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Effective algorithm for tomography imaging in three-dimensional problems

***Abstract**. This article presents a new effective imaging method that can be applied in ultrasonic and radio tomography. The proposed method by changing the shape of the voxels leads to a substantial simplification of the algorithm at the cost of small approximations of the voxels. As proved in the work, these approximations do not have a significant impact on the readability of the image which is several times faster.*

Streszczenie.** W pracy przedstawiono nową efektywną metodę obrazowania, która może mieć zastosowanie w transmisyjnej tomografii ultradźwiękowej lub radiowej. Proponowana metoda zmienia kształt voksela z sześciennego na kulisty dzięki czemu uzyskuje się znaczące uproszczenie algorytmu kosztem niewielkiej aproksymacji wokseli. Jak udowodniono w pracy, przyjęta aproksymacja nie ma znaczącego wpływu na wiarygodność obrazów, a jest kilkukrotnie bardziej efektywna.* ***Efektywny algorytm do konstrukcji trójwymiarowych obrazów tomograficznych

Keywords: Ultrasound Transmission Tomography, Algebraic Reconstruction Techniques, Singular Values Decomposition.
Słowa kluczowe: Ultradźwiękowa Tomografia Transmisyjna, metoda ART, Rozkład względem Wartości Osobliwych.

Introduction

There are many different methods to optimize the imaging problem solution [1-11]. Let us consider an Ultrasonic wave for which we can assume propagation along straight lines. Using this feature, an effective modification of an important part of image reconstruction algorithm (called Algebraic Reconstruction Techniques - ART), invented in 1938 by S. Kaczmarz, has been proposed in this paper. The essence of proposed algorithm modification in 3D space is an approximation of the voxel, so far treated as cube, by a sphere inscribed in this cube.

The attempt of using sphere-shaped voxels instead of cubic voxels could be questionable. In case of cubes the whole object volume can be covered because any wall of one cubic cling to another wall of the neighbouring cube. In case of sphere, it touches another sphere only in one point. So, between spheres there exists volume of object outside spheres which is not contained in voxels. The authors refer to this issue later in this paper (for example Fig. 6 and Fig. 7) showing that it has no significant influence on imaging.

This modification is one more simplifying assumption for that kind of imaging. That is why the main goal of this paper is to prove effectiveness of such an approach by numerical experiment.

What is more, such modification allows to significantly accelerate the method for determining voxels, through which the ray passes in the considered region. In such a way we have got much faster imaging software, which is particular important in cases where „on line" imaging is needed. Time profit is particularly high for 3D problems [12-18].

Tomographic images construction in 3D space

In the case of the modelling area in 3D space, you must make its discretization on a cube voxel, in which the points common for rays and voxel walls should be determined (see for example Fig. 1). Such calculations are pretty complicated. Ray passing through the voxel perforate two of six walls. Than in coefficient matrix for the position representing this voxel is placed the length of the ray enclosed within the voxel and normalized by the voxel diagonal. When the ray does not pass through the voxel the matrix coefficients remain equal zero. It is similar algorithm like in 2D space [3,17].

In order to construct an internal image of the 3D region the sensor matrix must be large enough. In Fig. 2 transmitters are marked by the circles but receivers by the squares.

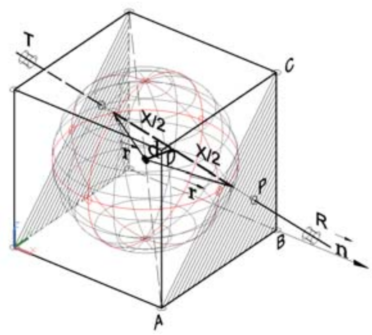

Fig.1. The ray passing through the cube voxel and inserted inside it another spherical voxel

The rays connected transmitters and receivers should possibly cover the whole volume which have to be imaged (see Fig. 3).

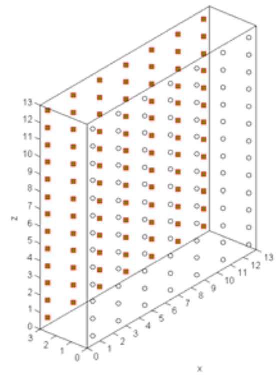

Fig. 2. Ultrasonic sensors location on the boundary of the region

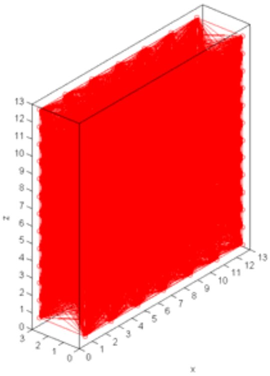

Fig. 3. Region under investigation and the rays between the sensors

New effective image algorithm for 3D problems

The traditional algorithm required for each of the six voxel faces designate the point of intersection of the ray (line section) with the voxel face (square). It was necessary to determine whether the ray is passing through the certain voxel or not. When there is a significant number of voxels this task meant a huge computer processor commitment. Therefore, the main task of this paper is to propose a much simpler method to identify the number of voxels, the ray is passing through. To simplify the process of identifying voxels, the authors gave up the idea of the traditional cubic shape of the voxels for spherical shaped ones. As reader would be convinced latter, such a sphere very well approximates the cubic voxels. Change the shape of the voxel from cubic to the spherical one in the sample region is presented in Fig. 4.

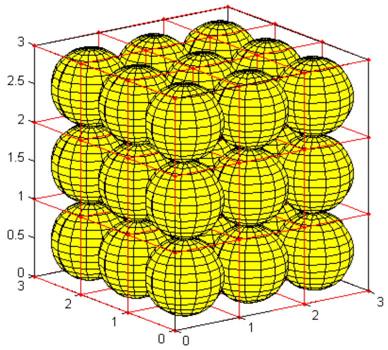

Fig. 4. Cube area filled up by spherical voxels inscribed in cubic voxels

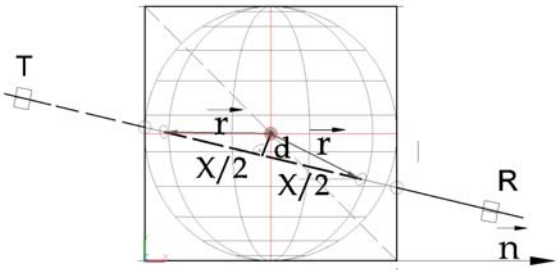

Fig. 5. Ray length differences passing through cubic like and sphere like voxels

Then instead of six tasks "the intersection of the line segment with a square" we have only one task to compare the distance d from the centre of the voxel with the radius of the sphere representing a simplified voxel. The idea of this approach is shown schematically in Fig. 5. This type of simplification entails several consequences.

The positive is that the algorithm has made everything much simpler, and therefore calculation time was drastically reduced. The complexity of the algorithm does not deviate from the case of 2D. To the negative effects of the spherical voxels, one can include the missing voxels on the way of rays, which graphically depict figures 6. It is clearly visible in these pictures that some spherical voxels are missing. It will set to zero these elements of matrix coefficients, which in the classical approach would had small values.

This fact does not have a negative influence on the quality of the images. Double-density grid does not improve radically this situation, as can be seen in Fig. 7. In addition, non-zero values of the coefficients of the matrix will be slightly different than in the case of cubic like voxels.

Approximation by circle like pixels in 2D or approximation by sphere like voxels in 3D, as proven herein by carrying out the numerical experiment, does not change the effectiveness and quality of imaging.

This simplification has changed the pseudo-rank of matrix coefficients, and slightly has changed the distribution of singular values, however obtained images have retained their essential properties (for example, spatial resolution). Now the task is to determine whether the centre of the voxel (cube) from ray (line segment) is less than the radius of the sphere representing this voxel. Therefore, such task is much simpler than the previous one for classical (cubic like) voxels (see for example [3]).

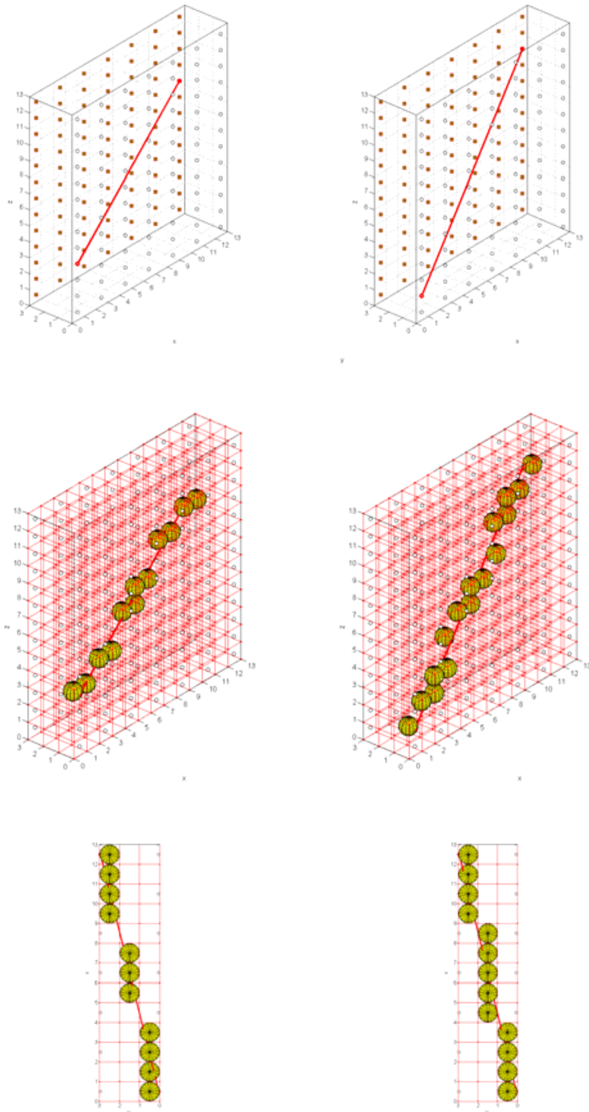

Fig. 6. Two different rays and the missing spherical voxels on their way for a rare discretization of the region

Just as it was in the 2D problem, one can easily calculate the ratio of the ray length passing through the voxel to the length of the diameter of the sphere which represents the voxel (in the classic algorithm for voxel diagonal $\sqrt{3}\ l$, where l is the length of voxel cube-shaped side, see for example Fig. 5).

The length of the ray passing through the voxel could be calculated using the following relation:

(1) $$x = 2\sqrt{R^2 - d^2}$$

where: x – is the length of the ray passing through the voxel, d is the distance from the centre of voxel to the ray.

Just as it was in 2D problems determination of distance d is done using the built-in MATLAB function named `distancePoint2Line` [19]. This function is called with the parameter `'segment'` because the ray is a straight-line segment connecting the transmitter (T) and the receiver (R). Next, the coefficients of the matrix $\mathbf{W}$ are calculated using identical formula as in 2D case:

(2) $$W_{ij} = \sqrt{1 - \left(\frac{d}{R}\right)^2}$$

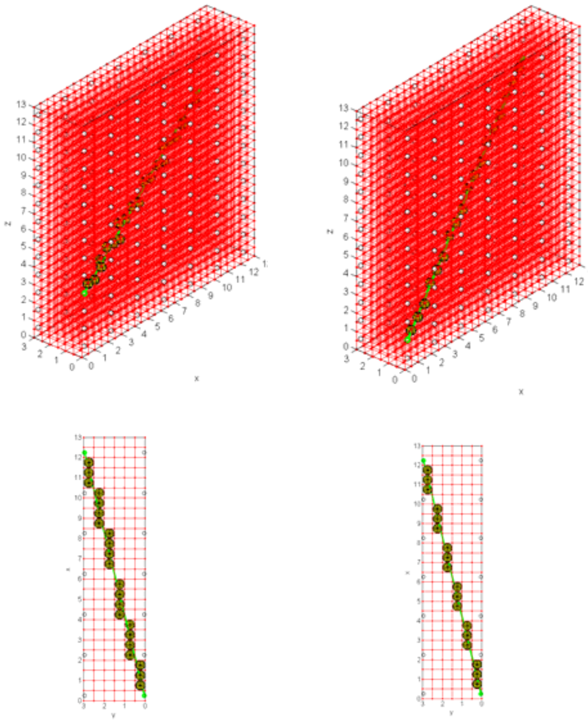

Fig. 7. Two different rays and missing spherical voxels on their ways in case of dense discretization

The 3D imaging examples

As the test object for three-dimensional imaging a cuboidal region has been selected with a rod inside which is shown in Fig. 8a. For the 3D problems two layers of the sensors have to be designed, each layer consists of 32 sensors (see Fig. 8b). The main goal of such selection of the inner object, was to check how the sensors should be placed in order the algorithm could produce a reliable image of the whole 3D region. The imaging results by the new algorithm are shown in the following figures. Singular values distribution is presented in Fig. 9. As it can be seen from this figure the matrix of coefficients $\mathbf{W}$ is pseudo-rank deficient. Therefore, it is necessary to reject more than three thousand, too small singular values.

As can be seen from the results, a new imaging method even in presence of 1% of noise, is able to provide the correct results (see for example Fig.10).

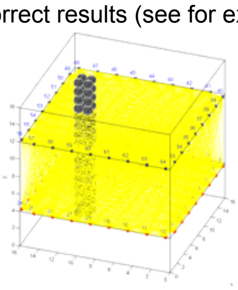

Fig. 8a. The Phantom of a rod located close to the one of the walls of container

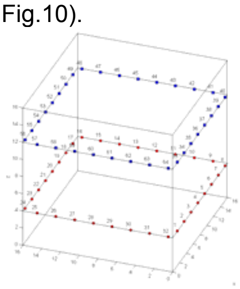

Fig. 8b. Two layers of 32 sensors in each layer

It is worth noting that the complexity of a new method for 3D calculations is similar to computational complexity of the 2D problems. One of the disadvantages of this imaging method, and regardless of the shape of the voxel, is a relatively high sensitivity to noisy data which was mentioned above.

The impact of such a small, because only 1% noise, illustrate the results shown below. For example in the Fig. 10 one can see some pale spherical voxels, even partially overrides the inner object marked in a little more dark colour. Such an image is not sufficiently clear and does not show if the internal sub region is identified correctly or not. The Fig. 10 may give the impression that the imaging is ineffective, because of the many extra voxels, which surfaces rendered in shades of grey do not accurately represent the reality.

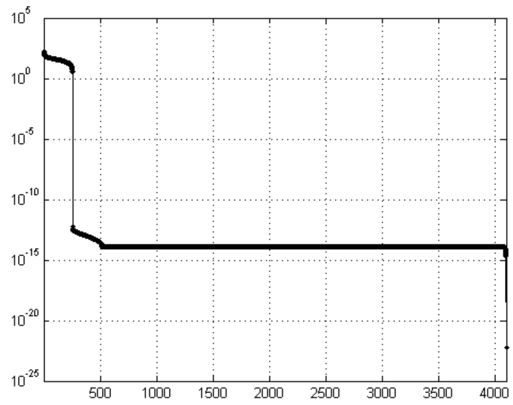

Fig. 9. Singular values distribution for 3D problem

Therefore, the results will be illustrated as 2D colour maps for the z plane placed in the half of the height of the region. Now the internal object is clearly visible on the noisy background (see Fig. 11).

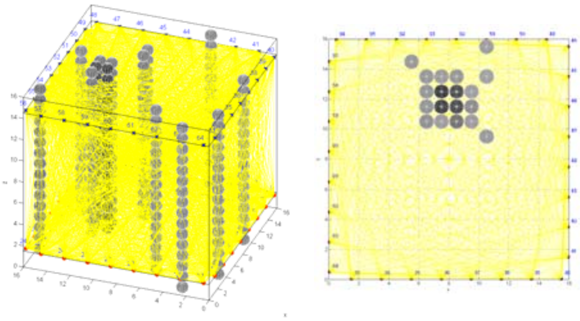

Fig. 10. Imaging results presented for the rod object presented in Fig. 8a for 1% noise

In Fig. 12 the image along the z axis is visible. The image is so clear because data was the noise free. However more interesting are noisy data. Reconstructions are presented in the Fig. 13.

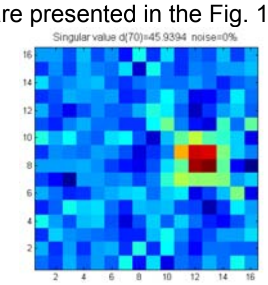

Fig. 11. 2D image of the rod in the x-y plane

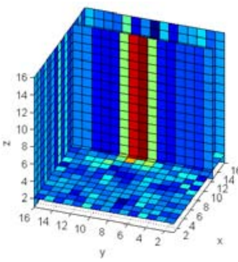

Fig. 12. The colour map along the z axis

Considering the 3D space always there is a question haw the sensors should be placed. There is many different ways but in this paper the authors selected two layes of 32 sensors. The question about the distance between them still remain open. That is why three different configuration have been selected as it is depicted in Fig. 13 (the upper raw).

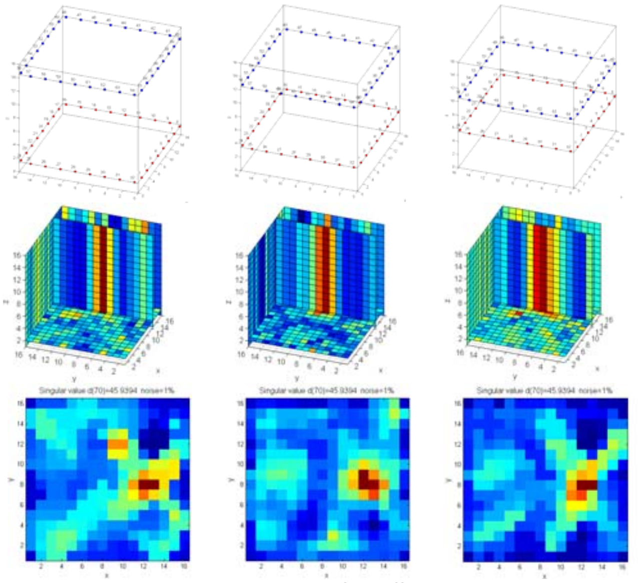

Fig. 13. Two dimensional images for different distances between sensors layers after median filtering

As ones can see from the Fig. 13 (the middle raw) the distance between the layers of sensors have no a significant impact on the quality of the imaging. Particularly there is no significant side effect along the z axis.

Conclusion

This research shows that the modified algorithm is suitable for imaging as well as the classic algorithm. The new algorithm possesses a very important advantage in the form of a drastic acceleration particularly in 3D problems. The more dense discretization the bigger acceleration.

Based on our research, we can conclude that the ART imaging method is susceptible to salt and pepper noise. Errors in the images are caused by approximations of cubic voxels by sphere-shaped voxels. Using a new voxel shape, the constructed coefficient matrix is slightly different, usually with a lower pseudo rank. Therefore, achieving the proper solution requires the rejection of more singular values, those which can be considered as equal to zero. The developed algorithm has one very significant disadvantage especially severe in the case of 3D problems. Usually, 3D tasks are characterized by arrays of a huge size. The heart of this algorithm is the SVD decomposition, which is a very time and memory consuming. Therefore, there is an urgent need to replace this passage of algorithm by quite another approach like reduction data size by the Principal Component Analysis (PCA) for example.

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