

Refining Induction Machine Characteristics at High Saturation of Steel

Abstract. The paper deals with an induction machine refined mathematical model taking into account the nonlinearity of the magnetization curve and steel losses at high saturation of the magnetic system. The steel losses increase influence on the characteristics of induction generators operating with connected high-value capacitors under the mode of high saturation of the magnetic circuit is demonstrated. The obtained calculated characteristics coincide with the results of the experimental research of a 2.2 kW induction machine with appropriate accuracy.

Streszczenie. Przedstawiono zmodyfikowany model matematyczny silnika indukcyjnego biorący pod uwagę nieliniowość krzywej magnesowania i straty przy nasyceniu. Obliczone parametry porównano z danymi eksperymentalnymi dla silnika 2.2 kW. (Model matematyczny silnika indukcyjnego uwzględniający nieliniowość charakterystyki magnesowania)

Keywords: induction machine, induction generator, mathematical model, steel losses, nonlinearity of magnetization curve, hysteresis, **Słowa kluczowe:** silnik indukcyjny, model matematyczny, krzywa magnesowania, straty

Introduction

At present induction machines (IM) are the most common type of electric machines (EM) due to the simplicity of design, high reliability and high dynamic performance. Mathematical models conventionally used to study the dynamic processes of IM are based on the Park-Gorev equations [1, 2], which do not quite correctly take into account steel losses in the high saturation mode of electrical steel (ETS). These results in noticeable deviations of IM calculated dynamic and static characteristics from real ones. High saturation of the magnetic circuit is characteristic of induction generators (IG) operating mainly in an over-excited state and variable-frequency induction motors when it is necessary to obtain increased starting torques or maximum overload capacity.

The importance of taking into account steel losses when calculating the characteristics of IM operating in high saturation mode is determined by these losses significant effect on the machine total losses. From [3-5] it is known that high saturation of IM steel is characterized by an abnormal increase in steel losses. In particular, a number of researchers [3-10] proved that losses in steel of magnetic cores that are magnetized in rotating magnetic fields make up a significant part of the total power losses in EM. Therefore, for induction motors, the steel loss in high saturation mode can reach up to 50 % of the total loss, for transformers – over 20%, for IG with capacitor excitation – about 20-60 % [3]. Consequently, in this mode, the steel losses influence on the characteristics of the machine is most noticeable, and their consideration is of primary interest.

In the well-known mathematical models of IM the classical method [6-7] takes into account steel losses by incorporating additional resistances parallel to the magnetization circuit of the T-shaped phase equivalent circuit. The drawbacks of the method are the increase in the number of differential equations, as well as the use of expressions based on the Steinmets dependence for determining the parameters of the additional contours of expressions [6]. It is known that the latter is valid only under the condition of an unsaturated and slightly saturated state of steel [3, 11] and its application affects the accuracy of the calculation of characteristics (Fig. 1).

Improving accuracy is necessary to determine the maximum load capacity of low- and medium-power generators.

In this regard, the urgent task is to develop a mathematical model and research the characteristics of IM operating in high saturation mode.

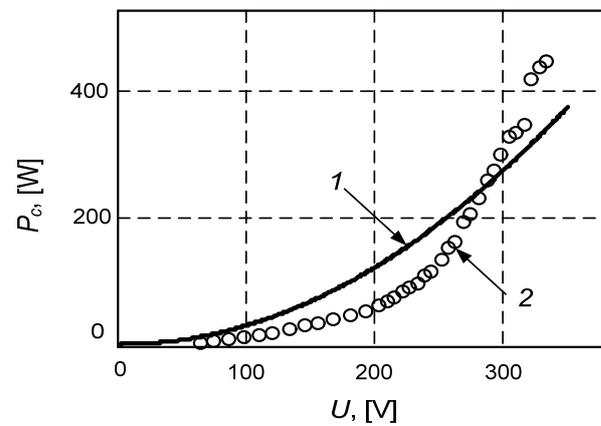


Fig. 1. Comparison of steel loss curves: 1 – curve obtained using the classical method [7], 2 – experimental curve

Theoretical research

A. The theory of describing processes in AG

When formulating differential equations for IM operating in the motor or generator modes and considering transient processes in them, the following generally accepted assumptions associated with the idealization of the machine are used: phase windings are symmetrical and located at an angle of 120° for 3-phase machines, there are no losses in steel, the air gap is uniform, the machine is unsaturated, magnetomotive forces are distributed sinusoidally along the circle of the air gap.

If it is necessary to take into account the design features of the EM (winding asymmetry), the nonlinearity of its magnetization curve (saturation of the magnetic circuit), the system of differential equations is significantly complicated.

When developing a mathematical model of IM, we make the equations of voltage of the windings and the equations of motion [11, 12].

The stress balance equations for a three-phase IM are represented by system (1).

$$\begin{aligned}
 \frac{d\Psi_A}{dt} &= U_A - i_A r_s, & \frac{d\Psi_B}{dt} &= U_B - i_B r_s \\
 \frac{d\Psi_C}{dt} &= U_C - i_C r_s \\
 0 &= i_a r_r + \frac{d\Psi_a}{dt}, & 0 &= i_b r_r + \frac{d\Psi_b}{dt} \\
 0 &= i_c r_r + \frac{d\Psi_c}{dt}
 \end{aligned}
 \tag{1}$$

where $\Psi_{A,B,C}$ and $\Psi_{a,b,c}$ – full flux linkage of the stator and rotor phases, respectively; $i_{A,B,C}$ and $i_{a,b,c}$ – current in the phase of the stator and rotor, respectively; $r_{A,B,C}$ and $r_{a,b,c}$ – resistance in the phase of the stator and rotor, respectively; $U_{A,B,C}$ – the value of voltage on the stator phases.

In turn, the flux linkage of any phase is determined by the values of self-inductance and mutual inductance with other EM windings:

$$(2) \begin{cases} \Psi_A = \left(L_{s\sigma} + \frac{2}{3}L_1 \right) i_A + \frac{2}{3}L_1 \cos(\rho) i_B + \frac{2}{3}L_1 \cos(2\rho) i_C \\ + Mi_a \cos \gamma + Mi_b \cos(\gamma + \rho) + Mi_c \cos(\gamma - \rho) \\ \Psi_a = \left(L_{r\sigma} + \frac{2}{3}L_2 \right) i_a + \frac{2}{3}L_2 \cos(\rho) i_b + \frac{2}{3}L_2 \cos(\rho) i_c \\ + Mi_A \cos \gamma + Mi_B \cos(\gamma - \rho) + Mi_C \cos(\gamma + \rho), \end{cases}$$

where $L_1 = L_2 = M = L_\mu$ – the maximum value of the mutual inductance, when the axes of the corresponding stator “A” and rotor phase windings “a” coincide; $L_{s\sigma}$ and $L_{r\sigma}$ – the stator and rotor phase inductance, respectively; $\rho = 2\pi/3$. Inductances $L_{s\sigma}$, $L_{r\sigma}$, L_μ – parameters of a traditional T-shaped IM equivalent circuit (Fig. 2).

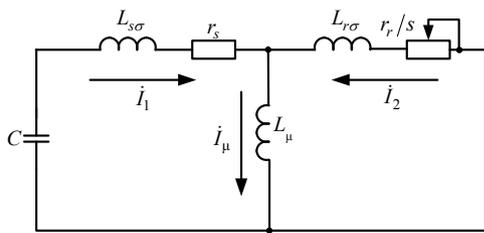


Fig. 2. IG equivalent circuit

Taking into account the symmetry condition of the motor windings and the symmetry condition of the stator and rotor currents ($i_A + i_B + i_C = 0$; $i_a + i_b + i_c = 0$), the system of equations (2) takes the form:

$$(3) \begin{cases} \Psi_A = (L_{s\sigma} + L_1) i_A + \\ Mi_a \cos \gamma + Mi_b \cos(\gamma + \rho) + Mi_c \cos(\gamma - \rho) \\ \Psi_a = (L_{r\sigma} + L_2) i_a + \\ Mi_A \cos \gamma + Mi_B \cos(\gamma - \rho) + Mi_C \cos(\gamma + \rho) \end{cases}$$

IM electromagnetic torque is defined as the derivative of the accumulated electromagnetic energy W_e of the machine and rotor rotation angle γ :

$$(4) \quad T_e = p \frac{dW_e}{d\gamma}$$

Electromagnetic energy is calculated as:

$$(5) \quad W_e = 0.5(\Psi_A i_A + \Psi_B i_B + \dots + \Psi_b i_b + \Psi_c i_c)$$

The equation of rotor speed is presented in the form:

$$(6) \quad \frac{d\omega_r}{dt} = (T_e - T_{meh}) / J,$$

where J – rotor inertia moment; T_{meh} – torque on the EM shaft.

To implement the IG self-excitation mode, capacitors are included in the stator circuit. If the system is symmetrical, instead of the power supply voltage, the voltage drop on the capacitors for all three phases of the

generator stator should be substituted into the equation of the IM mathematical model:

$$(7) \begin{cases} U_{Ac} = U_A = - \left(\frac{1}{C} \int_0^t i_{vA} + U_{vA}(0) \right), \\ U_{Bc} = U_B = - \left(\frac{1}{C} \int_0^t i_{vB} + U_{vB}(0) \right), \\ U_{Cc} = U_C = - \left(\frac{1}{C} \int_0^t i_{vC} + U_{vC}(0) \right), \end{cases}$$

where U_{Ac} , U_{Bc} , U_{Cc} – voltage on the capacitors of stator phases A, B, C, respectively; i_{vA} , i_{vB} , i_{vC} – currents flowing through the excitation capacitors of stator phases A, B, C, respectively; $U_{vA}(0)$, $U_{vB}(0)$, $U_{vC}(0)$ – voltage at the initial moment of time $t=0$ on capacities of stator phases A, B, C, respectively.

The saturation of the magnetizing inductance in IG is the main factor in the accumulation and stabilization of the generated voltage. Therefore, taking into account the magnetizing inductance L_μ as a function of changing the magnetizing current i_μ in the autonomous IG model is mandatory. In this case, the adequacy of the generator mathematical model will depend on the accuracy of the description of the magnetization curve.

The inductance nonlinear dependence on the current can be represented by various approximation expressions. Depending on the purpose, the required accuracy of research, various types of approximations can be applied (power or trigonometric polynomials; piecewise linear approximation, etc.).

The magnetizing circuit inductance dependence on magnetizing current of a 2.2 kW induction machine, obtained based on the no-load characteristic [8, 9], is shown in Fig. 3.

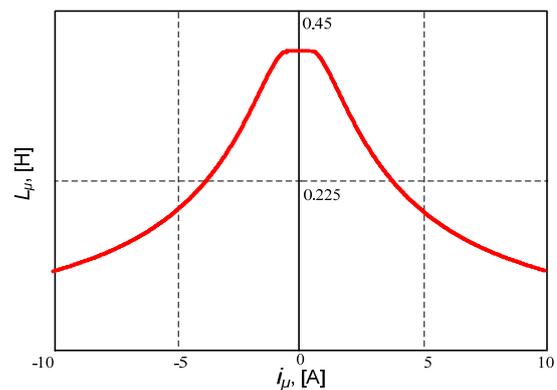


Fig. 3. The magnetizing inductance dependence on the magnetizing current

We describe the nonlinear dependence $L_\mu(i_\mu)$ (Fig. 3) by means of a power polynomial of the form:

$$(8) \quad L_\mu(i_\mu) = a_1 + 3a_3 i_\mu^2 + 5a_5 i_\mu^4 + \dots + (2n+1)a_{2n+1} i_\mu^{2n},$$

where $a_1, a_3, a_5, \dots, a_{2n+1}$ – approximation coefficients.

This mathematical form of representation of the magnetization inductance dependence on the current allows it to be described by a continuous function, and, accordingly, to exclude knees and discontinuities of the

sections of such dependence, as is the case when describing it by differential equations.

The magnetization curve can be described by polynomials with a lower degree. Then the number of approximation coefficients will decrease, but the accuracy of the dependence $L_{\mu}(i_{\mu})$ description will deteriorate accordingly. For example, in the case of approximating the magnetization curve by an eighth-order polynomial the maximum error increases to 9 % [15].

B. IM model taking into consideration hysteresis and eddy currents

The existing IM models do not take into account the fact that the change in the magnetic flux (inductance) lags behind the change in the magnetizing current due to the properties of the domain structure of ferromagnets [3]. This property can be taken into account by shifting the magnetic flux signal with respect to the magnetizing current signal by the value of the magnetic lag angle δ (the angle of magnetic losses).

Finding the instantaneous value of the magnetic lag angle δ is associated with difficulties in determining the instantaneous value of losses in steel in the high saturation mode of the IG magnetic circuit. Therefore, to determine angle δ at each integration step, we use the following relation [16, 17]:

$$(9) \quad \delta = \arctg\left(\frac{P_c}{P_{em.rms}}\right),$$

where P_c – the value of losses in stator steel at the current integration step; $P_{em.rms}$ – the effective value of electromagnetic power, which is at each step of integration;

$$(10) \quad P_{em.rms} = \sqrt{\frac{1}{T} \int_0^T P_{em}^2 dt},$$

$$P_{em} = T_e \cdot \omega_r.$$

In paper [3], the dependence for determining steel losses, taking into account the nonlinear dynamics of domain structures and the non-uniformity of their motion at different magnetization reversal cycles was substantiated, which ensures high convergence of calculated and experimental data:

$$(11) \quad P_c(I_{\mu}) = \frac{\xi}{dE(I_{\mu})/dI_{\mu}} \left(E(I_{\mu})\right)^2,$$

where I_{μ} – the effective value of magnetizing current; ξ – the coefficient depending on the characteristics of electrical steel; $E(I_{\mu})$ – EMF dependence on the magnetizing current.

Thus, expressions (3) - (11) form a mathematical model of IM in a three-phase coordinate system taking into account the inertia of the magnetization process, which manifests itself in the time and phase lag of the magnetization with respect to the magnetic field strength.

Block diagrams for modeling rotor contours can be obtained according to expression (3).

Fig. 4 contains the block diagram of one phase of the IM stator where the angle of magnetic lag is taken into account.

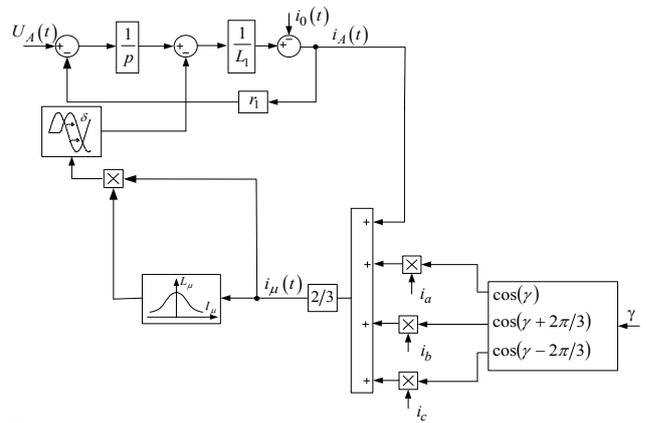


Fig. 4. IM stator phase block diagram taking into account the magnetic lag angle

The following designations are used in Fig. 4: $U_A(t)$ – the voltage of induction motors stator phase A; $i_A(t)$ – the current of induction motors stator phase A; i_a, i_b, i_c – rotor phase currents, respectively; δ – the magnetic losses angle, enabling taking into account the lag in the process of changing the magnetic flux with variable magnetization reversal; γ – rotor turn angle; $\frac{1}{p}$ – Laplace operator; $i_0(t)$ – zero sequence current.

All the other phases of the AM are modeled similarly.

Modeling results and experimental research

Applying the created mathematical model, we will analyze the characteristics of IG operating at high saturation of the magnetic system. To do this, we use induction motors operating in the generator mode with a squirrel-cage rotor with the following published data: $P_{nom} = 2.2 kW$; $p = 2$; $I_{nom} = 4.9 A$; $n_{nom} = 1430 rpm$; $\cos\phi_{nom} = 0.825$; $\eta_{nom} = 81\%$ and the equivalent circuit parameters $r_s = 2.5 \Omega$; $r_r = 1.8 \Omega$; $L_{s\sigma} = 0.008 H$; $L_{r\sigma} = 0.009 H$; $L_{\mu} = 0.366 H$; $C = 200 \mu F$.

Figure 5 contains the obtained dynamic hysteresis loops.

The expansion of the loops at the ends is caused by inertial phenomena during the movement of domain structures [3, 18, 19].

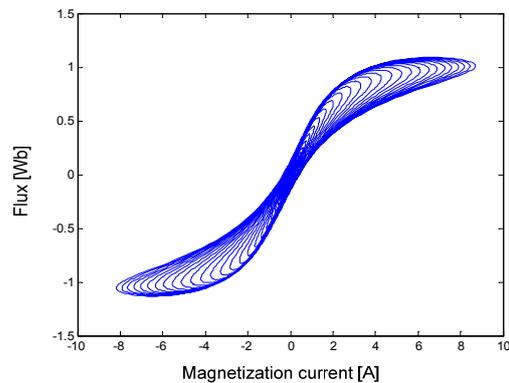


Fig. 5. Calculated hysteresis loops for self-excitation of an autonomous IG

It should be noted that taking into account steel losses in the model results in a prolongation of the process of self-excitation and a decrease in the value of the generated voltage (Fig. 6). Taking into account the steel losses during load surge significantly affects IG voltage. It is seen that

curve 1, calculated considering steel losses, is located slightly below curve 2, which was obtained using the classical method.

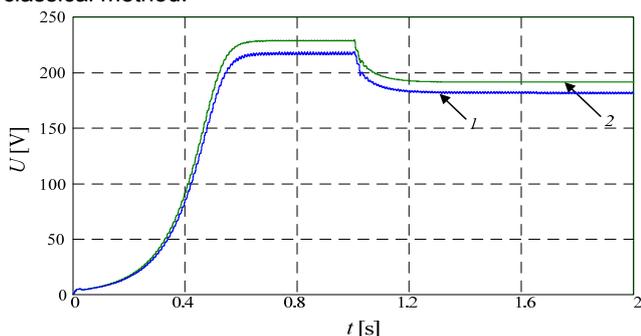


Fig. 6. The dynamic characteristics of IG with various methods of accounting for steel losses: 1 – according to expression (11), 2 – using the classical method

The conditions for the use of IG in the generator mode are determined by the maximum load power that can be connected to it. Therefore, research was carried out to find the steel losses effect on the external characteristics of IG, which made it possible to determine the maximum overload capacity of the generator.

The external characteristics of IG, obtained using various methods for determining steel losses, as well as when changing the value of the capacitance of the excitation capacitors, are shown in Fig. 7. The characteristics are presented in relative units for comparison. Power P_L and voltage U_L at the load are expressed as shares of respectively rated power and voltage of IM. Curve 2 was obtained using the classical method [7] for taking into account steel losses. Curve 1 is drawn using the proposed mathematical model.

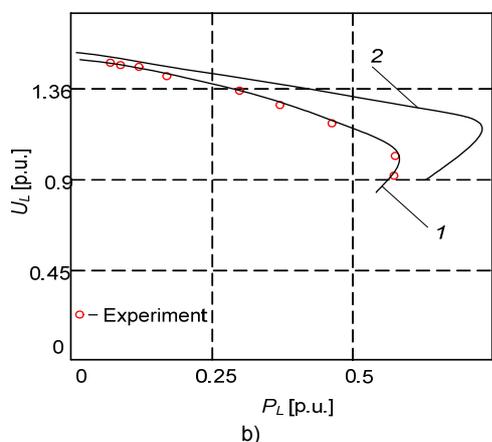
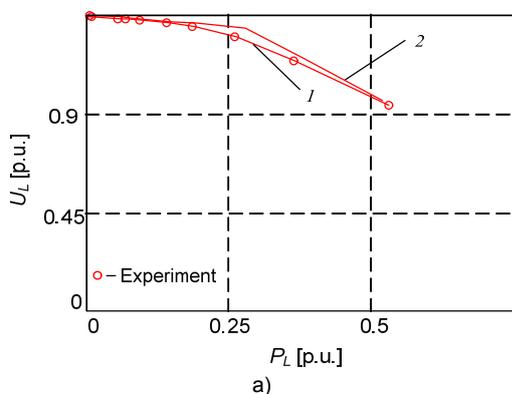


Fig. 7. Load characteristics of IG at different values of the capacitance of the excitation capacitors: a – 200 μF , b – 500 μF : 1 – by expression (11), 2 – by means of the classical method

The reliability of the obtained results is verified by comparing the calculated data with the data of an experimental research. It is seen that the discrepancy between the calculated and experimental characteristics is within 5 %.

The value of the maximum power at load, calculated according to the proposed method, taking into account steel losses and saturation, is 10 ... 20 % less than when calculated using conventional approaches. This is due to an increase in losses with growing saturation.

The analysis of the curves (Fig. 7) showed that the use of larger capacitors leads to an increase in load capacity. This ensures a relatively flat form of characteristics and a significant overload capacity of the machine. A further increase in capacitance from 200 μF to 500 μF does not enable obtaining the maximum generator power and results in rapidly decreasing curves (Fig. 7, b).

Conclusions

The results of the presented research make it possible to assess the steel losses effect on the characteristics of induction machines operating in the mode of high saturation of steel.

The use of the proposed method for accounting the steel losses when calculating the external characteristics of unregulated autonomous generators with capacitor excitation provides higher calculation accuracy compared with conventional methods for determining steel losses (the discrepancy between the calculated and experimental characteristics does not exceed 5 %). At the same time, the calculation results showed that taking into account the change in the properties of steel in the high saturation mode results in a decrease in the overload capacity of unregulated autonomous generators by 10 ... 20 %.

The proposed methodology for accounting steel losses at high saturation of induction machines does not increase the number of differential equations of the calculation model, is sufficiently accurate and provides the possibility of its practical application to engineering calculations of IM operating in various modes.

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