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Inductor shape determination for electromagnetic forming of sheet workpieces

Abstract. *The method of determining a cross section of an inductor which provides specified distribution of magnetic induction and consequently distribution of deforming forces on a flat sheet workpiece during the electromagnetic forming process is developed. The problem of continuation of a magnetic field from a flat boundary surface of an ideally superconducting half-space with given odd induction distribution is solved. Parameters, which allow us to minimize a difference between given and obtained induction distribution are determined.*

Streszczenie. *Analizowano kształt elektromagnesu umożliwiający równomierny rozkład deformacji cienkiego arkusza blachy podczas tłoczenia wyrobu. Analizowano układ z zasilaniem impulsowym.* ***Analiza kształtu elektromagnesu używanego do tłoczenia kształtu w arkuszu blachy***

Keywords: electromagnetic forming, sheet workpiece, inductor, inductor cross-section.
Słowa kluczowe: wytłaczanie elektromagnetyczne, elektromagnes.

Introduction

Electromagnetic forming is an impulse or high-speed forming technology, which uses a pulsed magnetic field to apply forces to a tubular or sheet metal workpiece [1]. In this technology, there is need to form parts with straight-line grooves of a complex shape [2]. A capacitor bank *C* is charged and then quickly discharged through a triggered controlled spark gap *S* into a special inductor *1* placed near a sheet metallic workpiece *2* (fig. 1). The inductor is made of high conductive and strong enough material (for example bronze, brass, etc.). Rapidly changing magnetic field induces eddy currents within the workpiece. Because of interaction between these currents and the outer magnetic field the workpiece is accelerated into a die *3*, becoming a detail *4* with required grooves. It’s needed to find a cross-section shape (profile) of an inductor which provides odd distribution of plane magnetic field on a flat sheet metal workpiece to obtain a needed groove shape.

The profile of such inductors can be defined through formulating and solving of the problem of magnetic field continuation from the flat workpiece boundary. Solutions of this problem are known and described in [3,4] for axisymmetric pulsed magnetic fields. Straight-line parallel parts *AB* and *CD* are similar to a system of two straight buses with opposite currents. These parts are much longer than the distance between them (fig. 1). Such inductor creates a magnetic field which is close to plane field in the working area, while induction distribution is an odd function of x-coordinate.

The objective is to solve the problem of magnetic field continuation from the flat boundary in elementary functions and determine an inductor shape for generating given distribution of magnetic field on the flat surface of a workpiece using this solution.

Main assumptions, formulation and solution of the problem by the method of partial solutions

Usually in the electromagnetic forming process the sharp skin-effect appears due to relatively high frequency (~10kHz and more) of the pulse through the inductor. So, currents flow in thin surface layers within the inductor and the workpiece. Therefore, the ideal skin-effect approximation is used, according to which we suppose that currents flow within the infinitely thin surface layers (such approximation is also right for ideal superconductors). Also assume that the magnetic field in the work area is plane, i.e. an influence of the edge parts *AD* and *BC* of the inductor is neglected.

Consider a plane magnetic field in a nonconductive and nonmagnetic space $y > 0$ above the ideally superconductive half-space $y < 0$, which we use to represent the effect of the workpiece (fig. 2). In Cartesian coordinates x, y, z magnetic induction $\vec{B}$ and vector potential $\vec{A}$ have such structures:

$$\vec{B}\{B_x; B_y; 0\},\ \vec{A}\{0;\ 0;\ A_z\},$$

where B_x, B_y, A_z are projections of the vectors onto the related axes of Cartesian coordinates.

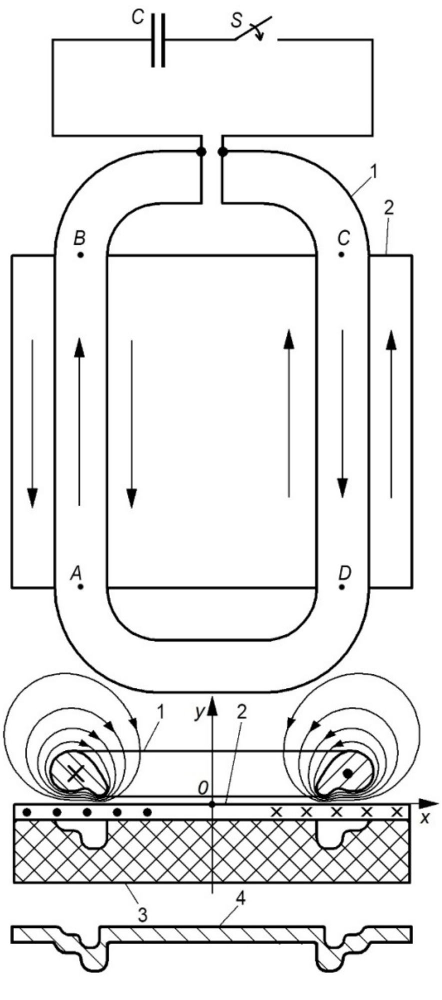

Fig. 1. A Device for electromagnetic forming of flat workpieces.

According to the structure the magnitude of $\vec{A}$ equals to A_z projection. Because the field is plane, it will be considered in coordinate plane xOy ($z = 0$). Formulation of the problem of magnetic field continuation into the top half-plane $y > 0$ from the x-axis consist of the Laplace equation

(1) $$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = 0$$

and two boundary conditions:

(2) $$A(x,0) = 0;$$

(3) $$\left.\frac{\partial A}{\partial y}\right|_{y=0} = B_x(x,0),$$

where $B_x(x,0)$ is given distribution of the magnetic field on the x-axis.

Notice that the formulation may be also prepared for the force function of the plane magnetic field $v_m(x,y)$ [5,6], that is connected with the vector potential by relation

$$A(x,y) = -\mu_0 v_m(x,y),$$

where μ_0 is magnetic constant.

The problem (1)-(3) is well known in mathematical physics as the Cauchy problem for the Laplace equation. Its numerical solutions may be unsustainable [7]. Therefore, let's get an analytical solution of the problem using the method of partial solutions that are continuously depended on λ-parameter [8]. Such solution for odd distribution $B_x(x,0)$ is

(4) $$A(x,y) = \sqrt{\frac{2}{\pi}} \int_0^\infty F_B(\lambda) \lambda^{-1} \sin(\lambda x) \operatorname{sh}(\lambda y) d\lambda,$$

where $F_B(\lambda)$ is the Fourier sine transform for the given induction distribution,

$$F_B(\lambda) = \sqrt{\frac{2}{\pi}} \int_0^\infty B_x(x,0) \sin(\lambda x) dx.$$

In this x-projection of magnetic induction at the boundary $y = 0$ is [9]

(5) $$B_x(x,0) = \left.\frac{\partial A}{\partial y}\right|_{y=0}.$$

On the other hand

(6) $$B_x(x,0) = \sqrt{\frac{2}{\pi}} \int_0^\infty F_B(\lambda) \sin(\lambda x) d\lambda.$$

It's necessary to know the Fourier sine transform $F_B(\lambda)$ of the given distribution $B_x(x,0)$ to use solution (4), but the number of such distributions is low.

Solutions of problem (1)-(3) in elementary functions

An idea of the method is that the parts *AB* and *CD* of the inductor can be replaced by a system of pairs of parallel axes with currents. In each pair of the top half-space $y > 0$ axes placed symmetrically about y-axis, and currents in these axes are equal in value and have opposite directions. According to the method of mirror images [9] the effect of the ideally superconducting half-space can be replaced by a system of axes placed in the lower half space $y < 0$ symmetrically about x-axis. Each initial pair of the axes has a reflected one with opposite currents. Next, we build magnetic field lines which cover all initial axes in right half-space $x > 0$. One of these lines may be chosen as the required contour of the inductor's cross section.

The simplest system consists of two pairs of axes (fig. 2): initial pair is located at points M_1 and M_2 in the top half-space $y > 0$ and image pair is located at points M_1' and M_2' in the lower half-space $y < 0$. Magnetic vector potential in area $y > 0$ equals

(7) $$A(P) = \frac{\mu_0 I}{2\pi} \ln\left(\frac{r_{M_1'P}}{r_{M_1P}} \frac{r_{M_2P}}{r_{M_2'P}} \right),$$

where P is an observation point which location is defined by x and y coordinates; r_{M_1P}, $r_{M_1'P}$, r_{M_2P}, $r_{M_2'P}$ are distances between corresponding axes and observation point P in the plane xOy.

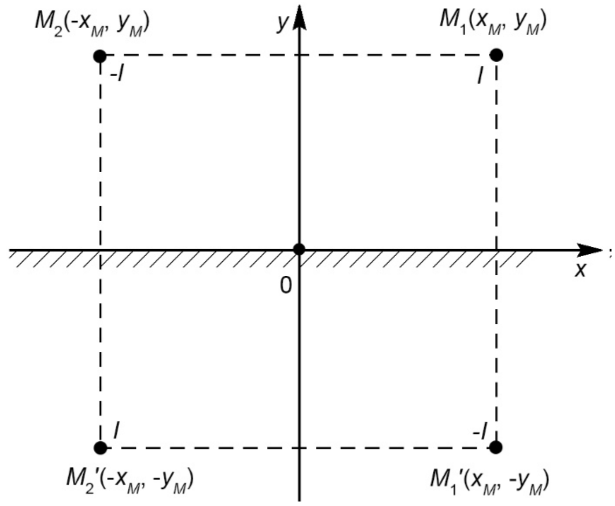

Fig. 2. Calculation system of one initial pair of the axes with currents

Putting solution (7) in formula (5) leads to the next relation for considering system of parallel axes with currents

(8) $$B_x(x,0) = \frac{\mu_0 I y_1}{\pi} \left[\frac{1}{(x - x_1)^2 + y_1^2} - \frac{1}{(x + x_1)^2 + y_1^2} \right],$$

where x_1, y_1 are coordinates of point M_1.

Can be seen that formula (7) fulfill equation (1) anywhere, besides points M_1, M_1', M_2, M_2', and boundary condition (2). Suppose that the function $B_x(x,0)$ from boundary condition (3) accepts values defined by formula (8). Then, it can be argued that relation (7) is the solution of the magnetic field continuation problem from the x-axis into the top half-space $y > 0$ for corresponding given distributions $B_x(x,0)$.

Fourier sine transform is known for distribution (8) [10]:

(9) $$F_B(\lambda) = \mu_0 I \frac{\sqrt{2}}{\pi} e^{-\lambda y_1} \sin(\lambda x_1).$$

Putting (9) into (6) leads to the next relation

(10) $$B_x(x,0) = \frac{2\mu_0 I}{\pi} \int_0^\infty e^{-\lambda y_1} \sin(\lambda x_1) \sin(\lambda x) d\lambda.$$

The induction distributions on the boundary $y = 0$ that are calculated using formulas (8) and (10) don't differ significantly. It confirms correctness of considered methods. However much simpler formula (8) is more preferred.

According to the linearity of the problem and the superposition principle we can use initial system of *n* pairs of parallel axes with currents I_k, $-I_k$, $k=\overline{1,n}$, which generates required odd distribution of magnetic induction. In this

(11) $$A(P)=\frac{\mu_0}{2\pi}\sum_{k=1}^{n} I_k \ln\left(\frac{r_{M'_{k,1}P}}{r_{M_{k,1}P}}\frac{r_{M_{k,2}P}}{r_{M'_{k,2}P}}\right),$$

(12) $$B_x(x,0)=\frac{\mu_0}{\pi}\sum_{k=1}^{n} I_k y_k\left[\frac{1}{(x-x_k)^2+y_k^2}-\frac{1}{(x+x_k)^2+y_k^2}\right]$$

where $M_{k,1}$ is a point in the first quadrant of coordinate plane xOy ($x>0$, $y>0$) with coordinates x_k, y_k, which determines position of the first axis with current I_k of *k*-th axes pair; point $M_{k,2}$ is placed symmetrically about *y*-axis with coordinates $-x_k$, y_k and determines position of the second axis with opposite current $-I_k$.

Examples of calculation.

Examples of given distributions $B_x(x,0)$ are shown in fig. 3 and 5 only for $x > 0$. These distributions have odd continuation in the area $x < 0$. The curves in fig. 3 corresponds to the simplest system of axes shown in the fig. 2 and are calculated by formula (8): for curve *1* x_1*, y_1* equal to 0.1 and 0.1, *2* – 0.5 and 0.1, *3* – 0.25 and 0.05, *4* – 0.5 and 0.05. Designations: $x^* = x / l_b$; $y^* = y / l_b$; $B^* = B / B_b$; $B_b = \mu_0 I / l_b$; l_b – basis length.

Can be seen that decrease in parameter y_1 leads to increasing of the amplitude value of induction, while change in x_1 leads to offset of the peak. Therefore, coordinates of point *M* are parameters of distribution (8). Changing them we can change the shape of given magnetic induction distribution.

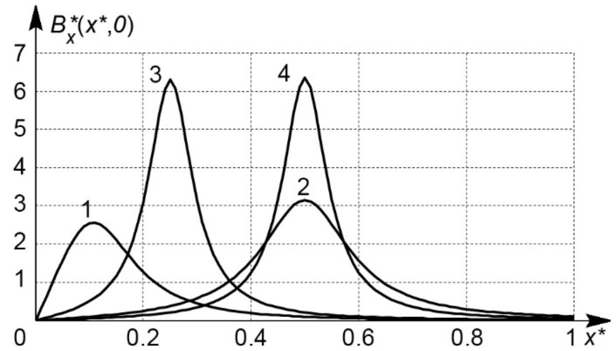

Fig. 3. Distributions of magnetic induction on the flat boundary in case of one initial pair of axes with currents

Magnetic field lines for one pair of axes with currents and coordinates $x_1^* = 0.1$, $y_1^* = 0.1$ which are calculated using (7) are shown in fig. 4. These lines correspond to distribution *1* in the fig. 3. For curve *1* - $A^*(x^*, y^*)$ = 0.05, *2* – 0.1, *3* – 0.15, *4* – 0.2. In this

$$A^*(x^*,y^*)=\frac{A(x^*,y^*)}{\mu_0 I}.$$

Inductor profile shown in the fig. 1 is obtained using initial system of two pairs of axes with currents (n = 2) and parameters x_1*= 0.75, y_1*= 0.1, I_1*= 1.0, x_2*= 1.0, y_2*= 0.2, I_2*= 1.5 ($I_k^* = I_k / I_1$). In fig. 5 it's shown induction distribution which provide required groove shape (fig. 1) and magnetic field lines that cover all initial axes in the right half-space x > 0 (*b*). These data were calculated by (11), (12). Line *1* corresponds to $A^*(x^*, y^*)$ = 0.1, *2* – 0.15, *3* – 0.2, *4* – 0.25, *5* – 0.3. Line *5* was used to define inductor profile which is shown in fig. 1 with some other magnetic field lines.

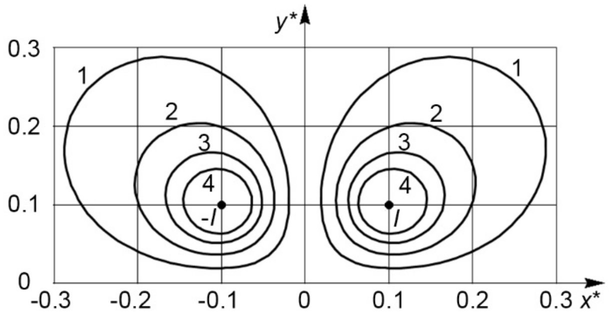

Fig. 4. Magnetic field lines above the flat boundary in case of one initial pair of axes with currents

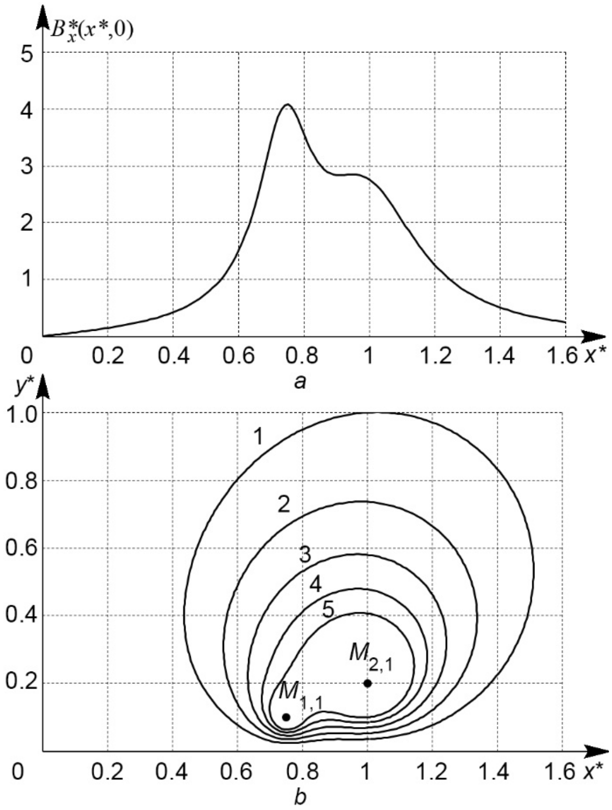

Fig. 5. Given induction distribution on the flat workpiece boundary (*a*) and magnetic field lines of system of two initial pairs of axes with currents in the first quadrant (*b*)

Conclusions

1. Solution of the problem of plane pulse magnetic field continuation from the flat boundary of a conductor with ideal skin effect approximation and odd given distribution of magnetic induction can be presented as magnetic vector potential of two systems of parallel axes with currents. Each pair of initial system is symmetrically about the *y*-axis and has a reflected pair with opposite currents that is placed symmetrically about the *x*-axis in the lower half-space.

2. Currents and coordinates of the axes are parameters that can be varied to achieve required distribution of magnetic induction on the processed surface of a flat sheet metal workpiece.

3. Any of these lines which cover all initial axes located in the first quadrant ($x > 0$, $y > 0$) can be chosen as a cross-section contour of an inductor which generates required given induction distribution.

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