase is as the angle difference of the first order waveform peak and the zero angle reference point. By calculating the first order magnitude and phase of the vibration signal both the magnitude and location of the unbalance can be estimated [6].

There are two phase measuring conventions, phase lead and phase lag, and it is very important not to confuse them. Fig. 6 shows the difference between them. The rotor is spinning counter-clockwise, the blue box is representing the notch and the red circle is representing the heavy spot. As the rotor spins, the analog vibration signal can be viewed either as leading the notch detection signal (peak of the vibration signal before the notch signal) or lagging behind it (peak of the vibration signal behind the notch signal). Generally speaking both conventions can be used interchangeably as long as only one of them is used throughout all the calculations. The presented method, and therefore all the software associated with it, uses phase lag convention.

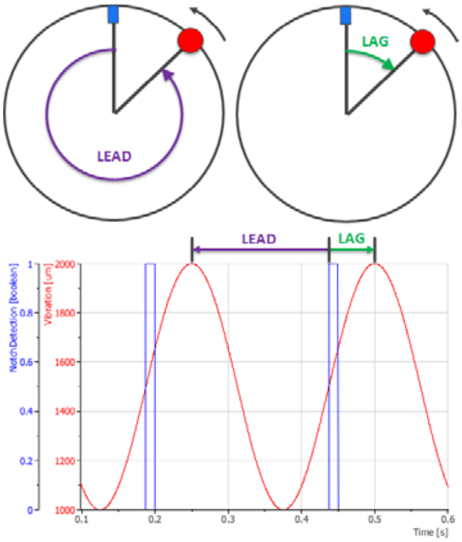

Fig. 6 – phase lead versus phase lag comparison

Bode and polar plots

When analyzing a single order magnitude and phase data of a signal acquired during rotor run up or coast down it is very useful to plot the data on a Bode plot. This kind of plot is a perfect tool to examine a rotating system behavior and to identify the resonance speeds. Magnitude and phase of the given order are plotted as a function of time, frequency or speed [6]. Fig. 7 shows a Bode plot of data acquired during coast down of the test stand with all eight steal wheels installed. The resonance speed is clearly visible around 2830 rpm.

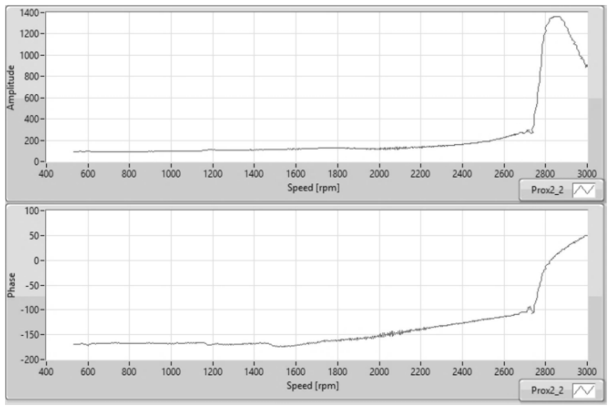

Fig. 7 – Bode plot example

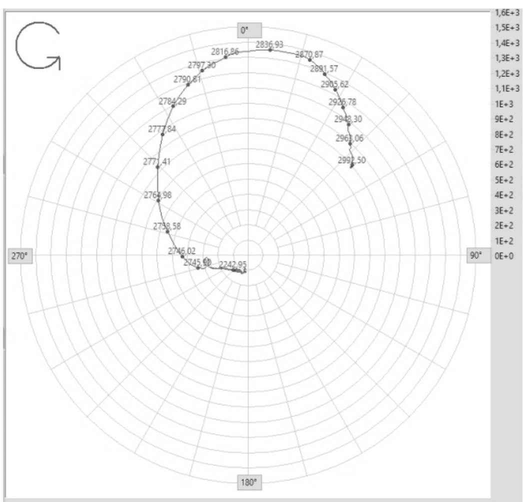

Fig. 8 – polar plot example

The data from the Bode plot can be viewed in a different, very useful form. While the Bode plot is a very powerful tool when looking at the magnitude and phase data with respect to time, frequency or speed, a polar plot is better at representing the magnitude with respect to the phase. As the name suggest, on a polar plot the data is displayed using polar coordinates with phase always wrapping within 0÷360° range. Fig. 8 shows the data previously plotted on the Bode plot, this time displayed in polar coordinates. The location of the 180° phase shift that occurs while going through a resonance is clearly visible when the data is plotted in this manner.

Balance weight calculation

First step of the method is to run the rotor up to the maximum operating speed while acquiring the data from both PNP proximity sensor and analog proximity probe. A separate application has been prepared for control of the test stand used to develop the method, which allows to turn on/off the variable frequency drive, turn on/off the motor, set the motor speed, set the acceleration ramp and choose the direction of rotation. The same piece of software performs also a more generic function of data acquisition and rotor monitoring during the run. The use of the control section of the application is optional – if the rotor is equipped with its own control system, then only the two proximity probes need to be installed on the rotor. If there is no notch or tooth on the shaft for the purpose of the zero degree angle detection, a piece of reflective tape can be adhered to the shaft and the inductive PNP proximity sensor can be exchanged with an optical PNP sensor. When running that kind of a system the data acquisition and monitoring part of the software can be used, while the control section remains inactive. The data is stored in TDMS file format.

Second step of the method is to analyze the data from both proximity probes. Figure 9 shows the controls of the order analysis application that has been prepared to perform this step automatically. First the notch detection signal has to be converted into a proper tachometer signal by setting correct values in the *Tacho Signal Parameters* cluster. The notch is detected when signal from the PNP proximity sensor changes value from 0 to 1 (a rising edge), therefore a value exactly in the middle should be used as a threshold. There is only one notch in the shaft so the signal has one pulse per each revolution. If the notch detection signal would be analog, then the hysteresis should be also provided. In case of digital signal, hysteresis can be set to zero. When the tacho signal is loaded it can be checked on a time chart in the order analysis application. If the tacho signal appears correct, then order analysis should be properly set in the *Bode Analysis Parameters* cluster. We are interested only in the first order, with a very narrow bandwidth and we want the rotor speed to be the X axis of the Bode plot. Both PNP proximity sensor and analog proximity probe are installed on the same angular location so there is no shift between them (0° in the *Ang Location* control). The direction of the rotor spinning was counter-clockwise. The data was recorded starting at the minimum speed of 0 rpm and finishing at maximum speed of 3000 rpm with a fairly steep acceleration ramp, therefore the rotation speed was changing quickly. Taking this into account the order magnitude and phase should be calculated very often to populate the whole speed range. In the example shown in Figure 9 the time width of the analysis was set to 0.01 s meaning the first order magnitude and phase was calculated with 10 ms intervals – with the signal sampling rate of 20 kS/s this is equal to 200 samples in the time domain. When all the parameters are set the data can be viewed on the Bode plot and the polar plot within the order analysis application (Fig. 7 and Fig. 8 respectively). Before continuing the first order magnitude and phase at the critical speed should be recorded. If the resonance didn't occur then first order magnitude and phase should be recorded for any arbitrary chosen speed.

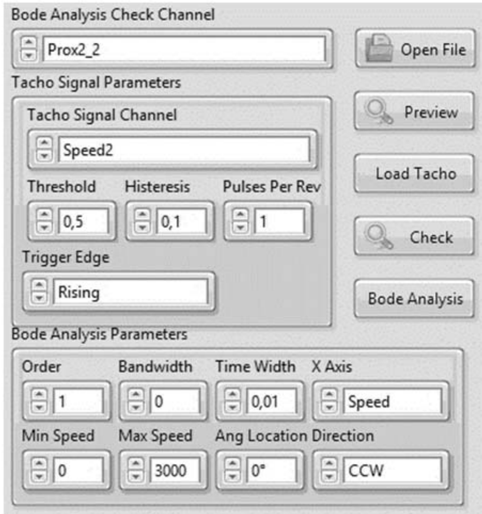

Figure 9 – front panel of the order analysis application

Third step of the method is to install a calibration balance weight. It shouldn't be heavy, as adding too much weight could have catastrophic results when rotor is accelerated to the maximum operating speed. The unit of the balance weights should also be chosen. Any metric or imperial weight unit can be used as long as it is not changed anytime during the calculations. It is a common practice to specify the balance weights as a product of mass and eccentricity and therefore use units such as gram-millimeters, pound-inches or even a combination of the two, gram-inches [7]. If not stated otherwise, all the balance weight values in this paper are given in gram-millimeters (abbreviated g-mm). For example a balance weight of 200 g-mm means a 5 g bolt installed on a 40 mm radius. The angle of balance weight location might be chosen at random or it might be estimated from the Bode and polar plots. We know that before the critical speed the heavy spot of the rotor flies out so the first order phase should indicate the unbalance location. However, using a proximity measurement as the source of vibration information needs to be taken into account. Signal from the proximity probe has a positive peak when the rotor is far from the probe and a negative peak when the rotor is close to the probe. When the heavy spot flies out its angular location can be estimated when it is closest to the probe so at the negative peak of the signal. Order analysis calculates the phase as a location of the positive peak, therefore 180° needs to be subtracted from the phase to estimate the location of the unbalance. Balance weight should be installed opposite side to the unbalance so another 180° needs to be subtracted. Together those two subtractions rotate the phase a full circle so a rough estimation of the calibration balance weight location can be read directly from the Bode plot as the first order phase before the critical speed.

Fourth step of the method is to perform steps one and two again, this time with the calibration balance weight added to the rotor. If during the first run the resonance didn't occur, then during the second run the first order magnitude and phase should be recorded for the same speed as before. To calculate the final balance weight, the following equations shall be used [8]:

(1) $$\vec{V}_{\Delta} = \vec{V}_2 - \vec{V}_1$$

(2) $$\vec{S} = \frac{\vec{V}_{\Delta}}{\vec{W}_C}$$

(3) $$\vec{W}_B = \frac{\vec{V}_1}{\vec{S}}$$

where: $\vec{V}_1$ – magnitude and phase of the first order vibration recorded during the first run (represented as a vector); $\vec{V}_2$ – magnitude and phase of the first order vibration recorded during the second run (represented as a vector); $\vec{V}_{\Delta}$ – magnitude and phase of the vibration delta between second and first run (represented as a vector); $\vec{S}$ – magnitude and phase of the sensitivity (represented as a vector); $\vec{W}_C$ – magnitude and phase of the calibration balance weight installed (represented as a vector); $\vec{W}_B$ – magnitude and phase of the final balance weight (represented as a vector)

The balance weight calculation application has been prepared to perform this step automatically. Fig. 10 shows a front panel of this application. To calculate the final balance weight, the magnitudes and phases of the first order vibration (recorded during the first and second runs) and the calibration balance weight have to be set in the *Calculation Data* cluster. The application then calculates the final balance weight prediction and writes its magnitude and phase into a *Trim Balance* cluster. It also plots all the values (first run response, second run response, calibration balance weight and final balance weight) in vector form on a compass plot.

The final step of the method is to remove the calibration balance weight, install the final balance weight and perform a balance check run. If the vibration level isn't low enough the method can be used again. At the end of each method iteration the new final balance weight should be added to the rotor without removing the final balance weight added during the previous iteration. Different approach would be to write the magnitude and phase of the previously calculated balance weight into a *Trim Offset* cluster of the balance weight calculation application and let the software calculate the combined balance weight of current and previous iterations. Table 1 presents the values recorded during the balancing of the test stand rotor with all eight steel wheels installed. Three iterations of the method has been performed with calibration balance weights added during runs 2, 4 and 6. Overall the vibration level at the resonance speed has been reduced to 25.6% of the initial vibration level.

Table 1 – data recorded during balancing of the test stand with all eight steel wheels installed

Run	Vibration at resonance speed		Calibration balance weight		Iteration final balance weight		Combined final balance weight	
	Mag.	∠	Mag.	∠	Mag.	∠	Mag.	∠
1	1362	13.5	-	-	-	-	-	-
2	1628	184	202.5	270	-	-	-	-
3	987	192	-	-	92.6	275.2	92.6	275.2
4	1370	188.5	36	0	-	-	92.6	275.2
5	536	76.2	-	-	91.2	192.4	137.9	234.2
6	1079	31.5	36	135	-	-	137.9	234.2
7	349	187	-	-	24.3	28.1	116.6	239.5

Balance weight distribution

It is seldom the case that the exact calculated value of balance weight can be installed at the exact calculated angle location. Usually there is a finite number of locations distributed evenly over a specified radius where bolts and washers can be installed. This means that there is also a finite number of possible weight configurations that can be installed in one place. This presents a challenge when trying to distribute the weights in order to match the calculated balance weight. Table 2 shows the possible weight configurations that were available on the test stand used to develop the method.

Table 2 – balance weight configurations

No	Configuration	Radius	Weight
1	1 bolt	45 mm	202.5 g-mm
2	1 bolt, 1 small washer	45 mm	238.5 g-mm
3	1 bolt, 2 small washers	45 mm	274.5 g-mm
4	1 bolt, 3 small washers	45 mm	310.5 g-mm
5	1 bolt, 1 big washer	45 mm	337.5 g-mm
6	1 bolt, 1 big washer, 1 small washer	45 mm	373.5 g-mm
7	1 bolt, 1 big washer, 2 small washers	45 mm	409.5 g-mm
8	1 bolt, 1 big washer, 3 small washers	45 mm	445.5 g-mm
9	1 bolt, 2 big washers	45 mm	472,5 g-mm
10	1 bolt, 2 big washers, 1 small washer	45 mm	508.5 g-mm

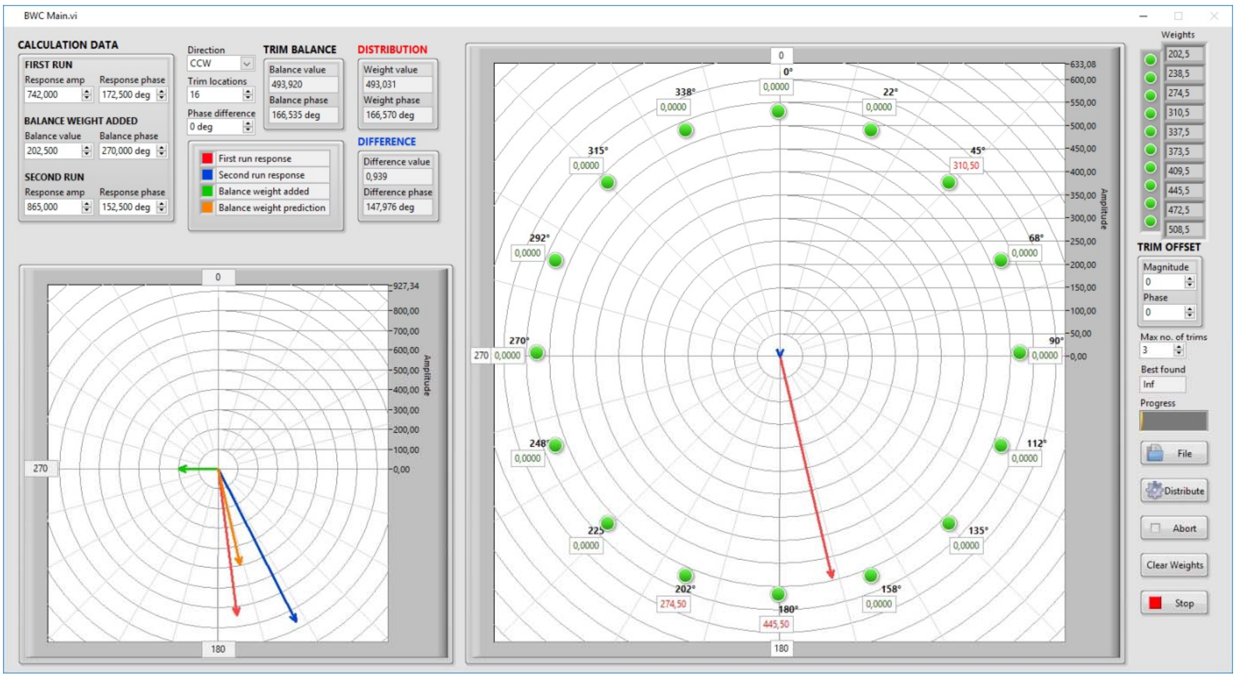

Fig. 10 – front panel of the balance weight calculation and distribution application

To facilitate easy balance weight distribution, the balance weight calculation application is also capable of performing this step automatically. Fig. 10 shows the front panel of the application, where the whole right-hand side is dedicated to the balance weights distribution. The application allows to specify how many evenly distributed angular locations there are and to offset the zero angle position. User can then manually input a specified balance weights into the boxes representing each angular location and observe the overall weight distribution (red arrow on the right-hand side compass plot) and how much it deviates from the sought balance weight distribution (blue arrow on the right-hand side compass plot). The easier way would be to use the implemented algorithm that uses a brute force approach and tries every possible balance weight combination to find the best match. To use this method first step is to read the possible balance weight configurations from a spreadsheet file. If for some reason any of these configurations is not possible to use then it can be disabled via a button right next to it. The same goes for the angular locations – if it is not possible to install balance weight in any particular location, then it can be disabled. By clicking the *Distribute* button the user can start the algorithm. At any given time the combination with the smallest deviation to the sought distribution is being displayed in the *Best found* box, therefore the user can stop the algorithm if the value is below some satisfactory level. The obvious drawback of this approach is the number of combinations that need to be tested. To try all the combinations it is necessary to use the variations with repetition [9] as the goal is to arrange the elements that can repeat (the balance weights configurations) in a specific order (angular locations). By using equation (4), the number of such variations can be calculated.

$$\bar{V}_n^k = n^k \tag{4}$$

where: n – number of balance weight configurations; k – number of angular locations

Given the example shown on the front panel of the application in Fig. 10, there are 11 balance weight configurations (10 that were read from a spreadsheet file plus an absence of balance weight or, looking at it differently, a balance weight value of 0) that can be installed at 16 different angular locations, so there are around 4.6×10^{16} possibilities to try. That's way too many even for the fastest computers. However, if we decide that out of all the available angular locations, the algorithm is only permitted to put balance weights in couple of them at the time, then the number of possibilities reduces dramatically. In order for this to work first we need to find all the combinations of balance weights in the reduced maximum number of angular locations so we need to use the so called combinations with repetition [9]. Equation (5) gives a total number of such combinations. Then the balance weight combinations need to be placed in all possible reduced combinations of the angular locations, therefore the so called variations without repetition [9] need to be found (there are a k number of all the angular locations but only an m out of them can be used at the time). Equation (6) gives a total number of such combinations. The total number of possibilities to try is given by eq. (7).

$$\bar{C}_n^m = \frac{(m+n-1)!}{m!(n-1)!} \tag{5}$$

$$V_k^m = \frac{k!}{(k-m)!} \tag{6}$$

$$C_{n,m,k} = \bar{C}_n^m \cdot V_k^m \tag{7}$$

where: n – number of balance weight configurations; k – number of all angular locations; m – maximum number of angular locations to use

Again using the data from the front panel of the balance weight calculation application shown in the Fig. 10 (n=11, k=16, m=3), there are 286 balance weight combinations to be distributed in 3360 angular locations combinations, therefore there are total of 960 960 possibilities for the algorithm to try. That's easy for any modern computer. Additional benefit of this approach is that it is also easier to install balance weights in fewer locations. As Table 3 shows the number of combinations to try rapidly increases when the maximum number of allowed angular locations is increased, however the balance weight distribution error only changes slightly. Usually setting the maximum number of permitted angular locations to 3 gives good enough balance weight distribution.

Table 3 – number of combinations and balance weight distribution change with different number of maximum permitted angular locations

Max	Comb.	Calculated balance weight		Balance weight distribution		Distribution error	
		Mag.	∠	Mag.	∠	Mag.	∠
1	176	493.9	166.5	472.5	157.7	79.1	236.3
2	15 840	493.9	166.5	486.8	166.4	7.2	176.0
3	960 960	493.9	166.5	493.0	166.6	0.9	148.0
4	4.37×10^7	493.9	166.5	493.8	166.5	0.1	197.4
5	1.57×10^9	493.9	166.5	493.9	166.5	0.07	297.0

Summary

The main advantage of the presented method is its simplicity. There is no need to disassemble the rotor from the rotating machine or to access the supports and bearings for accelerometers installation. All the measurements needed to perform the balancing can be acquired by just two non-contact probes which is a huge advantage. Furthermore, when the final balance weight is calculated it can be easily distributed in a broad range of different rotors thanks to the very versatile balance calculation and distribution application. Other than the test stand used to develop the method, the presented approach was also used during balancing of the low pressure compressor (the fan blades) of a turbofan jet engine while preparing for its overspeed test and several bird strike tests. The method proved to be both reliable and accurate what proves its validity.

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