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Mathematical modeling start-up and steady state modes of asynchronous motors operation with capacitive compensation of reactive power

***Abstract**. Mathematical models for the analysis of processes during start-up and steady-state operation of three-phase asynchronous motors with short-circuit rotor with capacitive reactive power compensation have been developed. Based on them, algorithms have been developed to determine the capacitance of capacitors in these modes and to investigate them for self-oscillation and resonance. The developed algorithms are based on a mathematical model of an asynchronous motor, which takes into account the saturation of the magnetic circuit and the phenomenon of the skin effect in the rotor bars. The problem is solved in orthogonal coordinates.*

***Streszczenie.** Przedstawiono model matematyczny umożliwiający analizę procesu startu I ciągłej pracy trójfazowego silnika asynchronicznego z konensatorem dołączonym do wirnika w celu kompensacji mocy biernej. W modelu uwzględniono nasycanie się materiału magnetycznego i efekt naskórkowy w prętach wirnika. (**Model matematyczny silnika asynchronicznego umożliwiający analizę startu i pracy ciągłej przy dołączonym kondensatorze do kompensacji mocy biernej**)*

Keywords: Asynchronous motor, start-up mode, reactive power compensation, mathematical modeling, static characteristics.
Słowa kluczowe: silnik asynchroniczny, model matematyczny, kompensacja mocy biernej.

Introduction

An important problem of modern electrical power engineering is the improvement of energy efficiency, in particular, the reduction of losses, primarily due to the reactive power compensation [1]. Its main customers are transformers and asynchronous motors (AM). They are the most widespread in the industry, and most of them are squirrel cage AM of high-power. The start-up of such AM is accompanied by large currents that can reach eight times the rated current [2, 3]. This leads to a significant reduction in the voltage across the supply network and, as a consequence, an increase in the start-up time or the motor non-startup [3]. And in the case of low power of the supply network will cause an avalanche voltage drop with all the consequences. However, the starting current is dominated by a reactive component that creates a magnetic field in the AM. In powerful AM, it is economically feasible to generate reactive power directly at the point where it is consumed, that is, at the point where the motor is connected to the power line. [4, 5].

Essence of the problem

An effective way to reduce the reactive power magnitude and therefore the currents magnitude in the power line in the AM start-up modes is series connection with the stator windings of the capacitors. Series compensation solves two problems: it significantly reduces the currents in the line that supplies AM, compensating for the reactive component of the current, and leads to an increase in motor voltage during start-up, compensating for the voltage drop in the line. Switching on the capacitors in the stator circuit can cause resonance phenomena [6, 7] and overvoltages, which are dangerous for both the motor and the capacitors, and their poor selection may result in the motor starting electromagnetic torque non-starting. Capacitors in series connected cause resonance phenomena mainly in the area of small slips, so after their start-up they are switched to work in parallel with the supply network in order to provide reactive power compensation in the operating mode [8, 9]. Since the value capacitors capacitance in the starting mode is significantly different from its value in the nominal mode, specially designed starting-compensating devices are used for this purpose [8–12]. This allows you to switch the way capacitors are connected accordingly. The correct choice of the necessary capacitance for the start-up of AM and for the long-term operation of the capacitors should provide the necessary operating performances in both specified modes and avoid the occurrence of overcurrents on the current and electromagnetic torque, as well as the possibility origination of overvoltage, the process of self-excitation and resonance [13 - 4].

With the help of capacitors, it is possible to achieve not only an increase in the driving torque and, as a consequence, a reduction in the duration of starting the motor, but also significantly reduce the currents in the power line. However, the use of capacitors to improve the starting properties of powerful asynchronous motors requires the development of programmable start-up devices [11, 12] that control the operation of the motor during start-up and steady-state operation. For this purpose it is necessary to have mathematical models that allow to perform preliminary calculations and to determine the necessary capacitor parameters of starting-compensating devices in order to provide microprocessor control of their work. This can be done based on the calculation of transients and steady-state modes, taking into account the possibility of determining overvoltages, self-excitation and the appearance of resonance, as well as preventing overcompensation in the operating mode, etc.

The reliability of the results of mathematical modeling is determined by the mathematical model of AM. Mathematical models are known to differ in their basis output assumptions, systems of coordinate axes, methods of determination of electromagnetic parameters and the like. It is obvious that the high accuracy of mathematical modeling of AM modes cannot be achieved using simple mathematical models which are based on equivalent circuits [2, 3], and the use of high complexity levels of AM models complicates the analysis of dynamic modes. Another important issue is the choice of a coordinate system in which the electric state equations of loops are written.

The mathematical models developed in [7–10] are based on linear mathematical models of AM, and therefore can be regarded as approximate. The main factors that influence the adequacy of the mathematical model of AM are the saturation of the magnetic circuit and the current displacement in the rotor bars. The developed mathematical models of the motor use the characteristics of magnetization by the main magnetic flux and leakage fluxes to determine the matrix of electromagnetic parameters [16]. In order to take into account the change in the active resistance of the rotor winding due to the phenomenon of skin effect, the short-circuited rotor winding is represented by n windings formed by dividing the heightwise of the rotor slot into elementary windings with an equivalent total cross-sectional area [13, 18].

The purpose of the article is to develop mathematical models that would be able to determine with high accuracy the values of the capacities required for start-up and operating mode and to investigate the behavior of the asynchronous electric drive system with the selected values of the capacities, including the possibility of self-excitation and resonance phenomena.

Mathematical modeling of start-up mode with series compensation

Three-phase AM with capacitors in each phase is symmetric, so for the analysis of electromagnetic processes we use transformed to orthogonal axes x, y coordinate system, which allows to solve the problem of analysis with a minimum amount of calculations, while adequately taking into account the saturation of the magnetic circuit and current displacement in the rotor bars. In this system, the short-circuit rotor winding is replaced by an equivalent three-phase [2] and, together with the three-phase stator winding, is reduced to two-phase by known formulas, which allows adequate description of the processes in these windings, and the three-phase rotor winding is replaced according to known methods two-phase equivalents.

Electromechanical processes in AM which powered from a supply network with a symmetric system of voltages through series connected capacitors (Fig. 1) are generally described by a nonlinear system of differential equations (DE), which in the system of orthogonal axes *x*, *y* has the form [16]

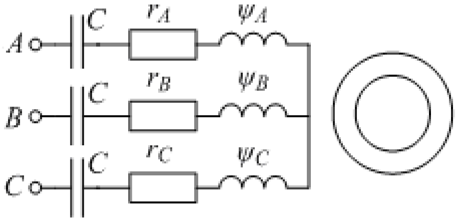


Fig.1. Electric circuit of AM with series connected capacitors

$$(1)\qquad \begin{aligned}
&\frac{d\psi_{sx}}{dt} = \omega_0\psi_{sy} - r_s i_{sx} - u_{cx} + u_{sx}\,;\\
&\frac{d\psi_{sy}}{dt} = -\omega_0\psi_{sx} - r_s i_{sy} - u_{cy} + u_{sy}\,;\\
&\frac{d\psi_{1y}}{dt} = -s\omega_0\psi_{1x} - r_1 i_{1y}\,;\\
&\frac{d\psi_{nx}}{dt} = s\omega_0\psi_{ny} - r_n i_{nx}\,;\\
&\frac{d\psi_{ny}}{dt} = -s\omega_0\psi_{nx} - r_n i_{ny}\,;\\
&\frac{du_{cx}}{dt} = \frac{i_{sx}}{C}\,; \qquad \frac{du_{cy}}{dt} = \frac{i_{sy}}{C}\,;\\
&\frac{ds}{dt} = \frac{p_0}{\omega_0 J}\left(M_c - M_e\right),
\end{aligned}$$

where i_{jk}, ψ_{jk} $(jk = sx, sy, 1x, 1y, ..., nx, ny)$ – currents and fluxes linkage of loops; r_s, r_r – resistance; u_{sx}, u_{sy} – applied voltageы to transformed loops (if the *x*-axis is combined with the phase *A* axis, the voltage $u_{sx} = U_{\mathrm{m}}$, and $u_{sy} = 0$); $s = (\omega_0 - \omega)/\omega_0$ – the rotor slip; ω_0 – the angular frequency of the supply voltage; ω – the rotor angular velocity of rotation; J – the inertia moment normalized to the motor shaft; p_0 – number of pairs of poles AM; M_c – load torque AM; M_e – electromagnetic torque AM, which in the coordinate axes *x*, *y* is determined by the formula

$$(3)\qquad M_e = 1{,}5p_0\left(\psi_{sx}i_{sy} - \psi_{sy}i_{sx}\right).$$

By equations (1), you can numerically calculate the transient or any other dynamic mode at a given supply voltage, the value of the capacitors capacitance *C* and depending on the time of load.

To determine the capacitors capacitance required to improve the starting properties of the AM (reducing current consumed from the supply network and increasing the driving torque), it is necessary to calculate the dependence of the coordinates on C when slip $s = 1{,}0$. It is important to avoid overcompensation [14, 15].

Fluxes linkage of the loops are constant with the constant slip, and therefore the electrical equilibrium equation of the system DE (1) becomes a nonlinear algebraic system

$$(3)\qquad \begin{aligned}
&\omega_0\psi_{sy} - r_s i_{sx} + i_{sx}/(\omega_0 C) = -u_{sx}\,;\\
&-\omega_0\psi_{sx} - r_s i_{sy} - i_{sy}/(\omega_0 C) = -u_{sy}\,;\\
&s\omega_0\psi_{1y} - r_1 i_{1x} = 0\,;\\
&-s\omega_0\psi_{1x} - r_1 i_{1y} = 0\,;\\
&\qquad\vdots\\
&s\omega_0\psi_{ny} - r_n i_{nx} = 0\,;\\
&-s\omega_0\psi_{nx} - r_n i_{ny} = 0\,;
\end{aligned}$$

The system of equations (3) makes it possible to investigate the dependence on the capacitors capacitance of electromagnetic torque, current, consumed active and reactive power and other coordinates at any slip value, or investigate the influence on the specified dependences of the slip change at a constant value of the capacitance.

Consider the problem of determining the value of capacitance of the series connected capacitors required to start the AM. This problem can be solved by using algebraic equations of electric equilibrium (3), which, in order to reduce the presentation of the developed method, we write in vector form

$$(4)\qquad \boldsymbol{\Omega}_0\vec{\psi} + \boldsymbol{Z}\vec{i} - \vec{u} = 0\,,$$

where

$$\boldsymbol{\Omega}_0 = \begin{array}{|c|c|c|c|c|c|c|}
\hline
 & -\omega_0 & & & \cdots & & \\
\hline
\omega_0 & & & & \cdots & & \\
\hline
 & & & -s\omega_0 & \cdots & & \\
\hline
 & & s\omega_0 & & \cdots & & \\
\hline
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\hline
 & & & & \cdots & & -s\omega_0 \\
\hline
 & & & & \cdots & s\omega_0 & \\
\hline
\end{array}\,;$$

$$\boldsymbol{Z} = \begin{array}{|c|c|c|c|c|c|c|}\hline r_s & -(\omega_0 C)^{-1} & & & \cdots & & \\ \hline (\omega_0 C)^{-1} & r_s & & & \cdots & & \\ \hline & & r_1 & & \cdots & & \\ \hline & & & r_1 & \cdots & & \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \hline & & & & \cdots & r_n & \\ \hline & & & & \cdots & & r_n \\ \hline \end{array} ;$$

$\vec{i} = i_{sx}, i_{sy}, i_{1x}, i_{1y}, ..., i_{nx}, i_{ny})^*$; $\quad \vec{u} = (U_m, 0, ..., 0)^*$;

$\vec{\psi} = (\psi_{sx}, \psi_{sy}, \psi_{1x}, \psi_{1y}, ..., \psi_{nx}, \psi_{ny})^*$ – current vectors, voltages and fluxes linkage of AM loops.

Equation (4), in addition to the components of the vector of fluxes linkage and currents, includes the values of slip s and capacitance *C*, which make up a set of interconnected coordinates, each of which can be taken as an independent variable. That is, equation (4) can be written in the form

$$\vec{y} = \vec{F}\left(\vec{\psi}, \vec{i}, \vec{u}, C, s\right). \tag{5}$$

The rest of the coordinates can be considered as dependent on the variable that is assumed to be independent. Since the fluxes linkage of each loop depends on the currents of all loops $\vec{\psi} = \vec{\psi}\left(\vec{i}\right)$, under the condition of constant slip and the applied supply voltage of equation (5) can be written as a vector function of parameter C

$$\vec{y} = \vec{f}\left(\vec{\psi}\left(\vec{i}\right), C\right). \tag{6}$$

The influence of value capacitors capacitance on other coordinates can be investigated by differential method [16, 17]. To do this, we differentiate system (6) by C.

$$\boldsymbol{A}\frac{d\vec{i}}{dC} = \frac{\partial \vec{f}}{\partial C}, \tag{7}$$

where $\boldsymbol{A}$ – the Jacobi matrix (6).

Integrating the system DE (7), we obtain the dependences of the components of the current vectors $\vec{i}$ and flux linkage $\vec{\psi}$ on the capacitance *C* of the capacitors and based on them of powers and electromagnetic torque with constant slip (for example $s = 1,0$) and applied voltage *U*, which is revised by the Newton method Similarly, static characteristics can be calculated as the dependence of coordinates on slip. In this case, DE has the form

$$\boldsymbol{A}\frac{d\vec{i}}{ds} = \frac{\partial \vec{f}}{\partial s} \tag{8}$$

Active and reactive power are determined by the formulas

$$P = 1{,}5\left(u_{sx}i_{sx} - u_{sy}i_{sy}\right); \; Q = 1{,}5\left(u_{sy}i_{sx} - u_{sx}i_{sy}\right). \tag{9}$$

Mathematical modeling of operating mode with shunt compensation

In operating mode, the so-called shunt compensation of reactive power is mainly used (fig. 2).

The capacitors in parallel connected to the AM do not affect its currents, so they can be determined from the electrical equilibrium equations

$$\boldsymbol{\Omega}_0\vec{\psi} + \boldsymbol{R}\vec{i} = \vec{u} \tag{10}$$

where $\boldsymbol{R} = diag(r_s, r_s, r_1, r_2, ..., r_n, r_n)$ – resistance diagonal matrix of AM loops.

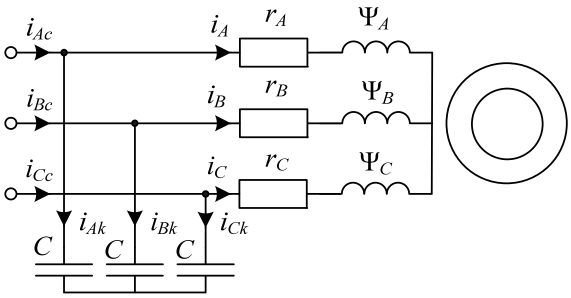


Fig.2. Electric circuit of AM with capacitors connected in parallel

We solve the nonlinear system of algebraic equations (10) by a parameter marching method whose essence in relation to the solvable problem is to increase the applied voltage in proportion to the parameter entered $\boldsymbol{\varepsilon}$ $(0 \le \varepsilon \le 1)$

$$\boldsymbol{\Omega}_0\vec{\psi} + \boldsymbol{R}\vec{i} = \varepsilon\vec{u}. \tag{11}$$

As a result of differentiation (11) with respect to *x* we get DE of the form

$$\boldsymbol{W}\frac{d\vec{i}}{d\varepsilon} = \vec{u}, \tag{12}$$

where $\boldsymbol{W} = \boldsymbol{X} + \boldsymbol{R}$ – Jacobi's matrix in which

$$\boldsymbol{X} = \begin{array}{|c|c|c|c|c|c|c|}\hline x_{sysx} & x_{sysy} & x_{sy1x} & x_{sy1y} & \cdots & x_{synx} & x_{syny} \\ \hline -x_{sxsx} & -x_{sxsy} & -x_{sx1x} & -x_{sx1y} & \cdots & -x_{synx} & -x_{syny} \\ \hline x_{1ysx} & x_{1ysy} & x_{1y1x} & x_{1y1y} & \cdots & x_{1ynx} & x_{1yny} \\ \hline -x_{1xsx} & -x_{1ysy} & -x_{1y1x} & -x_{1y1y} & \cdots & -x_{1ynx} & -x_{1yny} \\ \hline \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ \hline x_{nysx} & x_{nysy} & x_{ny1x} & x_{ny1y} & \cdots & x_{nynx} & x_{nyny} \\ \hline -x_{nxsx} & -x_{nxsy} & -x_{nxsy} & -x_{nx1y} & \cdots & -x_{nxnx} & -x_{nxny} \\ \hline \end{array}$$

– matrix of differential inductive resistances AM, determined [16] using the magnetization characteristics of the main magnetic flux ψ_μ and the leakage fluxes of the windings of stator $\psi_{\sigma s}$ and rotor $\psi_{\sigma r}$

$$\psi_\mu = \psi_\mu(i_\mu); \;\; \psi_{\sigma s} = \psi_{\sigma s}(i_s); \psi_{\sigma r} = \psi_{\sigma r}(i_r),$$

where i_μ, i_s, i_r – modules of generalized vectors of corresponding currents: magnetization, stator and rotor

$$i_\mu = \sqrt{\left(i_{sx} + i_{rx}\right)^2 + \left(i_{sy} + i_{ry}\right)^2};$$

$$i_s = \sqrt{i_{sx}^2 + i_{sy}^2}; \;\; i_r = \sqrt{i_{rx}^2 + i_{ry}^2}.$$

The currents of the rotor loops are defined as the sum of the currents of the elementary bars into which the rotor winding is divided

$$i_{rx} = (i_{r1x} + ... + i_{rnx}); \qquad i_{ry} = (i_{r1y} + ... + i_{rny}). \tag{13}$$

Currents i_c in the supply line are determined by the formulas

$$i_c = \sqrt{i_{cx}^2 + i_{cy}^2}\,, \tag{14}$$

where

$$i_{cx} = i_{sx} + i_{kx}\,; \qquad i_{cy} = i_{sy} + i_{ky}\,,$$

$$i_{kx} = -\omega_0 u_{1y}\,; \qquad i_{ky} = \omega_0 u_{1x}\,.$$

The effect of capacitor capacitance on the magnitude of the current in the supply line and the power factor of the electric drive as a whole can be investigated on the basis of the calculation of static characteristics as a sequence of steady-state modes that correspond to constant values of slip *s.*

Changing the value of C in the required limits, we get the dependence of the currents of the AM and the supply network, and therefore the power factor ($\cos\varphi$) on the capacitors capacitance. From this dependence we choose the value C, which provides the required value $\cos\varphi$ of the starting electromagnetic torque.

The calculation of each static characteristic is performed in two stages: in the first stage determines the coordinate value under the condition *s* = 1,0 and under the given value of the supply voltage, and in the second stage, the parameter marching method determines the static characteristic as the dependence of the coordinates on the parameter taken for the independent variable. The values of the coordinates defined in the first stage of the calculation are the initial conditions for the second stage.

Research results

According to the above algorithms, computer programs were developed to calculate the modes and characteristics of the startup and steady-state modes of AM with different capacitor capacitance values and different ways of connecting them on. The following are examples of calculation results for an asynchronous motor 4AP160S4Y3 (P_n = 15 kW, U_n = 380/220 V).

In Fig. 3 couched the dependence of the rms values of electromagnetic torque, current and reactive power on the capacitors capacitance when slip *s* = 1,0 of AM, which is supplied from a three-phase power network through series connected capacitors. As can be seen from the figure, this dependence of torque has a pronounced maximum, which makes it possible to select the necessary value of the capacitors capacitance to provide the required startup properties.

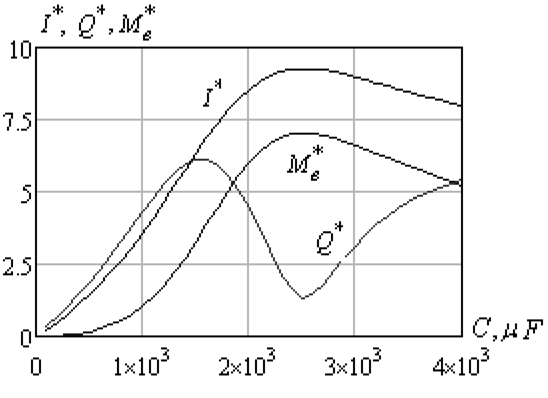


Fig. 3. Dependence of the driving electromagnetic torque (M_e^*), stator current (I^*) and reactive power (Q^*) at *s* = 1,0 on the value of the capacitors capacitance connected in series to the stator winding

Choosing capacitors requires not only determining the required capacitance but also determining the voltages on them. An example of the calculation results of the corresponding curves is shown in fig. 4.

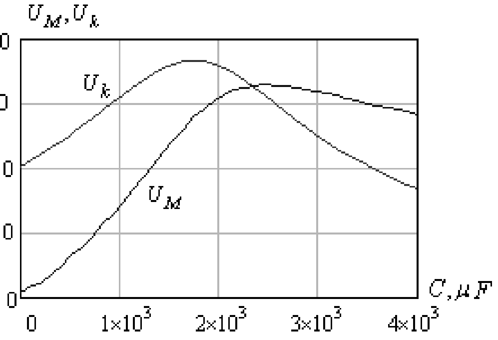


Fig. 4. Dependencies of voltage on series connected capacitors (U_k) and AM terminals (U_M) at *s* = 1,0 on capacitor capacitance.

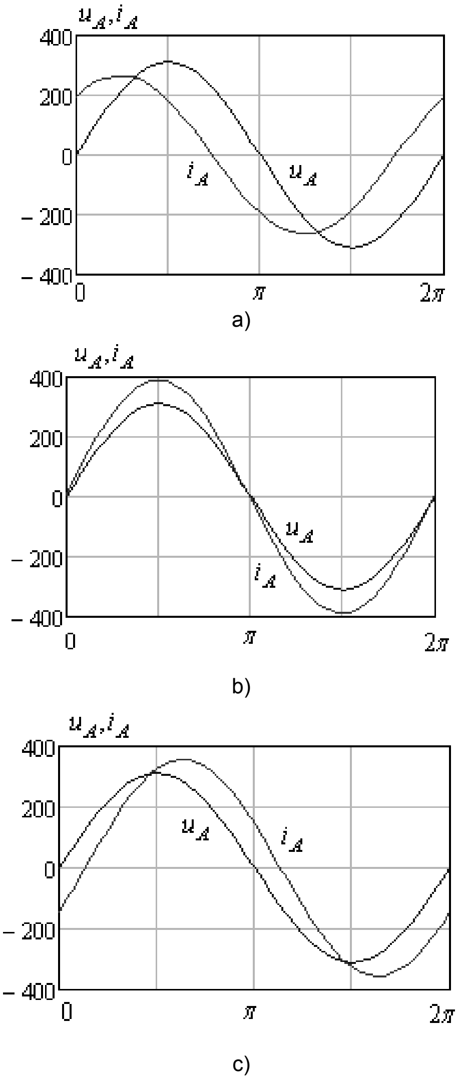
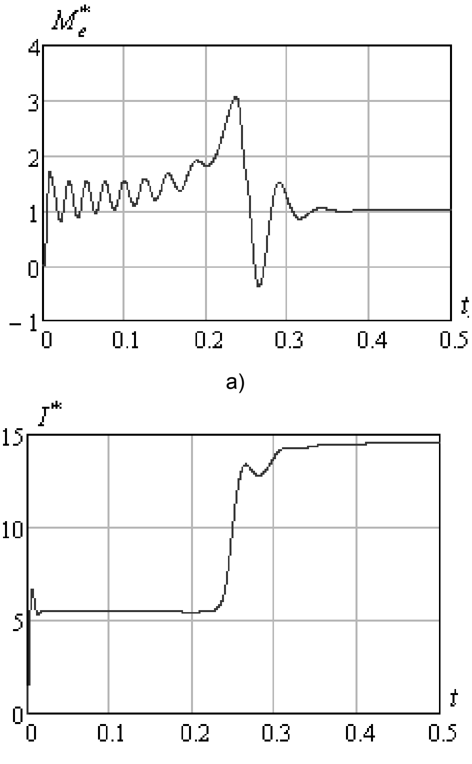


Fig. 5. Phase shift between the voltage and the current of the stator at *s* = 1,0 and different capacitance values connected on in series to the capacitor stator winding:
a) 1500 μF ; b) 2500 μF ; c) 3500 μF

As can be seen from fig. 3, at the value of the capacitance $C = 2500\mu F$ is full compensation of the reactive power Q of the motor. The maximum value of the electromagnetic torque and stator current corresponds to this point. Further increase in capacitor capacitance leads to overcompensation of reactive power. This is confirmed by the couched in fig. 5 by periodic voltage and current dependencies in phase *A* at three different capacitance values.

Having selected the capacitors, based on the condition providing of the driving torque and the permissible maximum values of current and voltages on the capacitors, it is necessary to check the startup mode for self-oscillation and resonance. To do this, it is necessary to calculate the transient process described by the electromechanical equilibrium DE (1) system. An example of the results of the performed calculations is shown in fig. 6.



a)

b)

Fig. 6. Dependences of the relative values of the electromagnetic torque (a) and the rms value of the stator current (b) AM 4A160M6Y3 on time during start-up with series connected the capacitors with capacitance *C*=1500 *μF*

As follows from fig. 6 a, b, in order to prevent negative phenomena during start-up, the series switching on of the capacitors must be replaced with a parallel one (fig. 2). Obviously, the capacitance of capacitor required for the working state mode must be different from the capacitance starting capacitor. To determine it, it is necessary to use a equations system of electric equilibrium, which describes the steady-state mode in the circuit of fig. 1. It consists of equations (10) and equations for the currents compiled by Kirchhoff's first law for an electric circuit (fig. 1), which in the x-axes have the form

$$i_{sx} + i_{kx} = i_x\,;\quad i_{sy} + i_{ky} = i_y \tag{15}$$

where i_x, i_y – current components in the AM power line; $i_{kx} = -\omega_0 C u_{sy}$; $i_{ky} = \omega_0 C u_{sx}$ – phase currents components in star-connected capacitors; *C* – capacitors capacity in one phase.

If we combine the *x*-axis with the generalized vector of the supply voltage, then $u_{sx} = U_m$; $u_{sy} = 0$, where U_m is the amplitude value of the phase voltage and the system of equations (15) takes the form

$$i_x - i_{sx} = 0\,;\quad i_y - i_{sy} = \omega_0 C U_m \tag{16}$$

The system of equations (11), (16) is solved according to the above algorithm.

By setting the values of the slip *s* in the range from one to the nominal value, it is possible to obtain a multidimensional static characteristic as a dependence of the vector of currents $\vec{i} = \vec{i}(s)$ on the slip, which makes it possible to calculate the dependence the capacitance value of the capacitors on the slip. To fully compensate for reactive power ($\cos\varphi = 1,0$) when changing the slip, the capacitance of the capacitors must be changed according to the formula

$$C(s) = \frac{i_y(s) - i_{sy}(s)}{\omega_0 U_m}\,. \tag{17}$$

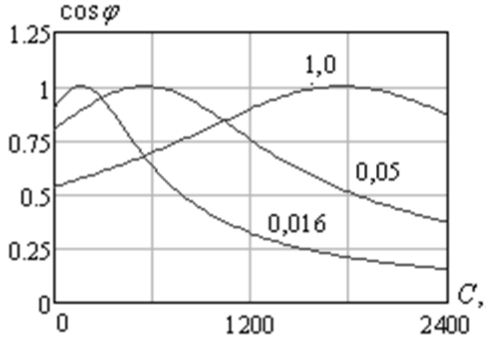


Fig. 7. Input dependencies of an asynchronous motor with in parallel switched capacitors on the value capacitance at three slip values *s*: 1,0; 0,05; 0,016.

Conclusions

The mathematical models developed allow us to study electromechanical processes in asynchronous electric drives with series and shunt reactive power compensation in starting and rated modes. On this basis, we can select the capacitance of the capacitors with series switched on capacitors for the successful startup of the motor, as well as determine their value for parallel switching on in operating modes in order to provide the required power factor. Using the developed mathematical models, it is possible to determine the optimal values of the capacitance from the point of view of providing the necessary characteristics of the electric drive and to identify those values that cause the appearance of resonant phenomena, leading to overcompensation of reactive power or significant amplitudes of oscillation of current and electromagnetic moment. They can serve as the basis for microprocessor control of a powerful asynchronous actuator by means of startup-compensating devices both during start-up and in operating mode.

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