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Application of the Frequency Symbolic Method for the Analysis of Linear Periodically Time-Varying Circuits

***Abstract**. The paper discusses the features of the formation of a system of differential equations describing the linear periodically time-varying circuit applying the nodal voltage analysis and their transfer to the frequency domain using L.A. Zadeh's equation. The features of formation of the transfer functions of the linear periodically time-varying circuit by the frequency symbolic method are considered. The results of solving the systems of linear algebraic equations formed during the analysis of such circuits by the frequency symbolic method are presented. The described approach is implemented in the system UDF MAOPCs (user-defined functions for multivariate analysis and optimization of parametric circuits), which is used for modelling circuits, particularly of parametric amplifiers in special purpose receivers.*

Streszczenie.** W artykule opisano opis matematyczny obwodu zmieniającego się w czasie z węzłem napięciowym oraz przejście z dziedziny czasu do dziedziny częstotliwości z wykorzystaniem ®ównań Zadeha. Analizowano trzymany system liniowych równań w dziedzinie częstotliwości. Prezentowana metoda.* ***Zastosowanie przejścia do domeny częstotliwości w analizie liniowych obwodów zmieniających się okresowo

Keywords: linear periodically time-varying circuits, frequency symbolic models, frequency symbolic method.
Słowa kluczowe: obwód zmieniający parametry okresowo, domena częsytotliwości, równania Zadeh'a.

Introduction

The frequency symbolic method (FS method) [1] is the method of formation of a time-varying transfer function $W(s,t) = y(s,t)/x(s)$ [1,2,3] of the linear periodically time-variable (LPTV) circuit in the steady-state mode in the frequency domain, where t is the time, s is the complex variable, $x(s), y(s,t)$ are the Laplacian images of the input $x(t)$ and output $y(t)$ signals. The basis for such a formation is the linear differential equation

$$(1) \quad \begin{aligned} &a_n(t)y^{(n)} + ... + a_1(t)y^{(1)} + a_0(t)y = \\ &= b_m(t)x^{(m)} + ... + b_1(t)x^{(1)} + b_0(t)x \end{aligned},$$

where $a_i(t)$, $b_j(t)$ are the known real-valued periodic functions defined by the parameters and structure of a certain circuit, m, n are positive integers, $m \le n$. According to the FS method, the transfer function $W(s,t)$ is determined from L.A. Zadeh's equation [1,2] written based on (1)

$$(2) \quad \begin{aligned} &\frac{1}{n!}\frac{d^nA(s,t)}{ds^n}\frac{d^nW(s,t)}{dt^n} + ... \\ &+ \frac{dA(s,t)}{ds}\frac{dW(s,t)}{dt} + A(s,t)W(s,t) = B(s,t) \end{aligned},$$

as a result of its approximation by a trigonometric or complex Fourier polynomial,

$A(s,t) = a_n(t)s^n + a_{n-1}(t)s^{n-1} + ... + a_1(t)s + a_0(t),$

$B(s,t) = b_m(t)s^m + b_{m-1}(t)s^{m-1} + ... + b_1(t)s + b_0(t)$.

The number of harmonic components taken into account in the Fourier polynomial defines the accuracy of the transfer function approximation.

The transfer function $W(s,t)$ determines the output signal $y(t)$. So if $x(s) = e^{st}$ and $s = j\omega$, then

$$(3) \quad y(t) = \mathrm{Re}[W(s,t) \cdot e^{st}].$$

In [4], the FS method is extended to the case when the circuit is not described by one equation (1) relative to the external variables $x(t)$ and $y(t)$, but by a system of linear differential equations (SLDE) describing such a circuit in relation to all its external and internal variables. This eliminates the need for the formation of the equation (1), which is usually formed from such SLDE, by excluding the internal variables from it. This approach leads to the formation and necessity of solving the L.A. Zadeh's equation in a matrix form and transfers the elimination of the SLDE internal variables from the time domain to the frequency domain. It is known that the elimination of variables in the frequency domain is simpler, as it is performed in the system of linear algebraic equations (SLAE), not differential ones. As practice shows [5], the elimination of variables in SLAE is much simpler in terms of algorithmization and significantly reduces the computation time required for this.

The important feature of the FS method is that some or even all the parameters of the circuit elements can be presented by symbols. Therefore, the transfer functions can be formed in a partially or completely symbolic form. Such symbolic transfer functions enable solving multivariate tasks, in particular statistical analysis or optimization of devices simulated by LPTV circuits.

However, the well-known drawback of symbolic methods is a sharp increase in the time of the transfer functions formation, as the simulated circuit becomes more complex. One of the most effective ways of combating this phenomenon is the application of the so-called sub-circuit method to solving the systems of linear algebraic equations (SLAE) formed as a result of using the FS method [5]. We believe that in the case under consideration, the most effective sub-circuit method is the so-called d-tree method [5]. The d-tree method is applicable only in case when the circuit is described by a system of equations written by the nodal voltage analysis [5]. This method, however, forms integro-differential equations, not differential ones [6,7]. Generally, this is due to the presence of constant or time-varying inductances in the circuit. Therefore, the task under consideration in this paper consists in eliminating the integrals from the integro-differential equations and reducing the equations to purely differential ones in the form of SLDE.

Elimination of integrals by differentiating the equations

The most common way of eliminating integrals from integro-differential equations is to differentiate them. However, the application of this method, especially in the presence of various parametric inductances $L_1(t), L_2(t), ...$ in the circuit, may not lead to the elimination of integrals. For example, the differentiation of the fragment of the equation with the nodal voltages $u_i(t), u_j(t)$

(4) $$\cdots + \frac{1}{L_1(t)} \cdot \int u_i(t)dt - \frac{1}{L_2(t)} \cdot \int u_j(t)dt + \cdots$$

does not eliminate the integrals in it. This situation can be avoided, for example, by adding low-ohmic resistors in the circuit model in series with the inductances. The resistors will move the terms in (4) to different equations, but will increase the number of equations in the system. If the structure of the circuit is such as to allow eliminating the integrals by differentiation, the corresponding equations should be pre-multiplied by the corresponding $L_n(t)$, which makes it possible to get rid of the product of the functions and then only perform the differentiation. However, such actions often increase the unwieldiness of the equations, and, as a result, unnecessarily extend the required computation time and reduce the complexity of circuits being analysed. Thus, the differentiation of equations is not always a desirable way of eliminating the integrals from the equations and should be avoided if possible.

Another way of eliminating integrals from equations is to replace the variables. The method of replacing the variables is presented below.

Elimination of integrals by replacing the variables

1. If the equations of the circuit model compiled by the nodal voltage method contain, for example, expressions with integrals of these nodal voltages

(5) $$\frac{1}{L_n(t)} \int u_i(t)dt \,,$$

then we replace the expression $\int u_i(t)dt$ for a new variable $V_i(t)$:

(6) $$V_i(t) = \int u_i(t)dt \,.$$

If there are also $u_i(t), u_i'(t), u_i''(t), \ldots$ in the system of equations, then, according to (6), they are replaced with $V_i'(t), V_i''(t), V_i'''(t), \ldots$, respectively.

The denominators in the expressions of the type (5), which significantly reduce the accuracy and speed of further computations, can be eliminated in two ways: by multiplying the equations by $L_n(t)$ or, the problem statement permitting, by making a replacement $\Gamma_n(t) = 1/L_n(t)$. In the second case, the expression (5) takes the form

(7) $$\Gamma_n(t) \cdot V_i(t) \,.$$

Thus, the given system of integro-differential equations of the circuit transforms into a system of purely differential equations, in which some or all the variables are new and, in addition, there are no denominators.

2. The obtained SLDE is solved using the FS method, which results in the formation of the corresponding transfer functions $W(s,t)$.

3. If the obtained transfer function contains a new variable $V_i(s,t)$:

(8) $$W(s,t) = V_i(s,t)/x(s) \,,$$

then, for example, for $x(s) = e^{st}$ according to (3)

(9) $$V_i(t) = \mathrm{Re}[W(s,t) \cdot e^{st}] \,.$$

Taking into account (6),

(10) $$\int u_i(t)dt = \mathrm{Re}[W(s,t) \cdot e^{st}] \,,$$

and, accordingly, $u_i(t)$ will appear as:

(11) $$u_i(t) = (\mathrm{Re}[W(s,t) \cdot e^{st}])' \,.$$

This approach, in our opinion, offers a number of advantages as compared to the differentiation of equations, as it

- does not require compulsory multiplication of the equations of the circuit by the expressions $L_n(t)$, except for cases when this reduces the bulkiness of the equations;
- does not cause an increased bulkiness of the equations of the circuit due to their differentiation;
- does not require the addition of low-ohmic resistors into the model of parametric inductance, due to which the number of equations in SLDE does not increase.

The differentiation according to (11) is performed only once for each formed transfer function, and only of its real-value part, and therefore presents no difficulties.

Application of the differentiation methods and replacement of the variables

Figure 1 shows the simplest LPTV circuit with a time-varying inductance, which nevertheless demonstrates the idea of the ways of eliminating the integrals from the equations by differentiation and replacement of the variables.

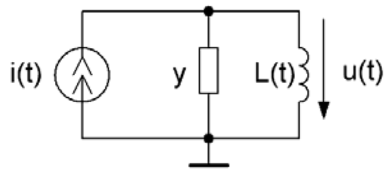

Fig1. Elementary LPTV circuit,
$L(t) = L_0(1 + m \cdot \cos(\Omega t)), L_0 = 0.2533e-6, \omega = 4 * pi * 1e8;$
$\Omega = 2 * pi * 1e8; m = 0.1; y = 1/2500; T = 2 * pi / \omega; m < 1$

The differential equation of such a circuit compiled by the nodal voltage method appears as:

(12) $$y \cdot u(t) + \frac{1}{L(t)} \cdot \int u(t)dt = i(t)$$

The method of differentiation of equations includes the following steps.

1. The equation (12) is multiplied by $L(t)$ and then differentiated, which results in:

(13) $y \cdot L(t) \cdot u'(t) + [y \cdot L'(t) + 1] \cdot u(t) = L(t) \cdot i'(t) + L'(t) \cdot i(t)$.

2. From (13), L.A. Zadeh's equation is obtained:

(14) $$\begin{aligned} &yL(t)W'(s,t) + [yL(t)s + yL'(t) + 1]W(s,t) = \\ &= L(t)s + L'(t) \end{aligned} \,,$$

where $W(s,t) = u(s,t)/i(s)$, $u(s,t)$ and $i(s)$ are the Laplacian images of the time variables $u(t)$ and $i(t)$, respectively.

3. The solution of (14) by the FS-method for

(15) $$L(t) = L_0(1 + m \cdot \cos(\Omega t))$$

and approximation of the transfer function by the Fourier polynomial for one ($k = 1$) harmonic component

$$W(s,t) = W_0(s) + W_{c1}(s) \cdot \cos(\Omega t) + W_{s1}(s) \cdot \sin(\Omega t) \quad (16)$$

makes it necessary to perform symbolic solution of the symbolic SLAE

(17) $$\begin{bmatrix} L_0 \cdot Y \cdot s + 1 & (L_0 \cdot Y \cdot m \cdot s)/2 & 0 \\ L_0 \cdot Y \cdot m \cdot s & L_0 \cdot Y \cdot s + 1 & L_0 \cdot Y \cdot \Omega \\ -L_0 \cdot Y \cdot m \cdot \Omega & -L_0 \cdot Y \cdot \Omega & L_0 \cdot Y \cdot s + 1 \end{bmatrix} \times \begin{bmatrix} W_0 \\ W_{c1} \\ W_{s1} \end{bmatrix} = \begin{bmatrix} L_0 \\ L_0 ms \\ -L_0 m\Omega \end{bmatrix}.$$

4. Based on the solutions of the system (17), we determine $u(t)$ for $i(s)=e^{st}$ in the form (3) $u(t)=\mathrm{Re}[W(s,t)\cdot e^{st}]$ for the time points $t=0.400e-6, 0.401e-6, 0.402e-6, 0.403e-6, 0.404e-6\ (s)$, which gives values $u(t)$, shown in A rows in Table 1, for the approximation of the transfer function $W(s,t)$ with one ($k=1$), two ($k=2$), three ($k=3$) and four ($k=4$) harmonic components, respectively.

Table 1. The values of the voltage $u(t)$ for the circuit in Fig.1

t, sec		0.400e-6	0.401e-6	0.402e-6	0.403e-6	0.404e-6
A	k=1	15.978	-112.972	-146.543	-143.357	-125.763
B		15.978	-112.972	-146.543	-143.357	-125.763
A	k=2	16.349	-113.279	-146.310	-143.610	-125.425
B		16.349	-113.279	-146.310	-143.610	-125.425
A	k=3	16.343	-113.280	-146.303	-143.611	-125.431
B		16.343	-113.280	-146.303	-143.611	-125.431
A	k=4	16.343	-113.280	-146.303	-143.611	-125.431
B		16.343	-113.280	-146.303	-143.611	-125.431
Micro-cap		16.343	- 113.279	-146.303	-143.611	-125.460

The actions required for the replacement of the variables requires the steps listed below.

1. By replacing $V(t)=\int u(t)dt$ in the equation (12) and multiplying it by $L(t)$, we obtain

(18) $$L(t)\cdot y\cdot V'(t)+V(t)=L(t)\cdot i(t).$$

The parameter $\Gamma(t)$, which is the inverse of $L(t)$, is not introduced here due to the need to compare the values $u(t)$ obtained by the two methods of eliminating the integrals from the equations.

2. From (18) we obtain L.A. Zadeh's equation of the circuit for the complex domain s

(19) $$L(t)\cdot y\cdot W'(s,t)+[L(t)\cdot y\cdot s+1]\cdot W(s,t)=L(t),$$

where $W(s,t)=V(s,t)/i(s)$, $V(s,t)$ and $i(s)$ are the Laplacian images of the time variables $V(t)$ and $i(t)$, respectively.

3. Solving (19) by the FS-method, considering (15) and (16), makes it necessary to perform a symbolic solution of the next symbolic SLAE:

(20) $$\begin{bmatrix} L_0 \cdot Y \cdot s + 1 & (L_0 \cdot Y \cdot m \cdot s)/2 & (L_0 \cdot Y \cdot m \cdot \Omega)/2 \\ L_0 \cdot Y \cdot m \cdot s & L_0 \cdot Y \cdot s + 1 & L_0 \cdot Y \cdot \Omega \\ 0 & -L_0 \cdot Y \cdot \Omega & L_0 \cdot Y \cdot s + 1 \end{bmatrix} \times \begin{bmatrix} W_0 \\ W_{c1} \\ W_{s1} \end{bmatrix} = \begin{bmatrix} L_0 \\ L_0 \cdot m \\ 0 \end{bmatrix}.$$

4. Solving (20) and determining $u(t)$ for $i(s)=e^{st}$ in the form $u(t)=(\mathrm{Re}[W(s,t)\cdot e^{st}])'$ for the same time points according to (8)-(11) results in the values $u(t)$ presented in B rows of Table 1 for the approximation of the transfer function $W(s,t)$ also with one ($k=1$), two ($k=2$), three ($k=3$) and four ($k=4$) harmonic components, respectively.

A separate line (the bottom one) in Table1 presents the values of the voltage $u(t)$ obtained using the Micro-Cap simulator.

Table 1 shows that:

1. The voltage values obtained both by the replacement of variables and by differentiation of equations for $k=3$ and $k=4$ are equal, indicating that for the approximation of the transfer function $W(s,t)$ of the circuit in Fig. 1 for the calculation of $u(t)$, three harmonic components will suffice.

2. The voltage values obtained both by the replacement of variables and by differentiation of equations for the same k and y in Table 1 are identical. Comparing all the significant digits of the numbers obtained in MATLAB, we determine the similarity in the computations using both approaches in 12 significant digits. This is a very satisfactory result.

3. The strong similarity of the voltage values obtained by the FS-method for $k=3$ and those generated by Micro-Cap [7] confirms the adequacy of the FS-method.

Given the similarity of the results of comparing both methods of eliminating integrals from systems of integro-differential equations, the further computer experiments will evaluate the speed they provide.

Computer experiments*

As said above, the method of replacing the variables, in contrast to the method of differentiation, allows an additional reduction of the bulkiness of the differential equations by eliminating the denominators in them by multiplying these equations by the corresponding $L_n(t)$ or by replacing $L_n(t)$ with $\Gamma_n(t)=1/L_n(t)$. So, if the problem being solved allows presenting the time-varying inductance in the form of a few harmonic components $\Gamma(t)$ or even one harmonic component

(21) $$\Gamma(t)=\Gamma_0(1+m\cdot\cos(\Omega t)),$$

then the differential equations describing the circuit can become even less bulky. So, for the above example from Fig. 1, the equation (18) in case of replacing $L(t)$ with $\Gamma(t)=1/L(t)$ will appear as

(22) $$y\cdot V'(t)+\Gamma(t)\cdot V(t)=i(t),$$

which results in L.A. Zadeh's equation

(23) $$Y\cdot W'(s,t)+(Y\cdot s+\Gamma)\cdot W(s,t)=1$$

and its solution in the form of SLAE

(24) $$\begin{bmatrix} \Gamma_0 + Y \cdot s & (\Gamma_0 \cdot m)/2 & 0 \\ \Gamma_0 \cdot m & \Gamma_0 + Y \cdot s & Y \cdot \Omega \\ 0 & -Y \cdot \Omega & \Gamma_0 + Y \cdot s \end{bmatrix} \times \begin{bmatrix} W_0 \\ W_{c1} \\ W_{s1} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$

which is less cumbersome than the equation (20), and, therefore, should be faster to solve.

It should be noted that the replacement of $L(t)$ with $\Gamma(t)$ in the method of differentiation is absolutely not advisable, since it does not eliminate the need to differentiate the products of the two time-dependent functions, one of which is an integral.

In general, the choice of the simplification method depends on a specific problem to be solved, which will be demonstrated in the computer experiments below.

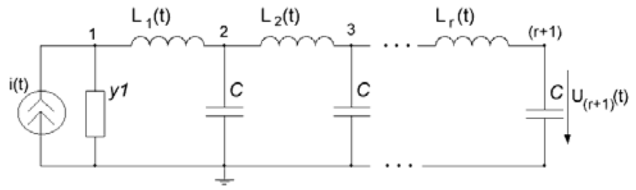

Fig 2. A test LPTV circuit consisting of r $L(t)c$ - elements

The objective of the experiment. Let the circuit with the identically time-varying inductances (15) consist of in-series $L(t)c$ - elements, as shown in Fig. 2. Let the number of the elements in the circuit be r . The task is to determine the output voltage of the circuit $u_{r+1}(t)$ and the dependence of the time of its formation on the number of elements in the circuit r by the FS-method applying both ways of eliminating the integrals from the equations, assuming that all the parameters of the circuit elements are designated by symbols in accordance with Fig. 2

The implementation of the experiment. The experiment was implemented for the two values of r : $r=1$ and $r=2$, which is demonstrated below.

For $r=1$, the system of integro-differential equations of the circuit is

$$\begin{aligned}&\frac{1}{L(t)}\int u_1(t)dt-\frac{1}{L(t)}\int u_2(t)dt=i(t),\\&-\frac{1}{L(t)}\int u_1(t)dt+\frac{1}{L(t)}\int u_2(t)dt+c\cdot u_2'(t)=0,\end{aligned}\tag{25}$$

where $u_2(t)$ is the output voltage of the circuit.

Eliminating the denominators, (25) is multiplied by $L(t)$, resulting in

$$\begin{aligned}&\int u_1(t)dt-\int u_2(t)dt=L(t)\cdot i(t),\\&-\int u_1(t)dt+\int u_2(t)dt+L(t)\cdot c\cdot u_2'(t)=0.\end{aligned}\tag{26}$$

Table 2. The value of the voltage u_2 and the time of its estimation

t, µs		8.00	8.001	8.002	8.003	8.004	Estimation time W, s
A	k=1	-0.0114	-0.0026	0.0073	0.0142	0.0157	44.1
B		-0.0114	-0.0026	0.0073	0.0142	0.0157	18.6
A	k=2	-0.0114	-0.0026	0.0073	0.0142	0.0157	74.0
B		-0.0114	-0.0026	0.0073	0.0142	0.0157	30.4
A	k=3	-0.0114	-0.0026	0.0073	0.0142	0.0157	135.8
B		-0.0114	-0.0026	0.0073	0.0142	0.0157	43.9
A	k=4	-0.0114	-0.0026	0.0073	0.0142	0.0157	561.2
B		-0.0114	-0.0026	0.0073	0.0142	0.0157	65.5
Micro-Cap		-0.0114	-0.0026	0.0073	0.0142	0.0157	

The method of differentiation.The equations (26) are differentiated and SLDE is written in a matrix form:

$$\begin{bmatrix}1 & -1\\ -1 & 1+cL'p+cLp^2\end{bmatrix}\times\begin{bmatrix}u_1\\ u_2\end{bmatrix}=\begin{bmatrix}(Lp+L')i\\ 0\end{bmatrix},\tag{27}$$

where the symbol p denotes the operation of differentiation d/dt .

From (27), L.A. Zadeh's equation is written as:

$$\begin{aligned}&\begin{bmatrix}0 & 0\\ 0 & cL\end{bmatrix}\times\begin{bmatrix}W_1''\\ W_2''\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & cL'+2cLs\end{bmatrix}\times\begin{bmatrix}W_1'\\ W_2'\end{bmatrix}+\\&+\begin{bmatrix}1 & -1\\ -1 & 1+cL's+cLs^2\end{bmatrix}\times\begin{bmatrix}W_1\\ W_2\end{bmatrix}=\begin{bmatrix}Ls+L'\\ 0\end{bmatrix},\end{aligned}\tag{28}$$

where s is the Laplace transform complex variable, $W_1=u_1(s,t)/i(s)$ and $W_2=u_2(s,t)/i(s)$. The value of the output voltage u_2 for the time points $t=8.000:0.001:8.004\mu s$ and its estimation time according to (3) are presented in A rows of Table 2.

The method of replacing the variables. From (26), taking into account (6),

$$\begin{aligned}&V_1-V_2=L(t)\cdot i(t),\\&-V_1+V_2+L(t)\cdot c\cdot V_2''=0.\end{aligned}\tag{29}$$

which in a matrix form appears as

$$\begin{bmatrix}1 & -1\\ -1 & 1+cLp^2\end{bmatrix}\times\begin{bmatrix}V_1\\ V_2\end{bmatrix}=\begin{bmatrix}Li\\ 0\end{bmatrix}.\tag{30}$$

L.A. Zadeh's equation written from (30) will appear as

$$\begin{aligned}&\begin{bmatrix}0 & 0\\ 0 & cL\end{bmatrix}\times\begin{bmatrix}W_1''\\ W_2''\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 2cLs\end{bmatrix}\times\begin{bmatrix}W_1'\\ W_2'\end{bmatrix}+\\&+\begin{bmatrix}1 & -1\\ -1 & 1+cLs^2\end{bmatrix}\times\begin{bmatrix}W_1\\ W_2\end{bmatrix}=\begin{bmatrix}L\\ 0\end{bmatrix},\end{aligned}\tag{31}$$

where $W_1=V_1(s,t)/i(s)$, $W_2=V_2(s,t)/i(s)$. The value of the output voltage u_2 for the time points $t=8.000:0.001:8.004\mu s$ and its estimation time according to (11) are presented in B rows of Table 2.

For $r=2$ by analogy with (26) the system of integro-differential equations of the circuit with the output voltage $u_3(t)$ is

$$\begin{aligned}&\int u_1(t)dt-\int u_2(t)dt=L(t)\cdot i(t),\\&-\int u_1(t)dt+2\int u_2(t)dt+L(t)\cdot c\cdot u_2'(t)-\int u_3(t)dt=0,\\&\int u_2(t)dt+\int u_3(t)dt+L(t)\cdot c\cdot u_3'(t)=0,\end{aligned}\tag{32}$$

The method of differentiation by analogy with (27) - (28) according to (32) results in L.A. Zadeh's equation

$$\begin{aligned}&\begin{bmatrix}0 & 0 & 0\\ 0 & cL & 0\\ 0 & 0 & cL\end{bmatrix}\times\begin{bmatrix}W_1''\\ W_2''\\ W_3''\end{bmatrix}+\begin{bmatrix}0 & 0 & 0\\ 0 & cL'+2cLs & 0\\ 0 & 0 & cL'+2cLs\end{bmatrix}\times\begin{bmatrix}W_1'\\ W_2'\\ W_3'\end{bmatrix}+\\&+\begin{bmatrix}1 & -1 & 0\\ -1 & 2+cL's+cLs^2 & -1\\ 0 & -1 & 1+cL's+cLs^2\end{bmatrix}\times\begin{bmatrix}W_1\\ W_2\\ W_3\end{bmatrix}=\begin{bmatrix}Ls+L'\\ 0\\ 0\end{bmatrix}.\end{aligned}\tag{33}$$

The value of the output voltage u_3 for the time points $t=8.000:0.001:8.004\mu s$ and its estimation time according to (3) are presented in A rows of Table 3.

The method of replacing the variables by analogy with (29)-(31) results in L.A. Zadeh's equation

$$\begin{aligned}&\begin{bmatrix}0 & 0 & 0\\ 0 & cL & 0\\ 0 & 0 & cL\end{bmatrix}\times\begin{bmatrix}W_1''\\ W_2''\\ W_3''\end{bmatrix}+\begin{bmatrix}0 & 0 & 0\\ 0 & 2cLs & 0\\ 0 & 0 & 2cLs\end{bmatrix}\times\begin{bmatrix}W_1'\\ W_2'\\ W_3'\end{bmatrix}+\\&+\begin{bmatrix}1 & -1 & 0\\ -1 & 2+cLs^2 & -1\\ 0 & -1 & 1+cLs^2\end{bmatrix}\times\begin{bmatrix}W_1\\ W_2\\ W_3\end{bmatrix}=\begin{bmatrix}L\\ 0\\ 0\end{bmatrix}.\end{aligned}\tag{34}$$

The value of the output voltage u_3 for the time points $t=8.000:0.001:8.004\mu s$ and its estimation time according to (11) are presented in B rows of Table 3.

Table 3. The value of the voltage u_3 and the time of its estimation

t , μs		8.000	8.001	8.002	8.003	8.004	Estimation time W, s
A	k=1	-0.0117	-0.004	0.00592	0.01378	0.01564	135.3
B		-0.0118	-0.004	0.0059	0.01377	0.01564	71.9
A	k=2	-0.0118	-0.004	0.0059	0.01378	0.01562	410.7
B		-0.0118	-0.004	0.0059	0.01378	0.01562	120.4
A	k=3	-0.0118	-0.004	0.0059	0.01378	0.01562	2033.5
B		-0.0118	-0.004	0.0059	0.01378	0.01562	180.7
A	k=4	over the allotted time					
B		-0.0118	-0.004	0.0059	0.01378	0.01562	290.9
Micro -Cap		-0.0118	-0.004	0.00594	0.01379	0.01462	

Table 4. The value of the voltage u_4 and the time of its estimation

t , μs		8.000	8.001	8.002	8.003	8.004	Estimation time W,s
A	k=1	-0.0305	-0.0993	-0.1338	-0.1114	-0.0464	350.3
B		-0.0217	-0.0746	-0.1039	-0.0839	-0.0329	208.9
A	k=2	-0.0367	-0.0678	-0.0756	-0.0508	-0.0069	1151.1
B		-0.0497	-0.0674	-0.061	-0.0279	0.01482	331.8
A	k=3	over the allotted time					
B		-0.0497	-0.0673	-0.0605	-0.0276	0.01512	496.6
A	k=4	over the allotted time					
B		-0.0497	-0.0673	-0.0605	-0.0276	0.01512	731.7
Micro-Cap		-0.05	-0.0678	-0.0603	-0.0276	0.01533	

Table 5. The value of the voltage u_5 and the time of its estimation

t , μs		8.000	8.001	8.002	8.003	8.004	Estimation time W,s
A	k=1	0.0111	0.0037	-0.0057	-0.0135	-0.0148	1081.9
B		0.0119	0.0036	-0.0066	-0.0144	-0.0161	692.3
A	k=2	0.0115	0.0036	-0.0063	-0.0138	-0.0158	3155.1
B		0.0119	0.0036	-0.0067	-0.0146	-0.0162	944.1
A	k=3	over the allotted time					
B		over the allotted time					
Micro-Cap		0.0119	0.0036	-0.0068	-0.0146	-0.0162	

The obtained output values of the voltages u_4 and u_5 for the selected time points and their estimation time by the method of differentiation for $r = 3$ and $r = 4$ are presented in A rows of Table 4 and Table 5, and those obtained by the method of replacing the variables in B rows of Table 4 and Table 5, respectively.

Table 2 – Table 5 data suggest the following.

1. The sameness of the output voltage values estimated by the FS-method for different numbers of k harmonic components in the transfer function suggests the sufficiency of the selected k values. For instance, in Table 2 the sufficient value is $k = 1$, in Table 3 $k = 2$, in Table 4 $k = 4$, respectively. As follows from Table 5, using the method of replacing the variables, the value of the input voltage u_5 cannot be evaluated for *k>2*.

2. The values of the voltage obtained both by the replacement of the variables and by the differentiation of equations for the same k values in Table 2 – Table 5 are very similar.

3. The strong similarity of the voltage values obtained by the FS-method and those generated by Micro-Cap [8] confirms the adequacy of the FS-method.

Conclusions

1. The results of the performed computer experiment assure that the method of replacing the variables provides the same accuracy as the method of differentiation, but is faster.

2. Therefore, we conclude that the method of replacing the variables is more feasible for the system UDF MAOPCs [9], which is intended for the simulation of LPTV circuits and a number of electronic devices simulated using such circuits, than the method of differentiating the equations.

3. In our opinion, the power law dependence of the time of the transfer functions formation on the complexity of the circuit requires the application of the sub-circuit method [5] and the introduction of this method into the UDF MAOPCs system, which can significantly reduce the computation time required for the formation of transfer functions and considerably extends the class of permissible analysable LPTV circuits in terms of complexity.

* Here and thereafter each value of the time was obtained by averaging the values of the time resulting from ten consecutive computations. The computer experiments were performed in MATLAB R2014a on Dell PC with Intel(R) Core(TM) i-5-3317U CPU, 1.70 Ghz, RAM:8.00Gb.MATLAB R2014a.

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