Łukasz GRĄKOWSKI, Andrzej Ł. CHOJNACKI, Katarzyna GĘBCZYK, Kornelia BANASIK

Department of Power Engineering, Kielce University of Technology, Poland

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Application of the queuing theory to the description of the customer service process by the electricity distributor company

Abstract: *The paper presents the possibility of using queuing theory as a tool to improve the work of distribution companies in the field of customer service. The analyzes were carried out to determine the optimal number of energy emergency brigades so that the costs incurred by the electricity distributors would be minimal while maintaining the shortest time of failure removal. The authors also presented the most important theoretical issues concerning the queuing theory.*

Streszczenie: *W referacie przedstawiono możliwość zastosowania teorii kolejek jako narzędzia usprawniającego pracę spółek dystrybucyjnych w zakresie obsługi odbiorców. Przeprowadzone analizy miały na celu wyznaczenie optymalnej liczby brygad pogotowia energetycznego tak, aby koszty poniesione przez dystrybutorów energii elektrycznej były minimalne przy jednoczesnym zachowaniu jak najkrótszego czasu usuwania awarii. Autorzy przedstawili również najistotniejsze zagadnienia teoretyczne dotyczące teorii kolejek.* ***(Zastosowanie teorii kolejek do opisu procesu obsługi odbiorców przez dystrybutora energii elektrycznej).***

Keywords: queuing theory, customer service, distribution companies, energetics.
Słowa kluczowe: teoria kolejek, obsługa odbiorców, spółki dystrybucyjne, energetyka.

Introduction

Electricity in the producer-consumer relationship is treated as a commodity, so its highest quality must be ensured. The quality of electricity supply to consumers can be divided into [8, 9]:

- quality of electricity supplied (voltage quality),
- reliability of electricity supply,
- quality of customer (consumer) service.

Distribution companies are bound by the high-quality standards of customer service specified in [11]. They set out the obligations of distributors, the procedures of conduct, information and request to customers [2]. The quality of customer service by distributors consists of the following factors:

- handling reports, claims and complaints,
- elimination of energy supply disruptions,
- request of scheduled breaks,
- discounts for failure to meet energy supply conditions,
- providing information on energy supply,
- determination of forms and mode of energy settlements,
- determination of the conditions for connecting consumers to the power grid.

From the point of view of consumers, the most important thing is to maintain the continuity of electricity supply, which is why each distribution company has several powerline technician teams that repair failures in the power grid. This raises the problem of proper handling of customer requests by teams of powerline technicians. This issue can be analyzed using the mass service theory, abbreviated to the queuing theory.

The functioning of queues is visible in many areas of everyday life. However, there are cases in which this issue requires specific analysis and, as a result, determining the exact procedure to be followed. The problem of queues' service is directly related to the issue of costs of implementing specific processes (tasks). Then we are dealing with a dilemma of cost - quality [10]. This problem is not unknown to power companies either.

Queuing theory is one of the branches of operational research and is based on the theory of probability and mathematical statistics. The aim of the queuing theory is to develop methods to determine the values of indicators characterizing the service process, allowing to assess the quality of the queuing system, as well as to select the optimal organization and structure of the system [6].

Using the queuing theory, it is possible to determine the optimal number of teams of powerline technicians, so that the costs incurred by electricity distributors are minimal while maintaining the shortest possible fault removal time.

This article presents the possibility of using the queuing theory as a tool to improve the work of distribution companies in the field of customer service.

Theoretical fundamentals of queuing systems

The basic concepts associated with the queuing theory include [1, 6, 7, 10]:

- request - a request for a specific activity to be performed by the system,
- service - meeting a specific need reported to the system,
- input stream - a sequence of events requiring handling, appearing at the system input,
- service regulations - a way of selecting pending requests for service,
- servicing device - number and configuration of operating channels (workstations),
- output stream - a stream of requests obtained at the output of the system.

The operation of the mass service system is shown in the block diagram in Figure 1.

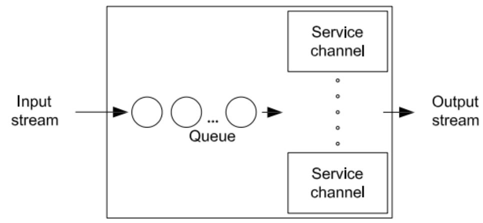

Fig. 1. Schematic diagram of the mass service system

Input stream is a statistical description of the process of inflow of requests to the service system. It is determined by the function of the distribution of time periods between successive requests. The number of requests received by the service system in the unit of time λ can be determined from the following relationship:

(1) $$\lambda = \frac{1}{\bar{t}_a} [requests\ per\ hour]$$

where: $\bar{t}_a$ – the average time between successive requests.

The service procedure is the next stage, following the request, describing the way of selecting requests for implementation. The basic service principles include [3]:

- FIFO (First-In, First-Out) – the oldest requests are directed to the service first,
- LIFO (Last-In, First-Out) – the latest requests are directed to the service first,
- SIRO (Selection in Random Order) – requests are handled randomly,
- preference-based selection – e.g. tasks with short service times can be handled first.

The service device is characterized by the number and configuration of service channels and the function of time schedule for one request or the number of requests handled in a unit of time. The number of requests handled in the unit of time *μ* can be determined from the following relationship:

(2) $$\mu = \frac{1}{\bar{t}_o} [interventions\ per\ hour]$$

where: $\bar{t}_o$ – average time of request handling.

Another very important parameter describing the queuing system is the utilization factor *ρ*, also called traffic intensity. It is the ratio of the average number of requests received to the system in a unit of time to the average number of requests handled in a unit of time; it can be determined from the following relationship:

(3) $$\rho = \frac{\lambda}{s \times \mu}$$

where: *s* – number of stations intended for service.

This parameter characterizes the stability of the system. A queuing system is considered to be stable when $\rho < 1$, which means that the number of requests handled in a unit of time is larger than the number of requests arriving to the service system in a unit of time.

In order to identify the queuing system and the mathematical model which corresponds to it, a code is used, in which information about the system's belonging to the relevant group is contained [4, 5]. When describing a queuing system, one can use the classification developed by D. Kendal, in which the queuing system is marked in the following way:

(4) $$X/Y/m$$

where:*X* – type of distribution of the input stream of requests to the system, *Y* – type of distribution of times for handling requests, *m* – number of service channels.

The following symbols are used to indicate the types of input stream distributions and service times:

D – determined or regular stream,

M – exponential distribution of service times or time intervals between adjacent requests, i.e. the Poisson distribution of arrivals,

E_k – Erlang distribution of the *k*-th order of service times or time intervals between adjacent requests,

H_r – hyperexponential distribution of the order *r*,

C_k – Cox distribution of the *k*-th order,

G_I – general type stream, unrestricted and independent,

G – a stream with unrestricted schedule of service times.

As mentioned above, service devices are characterized by the number and configuration of service channels. A distinction is made between systems with single service channels and with multiple service channels.

Figure 2 presents a schematic diagram of a mass service system with a single service channel.

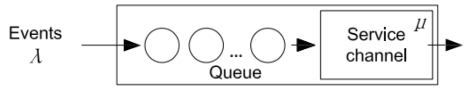

Fig. 2. Schematic diagram of a mass service system with a single service channel

The probability that there are no requests in system with a single service channel (the system is empty) can be determined from the following relationship:

(5) $$p_n = 1 - \rho$$

The probability of an event that there are *n* requests in queue is described by the following relationship:

(6) $$p_n = (1 - \rho) \times p^n, n = 0,1,2,...$$

The probability of an event that more than n_0 requests are waiting in queue can be determined as follows:

(7) $$p_{n>n_0} = p^{n_0+1}$$

The average queue length (number of requests) can be determined as follows:

(8) $$L_q = \frac{\lambda^2}{\mu \times (\mu - \lambda)} = \frac{\rho^2}{(1-\rho)} [requests]$$

The probability that a request will spend more than t_0 time units in queue is described by the following relationship:

(9) $$p_{t>t_0} = p \times e^{-t_0 \times (\mu - \lambda)}$$

The average queue waiting time can be determined as follows:

(10) $$W_q = \frac{\rho}{\mu \times (1-\rho)} [h]$$

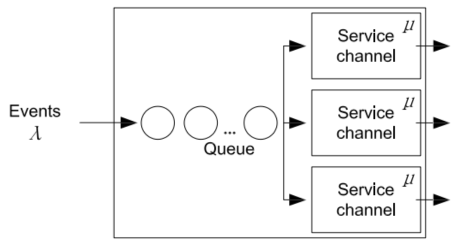

Fig. 3. Schematic diagram of a mass service system with multiple service channels

Figure 3 presents a schematic diagram of a mass service system with multiple service channels. The efficiency of a single-channel system can be increased by increasing the number of workstations. In a system with *s* service stations, the output intensity *μ* will increase *s*-fold. This has a significant impact on the intensity of traffic *ρ*, so that the system can increase the number of requests while remaining stable.

The probability of an empty system can be determined in this case from the following relationship:

(11) $$p_0 = \frac{1}{\sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \frac{\rho^s}{(s-\rho) \times (s-1)!}}$$

The probability of an event that there are n requests in queue is described by the following relationship:

$$p_n = \frac{\rho^n}{n!} \times p_0 = \frac{\lambda^n}{n! \times \mu^n} \times p_0, n < s \quad (12)$$

$$p_n = \frac{\rho^n}{s! \times s^{n-s}} \times p_0 = \frac{\lambda^n}{s! \times s^{n-1} \times \mu^n} \times p_0, n \geq s \quad (13)$$

The probability of an event that more than n_0 requests are waiting in queue can be determined as follows:

$$p_{n>n_0} = \frac{s^{s-n_0} \times \rho^{n_0+1} \times p_0}{(s-\rho) \times s!} \quad (14)$$

The average queue length (number of requests) can be determined as follows:

$$L_q = \frac{\rho^{s+1} \times p_0}{(s-1)! \times (s-\rho)^2} [requests] \quad (15)$$

The probability that a request will spend more than t_0 time units in a queue is described by the following relationship:

$$p_{t>t_0} = p(n > s-1) \times e^{-\mu \times t_0 \times (s-\rho)} \quad (16)$$

The average time a request spends in queue can be determined as follows:

$$W_q = \frac{L_q}{\lambda} [h] \quad (17)$$

In practice, one more variant of the mass service system can be found. Figure 4 presents a schematic diagram of a mass service system with both multiple queues and multiple service channels. This system can be treated as a combination of several systems with a single service channel. Therefore, in order to describe this system, relationships describing a system with a single service channel can be used.

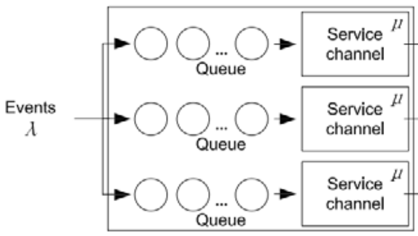

Fig. 4. Schematic diagram of a mass service system with both multiple queues and multiple service channels

Application of mass service systems for modeling of customer service systems by a distributor of electricity

Thus far, emergency calls have been handled locally by dispatchers, but more and more often, due to the use of state-of-the-art systems, distribution companies can receive and register calls 24 hours a day, 365 days a year. Requests are directed to a telephone consultant located in a request handling center. First, the customers calling the emergency number are serviced by an automatic system of informing about power outages. This system also provides information about the planned time of power restoration, in the event of power outages due to planned works, as well as the expected time of power restoration, in the event of power outages due to unforeseen failures. In the case of an urgent requests, customers can also be redirected to a consultant, such requests have the highest priority. Then, after verification, the consultant or dispatcher directs the request for implementation by a team of powerline technicians.

Distribution company's team of powerline technicians repair damaged distribution network components at any time of the day, 24 hours a day. Failures which endanger life and health, e.g. overturned power line poles, broken wires, cables lying on the ground, etc. are handled first. Then power is restored to industrial facilities and houses. It should be noted that each distribution company has several teams of powerline technicians operating in shifts. For financial reasons, there is only one team of powerline technicians on duty on Sundays and public holidays. Three or more teams of powerline technicians only work during the first shift, i.e. from 7.00 am to 3.00 pm. Despite the fact that there are three teams of powerline technicians on duty, usually only one of them handles current breakdowns. The other two are involved in scheduled work. Only in the case of a very large number of requests, all three teams of powerline technicians handle failures. This can lead to a significant increase in the duration of power outages. In this case, electricity distributors should maintain a "balance" between the cost of maintaining teams of powerline technicians and the acceptable duration of power outages.

In order to apply the queuing theory to the description of the process of customer service by electricity distributors it is necessary to make certain assumptions:

- the number of teams of powerline technicians should be understood as the number of service channels,
- only the power outages caused by failures in the LV grid were analyzed.

The observation period covered 9 years. At that time a total of 15576 failures occurred. The average duration of power outages was $\bar{t}p$ = 5.68 h. On the basis of these data, the input stream and the intensity of service were determined, which are as follows:

- *λ = 0.20 requests per hour,*
- *μ = 0.18 interventions per hour.*

First of all, a variant with one team of powerline technicians was considered, s = 1. For this case, the utilization factor is ρ = 1.11. Therefore, it can be concluded that the tested system is unstable and the probability of a long queue increases. Considering the fact that from 3.00 p.m. on Friday to 7.00 a.m. on Monday there is only one team of powerline technicians on duty, which means that on Monday at 7.00 a.m. the average queue waiting time is W_q = 1.28 h.

Reducing the time a request waits in queue is only possible if the operating conditions change. To this end, the number of teams of powerline technicians must be increased to two, s = 2. An increase in the number of teams of powerline technicians shall result in a reduction in the utilization factor, which in this case shall be ρ = 0.56. In addition, due to having two teams of powerline technicians the system becomes stable and further calculations can be made.

For a case with two teams of powerline technicians, the following values were determined:

- the probability of no queue:

$$p_0 = 0.56$$

- the probability that there are n request in the queue:

$$p_1 = 0.31; p_2 = 0.08; p_3 = 0.02; p_4 = 0.01; p_5 = 0.002$$

- the probability that a request will be queued up for more than half an hour:

$$p_{t>0.5} = 0.11$$

- average queue waiting time:

W_q = 0.25 h

- the average queue length:

L_q = 0.05 requests

Analyzing the results, one can notice a high probability of handling of a request without a queue *(0.56*) and a fairly high probability that only one request is waiting to be handled in queue *(0.31*). In addition, it was noted that there was a low probability of a request waiting longer than half an hour for handling *(0.11*) and the relatively short average waiting time *(0.25 h*). The average queue length is also low *(0.05 requests*).

In order to further improve the operation of the system, the number of teams of powerline technicians should be increased to three, *s = 3.* Then, the utilization factor is ρ = *0.37*. Therefore, it is even lower than in the case with two teams of powerline technicians.

For a case with three teams of powerline technicians, the following values were determined:

- the probability of no queue: $p_0 = 0.68$
- the probability that there are n request in queue: $p_1 = 0.05$; $p_2 = 0.01$; $p_3 = 0.006$
- the probability that a request will be queued up for more than half an hour: $p_{t>0.5} = 0.01$
- average waiting time in queue: $W_q = 0.005$ *h*
- the average queue length: $L_q = 0.001$ *requests*

Analyzing the results, one can notice a very high probability of handling of a request without a queue *(0.68*) and a very low probability that there are other request in queue waiting to be handled. In addition, it was noted that there was a very low probability of a request waiting longer than half an hour for handling *(0.01*) and the very short average waiting time *(0.005 h*). The average queue length is also very short *(0.001 requests*).

Conclusions

The paper presents the possibility of applying the queuing theory to the analysis of the process of customer service by distributors of electricity. The research shows that one team of powerline technicians is insufficient to ensure the adequate quality of customer service, making the system unstable, and the probability of a long queue increases. In this case, the average queue waiting time on a Monday morning is $W_q = 1.28$ *h*. For financial reasons, distribution companies leave only one team of powerline technicians on duty during night shifts, on Sundays and public holidays. What is more, despite the fact that there are three teams of powerline technicians on duty, usually only one of them handles current breakdowns. The other two are involved in scheduled work. In the light of the results obtained, it can be concluded that such a state of affairs leads to a significant extension of service time and an increase in the duration of power outages, i.e. interruptions in supplying energy to customers, which as a result does not create a positive marketing image of distribution companies. In order to ensure proper functioning of the system, current failures should be handled by at least two teams of powerline technicians. This solution allows the system to become stable. In the case of at least two teams of powerline technicians a high probability of handling of a request without a queue p_0 and a fairly high probability that only one request is waiting to be handled in queue p_1 can be seen. It has also been noted that there is a low probability that a request will wait longer than half an hour for handling $p_{t>0.5}$. For the sake of comparison, in a case with three teams of powerline technicians, the probability of handling of a request without a queue p_0 is even higher. Whereas, the probability that only one request is waiting for handling in queue p_0 and the probability that a request will wait longer than half an hour for handling $p_{t>0.5}$ is negligible, it practically equals zero. Differences can also be seen by analyzing the average time a request waits in queue W_q and the average queue length L_q. In case of two teams of powerline technicians $W_q = 0.25$ *h and* $L_q = 0.05$ *requests*. In case of three teams of powerline technicians, however, these values are even lower, i.e. $W_q = 0.005$ *h* and $L_q = 0.001$ *requests*. Despite the fact that the parameters characterizing the queue system for cases with two and three teams of powerline technicians are definitely satisfactory, it should be noted that on Saturdays and Sundays there is only one team on duty. As a result of this, queue on these days is constantly increasing and its decrease will only take place on Monday mornings after all teams arrive at work.

The research carried out confirmed the legitimacy of using the queuing theory to analyze the process of customer service by electricity distributors. The application of mass service theory can play a significant role in assessing the quality of functioning of existing service systems as well as in the modeling of new systems. The results of the analysis can be used to select an appropriate organization of the system's operation, which, as a result, may shorten service time significantly.

Authors: *Łukasz Grąkowski, M.Sc. Eng., Andrzej Ł. Chojnacki, PhD. Eng., Katarzyna Gębczyk, M. Sc. Eng., Kornelia Banasik, M.Sc. Eng., Faculty of Electrical Engineering, Automatic Control and Computer Science, Department of Power Engineering, Kielce University of Technology, Poland, e-mails: lgrakowski@tu.kielce.pl, a.chojnacki@tu.kielce.pl, kgebczyk@tu.kielce.pl, k.banasik@tu.kielce.pl*

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