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LQR controller with an integral action for Z-source DC-DC converter

***Abstract**. This paper presents a robust linear quadratic regulator with an integral action (LQR+i) designed for Z-source DC-DC converter (ZSC) operating in conduction continuous mode (CCM). Depending on converter's commutation states and using the electrical equivalent circuits, both switched and small-signal models of ZSC are built. The design procedure of LQR + i controller is described. The robustness of the controller is tested, using Matlab/Simulink software, considering circuit parameter (source and load) uncertainties and external signal (reference voltage) disturbance. A comparison study with classical PI controller are performed. It has been shown that the robustness of LQR + i controller is better than classical PI controller.*

Streszczenie.** W artykule zaprezentowano liniowy, kwadraturowy sterownik w włączonym LQR zaprojektowany do przekształtników DC-DC ze źródłem Z. Odporność kontrolera była testowania przy wykorzystaniu programu Matlab/Simulink. Porównano sterownik z klasycznym układem PI.* ***Sterownik z własną akcją LQR do przekształtników DC-DC ze źródłem Z

Keywords: DC-DC Converter, Z-Source, LQR Controller, PI Controller.
Słowa kluczowe: przekształtnik DC-DC, sterownika LQR

Introduction

Power electronics *DC-DC* converters became a key part of renewable energy conversion systems such as photovoltaic, wind and fuel cell [1-3]. Generally these systems are sources of low voltage and power, therefore high-voltage step-up *DC-DC* converters [4, 5] are required as an interface between the voltage source and output load in order to provide high output voltage. With conventional boost converters, it is complicated to obtain high voltage gain, mainly because requirement of extreme duty cycle, which a high stress on switching devices is produced [6]. Using extreme duty cycle may also lead to poor dynamic responses to line and load variations.

In order to increase the voltage gain and to avoid extreme duty cycle, the Z-source *DC-DC* converter (ZSC) has been appeared as an alternative power conversion topology that can both reduce and increase the input voltage using only a *LC* impedance network and one active switch, a thing that cannot carry out with the traditional converters [7]. Figure 1 represents the basic topology of ZSC, which consists of two inductors (*L*1 and *L*2) and two capacitors (*C*1 and *C*2) connected in X form for coupling the main circuit of converter to the power supply, which provides an amplification means of the input voltage. The ZSC may be open or shorted, without the risk of damage the switching devices [7, 8]. Due to this particular structure, ZSC has a switching state in which the load terminals are shorted to switch terminals. This state is called shoot-through (ST).

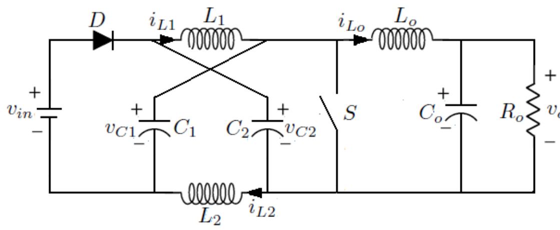

Fig. 1. Basic topology of Z-source DC-DC converter

ZSC is switched nonlinear system. It represents a major challenge in the control design. In the most studies that have addressed the ZSC control, conventional controllers were applied [9-11]. These controllers are designed using conventional linear control techniques in which the small signal model is derived from the linearization around a nominal point of space state average model [12, 13]. In these techniques the switching effects are averaged when the converter is operating in steady state and the system can be treated as a linear system. However, switching is not the only source of nonlinearities in *DC-DC* converters, the changes in system parameters, well-known as parametric uncertainty, can also be another non linearity sources. A problem of interest for *DC - DC* converters is to maintain stability and good regulation of the output voltage for parametric uncertainties and/or under external disturbances. One solution to overcome the parametric uncertainty problems is the use of the optimal control [14]. Linear quadratic regulator (*LQR*) which is one of methods of optimal control has been widely developed and successfully used in *DC-DC* converters [15-25].

In this paper, linear quadratic regulator with an integral action (*LQR*+*i*) technique is designed to regulate the Z-source DC-DC converter output voltage. For this purpose, the following contents are addressed: Section 2 describes both the switched model and the small-signal model of the converter; Section 3 presents the fundamentals and design method of *LQR+i* controller; Section 4 presents the simulation results for the designed controller and a comparison with a conventional PI controller. Finally, the general conclusion of the paper is presented in section 5.

Converter modeling

It is assumed that the ZSC operates in continuous conduction mode (CCM). Considering $L_1 = L_2 = L$ and $C_1 = C_2 = C$ then $i_{L1} = i_{L2} = i_L$ and $v_{C1} = v_{C2} = v_C$ [2]. Depending on the state of the switch S and during one cycle switch, ZSC has two operation modes : The Non Shoot-through mode (NST) (Fig.2-a) and Shoot-through (ST) mode (Fig. 2-b). The first mode occurs when diode D is closed (ON) and S is open (OFF). When D is OFF and S is ON occurs the second mode.

Both modes are described by affine time invariant differential equations $\dot{x} = A_1 x + B_1 v_{in}$ and $\dot{x} = A_2 x + B_2 v_{in}$ respectivily. x is the vector of state variables i_L, v_C, i_{Lo} and v_o. The switched model that describes the ZSC is given by:

$$\text{(1)} \qquad \begin{cases} \dot{x} = [(1-d)A_1 + dA_2]x + [(1-d)B_1 + dB_2] \\ y = v_o = Cx \end{cases}$$

where $d \in \{0, 1\}$ is the control signal, v_{in} is the input voltage, v_o is the output voltage and L_o, C_o and R_o are the parameters output filter and load respectively. The state matrices are given by:

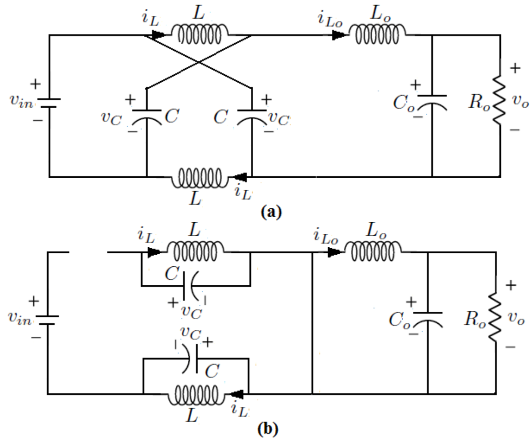

Fig. 2. (a) Non Shoot-through mode. (b) Shoot-through mode

$$A_1 = \begin{bmatrix} 0 & \frac{-1}{L} & 0 & 0 \\ \frac{1}{C} & 0 & \frac{-1}{C} & 0 \\ 0 & \frac{2}{L_o} & 0 & \frac{-1}{L_o} \\ 0 & \frac{1}{C_o} & 0 & \frac{-1}{R_oC_o} \end{bmatrix}, \quad B_1 = \begin{bmatrix} \frac{v_{in}}{L} \\ 0 \\ -\frac{v_{in}}{L} \\ 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & \frac{-1}{L} & 0 & 0 \\ -\frac{1}{C} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{L_o} \\ 0 & \frac{1}{C_o} & 0 & \frac{-1}{R_oC_o} \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$$

In the steady state X_{ss}, the values of voltages and currents are given by equation (2).

(2)
$$X_{ss} = \begin{bmatrix} I_{Lss} \\ V_{Css} \\ I_{Loss} \\ V_{oss} \end{bmatrix} = \begin{bmatrix} \left(\frac{1-D}{1-2D}\right)^2 \frac{V_{in}}{R_o} \\ \left(\frac{1-D}{1-2D}\right) V_{in} \\ \left(\frac{1-D}{1-2D}\right) \frac{V_{in}}{R_o} \\ \left(\frac{1-D}{1-2D}\right) V_{in} \end{bmatrix}$$

where D is the duty cycle given by $D = \frac{(V_{oss} - V_{in})}{(2V_{oss} - V_{in})}$. The control objective in power converter is to enforce v_o to track a given constant reference voltage V_{ref} (eq. (3)).

(3)
$$X_{ss} = X_{ref} = \left(\frac{V_{ref}^2}{V_{in}R_o}, V_{ref}, \frac{V_{ref}}{R_o}, V_{ref}\right)^T$$

As can be seen, the model obtained in equation (1) presents products between state variables and the control signal so that which is considered a nonlinear model. To linearize it, a common approach is to apply the perturbation and linearization technique around the operating point (steady state) to obtain the linear small signal model [12]. The state (x), input (v_{in}) and control signal (d) variables are decomposed into the sum of a steady state value (X_{ss}, V_{in} and D) plus a disturbed value ($\tilde{x}$, $\tilde{v}_{in}$ and $\tilde{d}$) as follow:

$$x = X_{ss} + \tilde{x};\ v_{in} = V_{in} + \tilde{v}_{in};\ d = D + \tilde{d}$$

The linearisation around the steady state (eq . (3)) gives the following small signal linear model:

(4)
$$\dot{\tilde{x}} = [(1-D)A_1 + DA_2]\tilde{x} + [(1-D)B_1 + DB_2]\tilde{v}_{in} + [(A_1 - A_2)X_{ss} + (B_1 - B_2)V_{in}]\tilde{d}$$

We can rewrite equation (4) in the following compact form:

(5)
$$\begin{cases} \dot{\tilde{x}} = A\tilde{x} + B\tilde{u} \\ \tilde{y} = C\tilde{x} \end{cases}$$

where $\tilde{u} = (\tilde{v}_{in}, \tilde{d})^T$ is the vector input and the matrices $A = (1-D)A_1 + DA_2$ and

$$B = \begin{bmatrix} (1-D)B_1 + DB_2 & 0 \\ 0 & (A_1 - A_2)X_{ss} + (B_2 - B_1)V_{in} \end{bmatrix}$$

Using the Laplace transform, the small-signal model (eq. (4)) can be used to derive power-stage transfer functions such as open-loop input-to-output voltage and control-to-output voltage transfer functions. By setting the small-signal perturbation $\tilde{v}_{in}$ = 0, the control-to-output voltage transfer function G_{vd} (eq. (6)) is derived.

(6)
$$G_{vd}(s) = \frac{\tilde{v}_0(s)}{\tilde{d}(s)} = k\frac{b_2s^2 + b_1s + b_0}{a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0}$$

where $k = \frac{1}{(1-2D)^2R_0}$, $b_2 = -(1-2D)LCR_0V_{in}$, $b_1 = -2(1-D)^2LV_{in}$, $b_0 = (1-2D)^2R_0V_{in}$, $a_4 = LL_oCC_oR_o$, $a_3 = LL_oC$, $a_2 = (1-2D)^2L_oC_oR_o + LCR_o + 2(1-D)^2LC_oR_o$.

***LQR* controller with an integral action**

LQR technique offers a regulator such that the system evolves in a way that physical constraints are satisfied, control objectives are met, and at the same time a previously defined cost function *J* (eq. (8)) is minimized [14]. Let us consider a continuous-time system defined as follow:

(7)
$$\begin{cases} \dot{x} = Ax + Bu \\ y = v_o = Cx \end{cases}$$

The aim is to find the stabilizing feedback control law *u* = -*Kx* that minimizes the following cost function:

(8)
$$J = \int_0^\infty (x^TQx + u^TRu)dt$$

where Q and R are state and input weighting matrices.

These matrices are considered as the tuning parameters of LQR by observing Q as state error penalty and R as penalty on control input. Q is required to be positive definite or positive semi-definite symmetry matrix and R is required to be positive definite symmetry matrix. One practical method is to Q and R to be diagonal matrix. The selection of elements of these matrices is normally based on iterative procedure using experience and physical understanding of the problems involved [14]. In this work, the trial and error method is used to set the elements of Q and R. The matrix gain K is defined as:

(9)
$$K = -R^{-1}BP$$

where P is a symmetric semi-definite matrix denotes the stabilizing solution of the *algebraic Riccati equation* (*ARE*):

(10) $$A^T P + PA - PBR^{-1}B^T + Q = 0$$

Since the objective of the LQR control is to bring the state x as close as possible to the reference state X_{ref} (eq. (3)), the feedback control law takes the form:

(11) $$u = -K\left(x - X_{ref}\right) = -Ke$$

where $e = x - X_{ref}$ is the vector tracking error. The orginal model is transformed to a following error dynamics model:

(12) $$\dot{e} = Ae + Bu$$

The system model may show deviations, which can be sources of disturbances in the control system and, therefore, steady state errors. It is well known, from the classical control theory [14], that the incorporation of an integral part in the controller allows to reject asymptotically the perturbations. It is therefore desirable to add integral action in order to override the steady state error. The block diagram of LQR with integral action ($LQR+i$) is shown in figure 3. The state equation of the integrator is given by equation (13).

(13) $$e_i = e_4 = v_o - V_{ref}$$

Now, if we define a new augmented state vector as $e_{aug} = [e\,, e_i]^T$ we obtain

(14) $$\dot{e}_{aug} = A_{aug} e_{aug} + B_{aug} u_{aug}$$

where the augmented matrices are given by:

$$A_{aug} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \quad \text{and} \quad B_{aug} = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

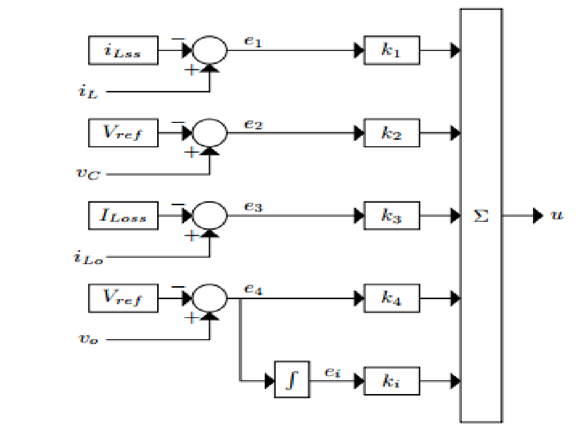

Fig. 3. LQR controller with integral action

If the system (eq. (14)) is controllable and observable, there is a single optimal control matrix K_{aug} such that the closed loop system with the control u (eq. (15)) is asymptotically stable.

(15) $$u_{aug} = -K_{aug} e_{aug}$$

where $K_{aug} = [K \;\; k_i]$. Based on the new system (eq. (14)), it is necessary to define the new matrices Q and R of the cost function J, which also increase its dimension. The elements of these matrices related to the original states condition the proportional part of the controller, whereas the elements related to the added states condition the integral part. K_{aug} is computed just like the original case.

Results and analysis

In this section, simulation results demonstrating the potential advantages of the proposed control methodology are presented. The circuit parameters expressed in the international standard system are given by $V_{in} = 24\ V$, $L = 100\ \mu H$, $C = 10\ \mu F$, $L_o = 100\ \mu H$, $C_o = 25\ \mu F$, $R = 12\ \Omega$. The desired output voltage is $V_{ref} = 48\ V$. Therefore the rest of desired operating point is (eq. (3)): $D = 0{:}333$, $I_{Lss} = 8\ A$, $V_{Css} = 48\ V$, $I_{Loss} = 4\ A$ and $V_{oss} = 48\ V$. We select matrices Q and R by trial and errors and are given by

$$Q = \begin{bmatrix} 25.10^5 & 0 & 0 & 0 & 0 \\ 0 & 10^2 & 0 & 0 & 0 \\ 0 & 0 & 10^2 & 0 & 0 \\ 0 & 0 & 0 & 10^2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad R = 1$$

We can calculate the solution to the LQR problem by using "*lqr*" function in Matlab's Toolbox. The matrix gain K_{aug} is given by

$$K_{aug} = [1581.2 \;\; -0.02 \;\; 0.01 \;\; -2 \;\; 0.01]$$

Firstly, the operation of the controlled converter to work at the nominal operating point ($V_{ref} = 48\ V$, $V_{in} = 24\ V$, $R = 12\ \Omega$) is checked and compared with open loop (*OL*) control converter (Fig. 4). We can clearly see that $LQR+i$ allows the system to track the reference voltage V_{ref} very quickly and without oscillations. The performance of $LQR+i$ controller to track V_{ref} is checked. Figure 5 depicts the inductor current i_L and the output voltage v_o for a V_{ref} change from 48 V to 57.6 V at t = 20 *ms*. We can easly see the good tracking of the controller. In order to test the robustness, V_{in} is varied from 24 V to 18V at t = 15 *ms* and from 18 V to 24V at t = 25 *ms*. Figure 6 shows the excellent recovery of $LQR+i$ controller features. Figure 7 despicts the validity $LQR+i$ scheme when R_o is subjected to a variation of 25% of its nominal value. This variation occurred during a time t = 10 *ms*. The robustness of $LQR+i$ controller is compared with a conventional PI controller. Taking into account the *LTI* model obtained in (eq. (6)), the PI controller for the converter is designed using classical frequency domain technique (phase margin $PM > 45^o$). A modulator pulse width (*PWM*) is used with a ramp frequency of 20 *kHz*. The transfer function of the PI controller is shown in equation (16).

(16) $$C(s) = 0.003 + \frac{0.001}{s}$$

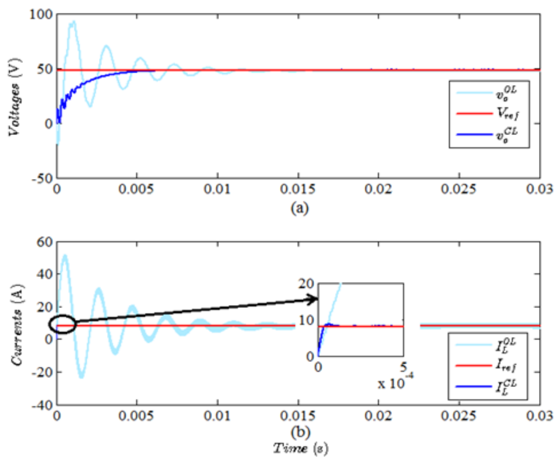

Fig. 4. Output voltage (a) and inductor current (b) waveforms for open loop and closed loop system

The comparison result is presented in Figure 8. The system was perturbed by changing V_{in} nominal value. We can see that $LQR+i$ response are more better than PI response and $LQR+i$ controller is more robust with respect to parameters variations than *PI* one.

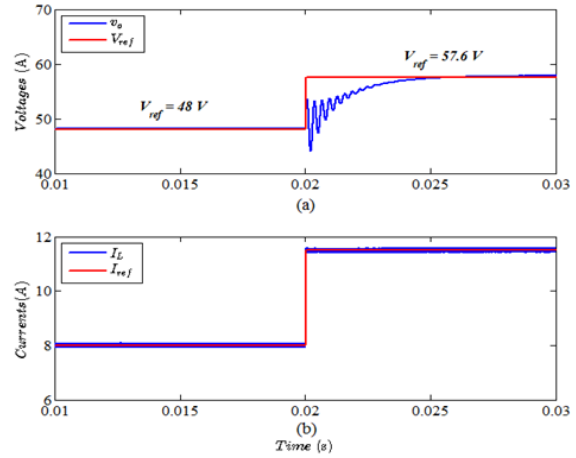

Fig. 5. Output voltage (a) and inductor current (b) waveforms for step change in V_{ref}.

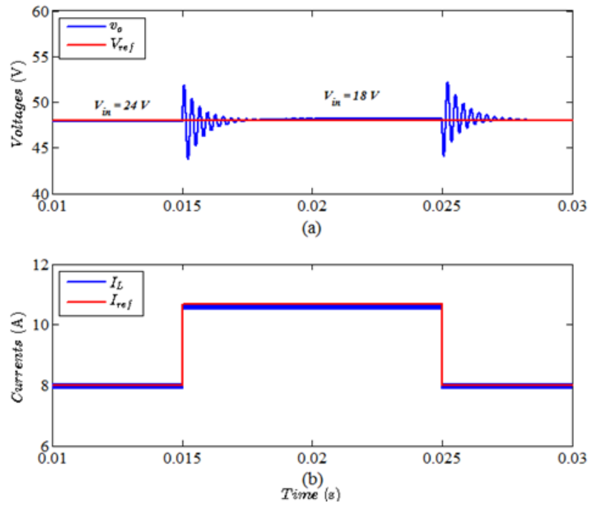

Fig. 6. Output voltage (a) and inductor current (b) waveforms for step change in V_{in}.

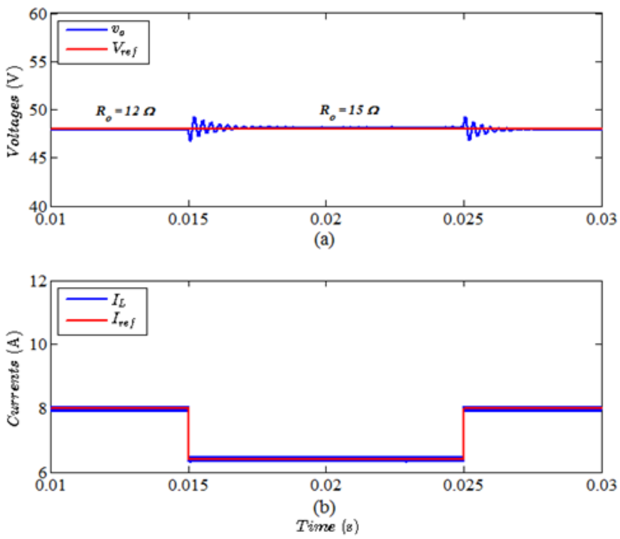

Fig. 7. Output voltage (a) and inductor current (b) waveforms for step change in R_O

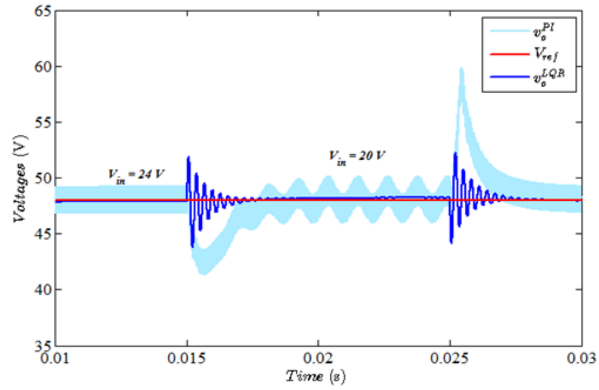

Fig. 8. Output voltage waveform for V_{in} variation

Conclusion

In this work, a *LQR* controller with an integral action is designed for Z-source DC-DC converter. After converter modeling and controller design, simulations have been done by Matlab/Simulink software. The simulation results show the validity of the overall converter-controller model. Through the robustness test of the controller, it has been shown that the system attains a robust output voltage to variations and changes imposed on voltage source and load. The performances of $LQR+i$ controller is compared to that of conventional PI controller in the case that line disturbance enter the system. The results showed that $LQR+i$ controller has good robustness against disturbances than PI controller.

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