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Measurements and calculation of self inductance of testing coils used in physical transformer model construction and its frequency analysis

Streszczenie. *W artykule dokonano przeglądu i porównania różnych metod i sposobów wyznaczania, najdokładniej jak to możliwe, indukcyjności własnej wielowarstwowej cewki spiralnej. Cewka poddana analizie stanowiła element wyposażenia pomiarowego w modelu fizycznym transformatora badanego pod względem charakterystyk częstotliwościowych. Uzyskane indukcyjności zostały porównane z wartością uzyskaną metodą pomiarową.*

Abstract*. The paper reviews and compares various methods of accurately determining the value of self-inductance of multilayer spiral coil, being the part of measuring equipment in physical model of the transformer tested for frequency characteristics. These values were compared to the inductance measured in the laboratory and the accuracy of calculation methods has been evaluated. (**Pomiar i obliczanie indukcyjności własnej cewki testowej użytej do budowy fizycznego modelu transformatora oraz jego analizy częstotliwościowej**)*

Słowa kluczowe: indukcyjność własna, cewka spiralna, model transformatora
Keywords: self-inductance; spiral coil; transformer model

Introduction

The aim of the paper is to present calculations and measurements of self-inductance of the flat spiral coils with thin internal insulation used to build a physical transformer model and its frequency analysis. The exact knowledge of self-inductance was necessary for further use of the real model and its equivalent circuit diagram as well. The authors decided to use numerous formulas available in literature.. The measure of this accuracy is the comparison of the results with the value obtained in the direct measurement.

Test coil

To build the transformer physical model [6], 5 coils were made of 20 mm wide copper tape with a thickness of 0,5 mm. Each coil (Fig. 1) had 30 turns separated by a thin insulation tape, what gave a square with 0,02 x 0,02 m side in the cross section. Other parameters were as follows: internal diameter D_w = 0,295 m; external D_z = 0,335 m and the mean diameter D = 0,315 m.

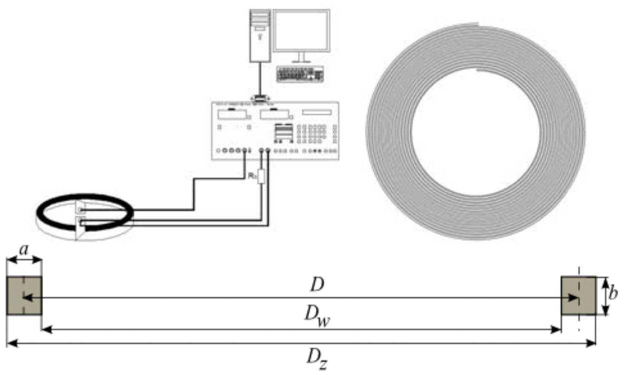


Fig. 1. The test coil on the measuring stand, sketch of its spiral and the cross-section with geometric size markings

The coils were wound manually on a specially built machine. It helped to keep the circular shape of the coil, its dimensions (internal diameter) and ensured proper tension of the copper tape during winding.

Review of methods and formulas for determining the inductance of current contours

Inductance is a property of an electrical conductor which opposes a change in current. It does that by storing and releasing energy from a magnetic field surrounding the conductor when current flows, according to Faraday's law of induction. It is connected with the magnetic flux Φ and the current I by the formula: $\Phi = LI$, which is most often used to calculate inductance as $L = \Phi/I$. Therefore, the calculation of inductivity becomes mainly the task of determining the magnetic flux (Fig. 1) with the induction vector $\mathbf{B}$, excited through the conductor 1, leading current I, therefore the fundamental formula might be used, [4]:

$$\Phi = \int_S \mathbf{B}\mathrm{d}\mathbf{s} \tag{1}$$

where: S is any surface spread over another, but closed contour 2 (not necessarily leading the current), with a positive orientation of its circulation, $\mathrm{d}\mathbf{s}$ - elementary surface vector normal to the surface S.

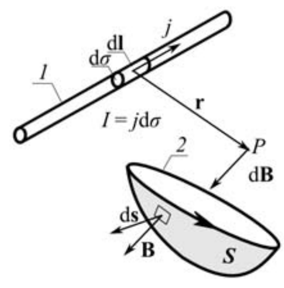


Fig. 2. Magnetic flux Φ associated with a current-free contour 2 and conductor 1 with current I.

The gain $\mathrm{d}\mathbf{B}$ of induction at any point P of the space, coming from the section $\mathrm{d}\mathbf{l}$, under the law of Biot-Savart [4], is given by the formula:

$$\mathrm{d}\mathbf{B} = \frac{\mu_0 I}{4\pi} \cdot \frac{\mathrm{d}\mathbf{l} \times \mathbf{r}}{r^3} \tag{2}$$

and integration over the entire length l of conductor 1 with current I gives induction $\mathbf{B}$ at the point P. Usually, however, we do not deal with a single current filament, but with their bunch filling the cross-section Σ of the conductor, and the current filament itself is considered to be closed, single turn of the coil (Fig. 3).

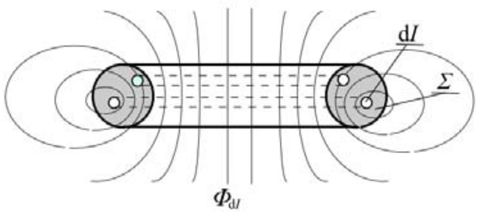


Fig. 3. Magnetic flux Φ_{dI} associated with the elementary current loop dI

Then the linkage flux Ψ [4] defined as the surface integral is widely use:

$$\Psi = \frac{1}{I}\int \Phi \mathrm{d}I = \frac{1}{\Sigma}\int \Phi \mathrm{d}\sigma \tag{3}$$

If we assume that the current has constant distribution density in the conductor cross section (dI = Idσ/Σ) and there is no external magnetic field originating from another contour. However, if it exists, there is a need to distinguish between the own contour and neighbour contours, which leads to the concept of self-induction (→ self-inductance L) and mutual induction (→ mutual inductance M). Unluckily, the direct use of formula (1) may prove to be a difficult task and therefore the notion of vector potential **A** is used, which binds with the induction, as $\mathbf{B} = \mathrm{rot}\mathbf{A}$. According to Stokes theorem, it is possible to replace the surface integral with the circular integral along the closed contour limiting this surface. In figure 3, it is, for example, the internal surface of the torus limited by a ring with a cross-section Σ.

In a general case, two separate loops 1 and 2 of the lengths l_1 and l_2 are being considered, respectively.

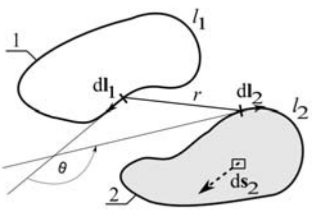


Fig. 4. Illustration for determining the mutual inductance of two current loops.

If in the first loop current I_1 flows, then we can specify the flux called Φ_{21} associated with the second contour. The quotient of this flux by the excitation current I_1 determines [4] the mutual inductance $M_{21} = \Phi_{21}/l_1$. So we have:

$$\Phi_{21} = \int_{S_2} \mathrm{rot}\mathbf{A}\mathrm{d}\mathbf{s}_2 = \oint_{l_2} \mathbf{A}\mathrm{d}\mathbf{l}_2 \tag{4}$$

where: d$\mathbf{l}_2$ - unitary vector along the contour 2; d$\mathbf{s}_2$ - unitary vector, normal to the surface S_2 limited by contour l_2, positively oriented relative to the direction of its circulation, i.e. in the direction of current flow I_2.

In turn, the vector potential **A** coming from the current I_1, at any point in the space [4] is defined by the formula:

$$\mathbf{A} = \oint_{l_1} \frac{\mu_0 I_1}{4\pi r}\mathrm{d}\mathbf{l}_1 \tag{5}$$

which can replace (4) by another one:

$$\Phi_{21} = \frac{\mu_0 I_1}{4\pi}\oint_{l_1}\oint_{l_2}\frac{\mathrm{d}\mathbf{l}_1 \cdot \mathrm{d}\mathbf{l}_2}{r} = \frac{\mu_0 I_1}{4\pi}\oint_{l_1}\oint_{l_2}\frac{\mathrm{d}l_1\mathrm{d}l_2}{r}\cos\theta \tag{6}$$

where: I_1 - current in loop 1; d$\mathbf{l}_k$, d$\mathbf{l}_m$ – unitary lenghts vectors of two current loops; θ - angle between the directions of contour lengths.

The mutual inductance then equals to:

$$M_{21} = \frac{\mu_0}{4\pi}\oint_{l_1}\oint_{l_2}\frac{\mathrm{d}\mathbf{l}_1 \cdot \mathrm{d}\mathbf{l}_2}{r}\cos\theta \tag{7}$$

The interaction between current loops is symmetrical, so $M_{12} = M_{21}$, that is why the double indexing is usually omitted, leaving one symbol M. There is a special case to be considered when only one conductor of non-negligible cross-sectional area carries the current I, for which the concept of self-inductance L (Fig. 5) needs to be explained.

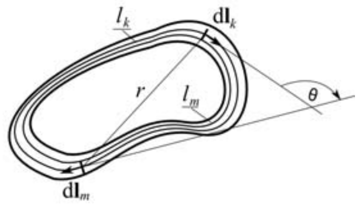


Fig. 5. Geometrical illustration for determining the inductance of the single current conductor.

The formula (6) will be referred to the k - th current filament of length l_k and will take form that respect the total share of all other current filaments l_m of the same conductor. To avoid singularities, the influence of an elementary segment dΔl_k with current ΔI_k on itself is omitted in integration, because then $r = 0$. For this reason, there is no formal integration, but sufficiently numerous summation of shares with exclusion of this one component.

$$\Phi_k = \frac{\mu_0}{4\pi}\sum_{I-\Delta I_k}\left(\oint_{l_k}\oint_{l_m}\frac{\mathrm{d}l_k\mathrm{d}l_m}{r}\cos\theta\right)\Delta I_m = \sum_{I-\Delta I_k} M_{km}\Delta I_m \tag{8}$$

where: $\Delta I_k = I/l_k$, $\Delta I_m = I/l_m$ – the entire current I of conductor referenced to the contour lenghts (linear density). In order to determine the linkage flux Ψ with the entire current (and hence the self-inductance L), one should apply formula (3) and integrate over the entire surface S of the conductor with current I. In practice, the integral (3) is again replaced by the sum of shares, as a result of which we get:

$$\Psi = \frac{1}{I}\int_I \Phi_k \mathrm{d}I_k = \frac{1}{I}\sum_I \Phi_k \Delta I_k \tag{9}$$

and finally the self-inductance

$$L = \frac{\Psi}{I} = \frac{1}{I^2}\sum_I\left(\Delta I_k \sum_{I-\Delta I_k} M_{km}\Delta I_m\right) \tag{10}$$

or alternatively

$$L = \frac{1}{\Sigma^2}\sum_{\Sigma}\Delta\sigma_k \sum_{\Sigma-\Delta\sigma_k} M_{km}\Delta\sigma_m \tag{11}$$

related to the elementary cross sectional area of fictitious current tube, assigning it a value resulting from the sequence of proportions: $\Delta I_k/I = \Delta I_m/I = \Delta\sigma_k/\Sigma = \Delta\sigma_m/\Sigma$.

The general formulas quoted here, applied to the most common coreless coils, but varied in terms of their shape and winding arrangement, resulted in many specific cases. The paper recalls a few of them that could be considered as approximate modeling of the analyzed test coil.

Inductance of rectangular cross-section ring

It is assumed that the ring is built of one solid turn of winding (Fig. 1) with uniform current density. According to

[3], we can use the formula (in μH):

$$L = \frac{\mu_0 R}{2\pi}\left(\ln\frac{8R}{a+b} - 0,5\right)$$

where: a, b – width and height of the ring; $D/2$ – radius of the axial line of the conductor.

By applying the above mentioned data, we obtain (in μH):

$$L = \frac{4\pi\cdot 10^{-7}}{2\pi}\left(\ln\frac{8\cdot 0,315}{0,02+0,02} - 0,5\right) = 0,584$$

The result concerns a solid coil so fitting to the test coil requires a multiplication of this value by the square of number of turns w^2. We then get $L = 0,584\cdot 30^2 = 525,5$ μH.

Inductance of circular cross-section ring

The replacement of a rectangular section with a circular one of radius $r = a/2 = b/2$ is intended only to show how significant is this change in affecting the previous result. According to [4], in case of $R >> r$ (here: 0,315 >> 0,2), we can use the formula

$$L = \mu_0 R\left(\ln\frac{8R}{r} - \frac{7}{4}\right)$$

where: w – number of turns, d - average diameter ($d = D$).

and like above we get:

$$L = 4\pi\cdot 10^{-7}\cdot 0,315\cdot\left(\ln\frac{8\cdot 0,315}{0,01} - \frac{7}{4}\right) = 0,562 \ \mu\text{H}$$

what after multiplication by w^2 gives $L = 0,562\cdot 30^2 = 505,8$

Inductance of square cross-section coil

In this case, the windings forms a square of side a, whose dimensions were used for previous calculations. The formula taken from [4] looks as below:

$$L = \frac{\mu_0}{8\pi}\cdot w^2\cdot d\cdot\Phi(\alpha)$$

where: w – number of turns; d – mean diameter ($d=D$).
Φ is a function of the parameter $\alpha = a/D$:

$$\Phi(\alpha) = 2\pi\left[\left(1+\frac{\alpha^2}{6}\right)\ln\frac{8}{\alpha^2} - 1,6967 + 0,4082\alpha^2\right]$$

and for $a = 0,0635$ returns $\Phi = 37,085$, so the inductance equals to (in μH):

$$L = \frac{\mu_0}{8\pi}w^2 d\cdot\Phi(\alpha) = \frac{4\pi\cdot 10^{-7}}{8\pi}\cdot 30^2\cdot 37,085 = 525,7$$

Inductance of circular section coil

In such a case the inductance L is given by the formula:

$$L = \frac{\mu_0}{4\pi}\cdot w^2\cdot d\cdot\Phi(\gamma)$$

where: r – radius of the circular section of the coil, $d = D$ – radius of axial line of wire (mean diameter); $\gamma = r/D$

$$\Phi(\gamma) = 2\pi\left[\left(1+\frac{\gamma^2}{2}\right)\ln\frac{4}{\gamma} - 1,75 + \frac{\gamma^2}{6}\right]$$

and for $\gamma = 0,01/0,315 = 0,03175$ returns the value $\Phi = 19,438$, so inductance of such a coil is equal to (in μH)

$$L = \frac{4\pi\cdot 10^{-7}}{4\pi}\cdot 30^2\cdot 0,315\cdot 19,438 = 550,2$$

The procedure presented so far was to find a coil of similar geometry and use a ready formula for its inductivity. However, none of the formulas took into account the spiral winding, i.e. the continuous increase of the coil radius rather than the stepwise coil, as for completely separate coils laid on each other.

Inductance of the flat spiral coil wound with round wire

There is a small amount of literature focussing directly to spiral coils. Most attention is paid by the author [4], where spiral coils are still treated in an approximate way as the concentricity of circular coils with a circular cross-section (Fig. 6).

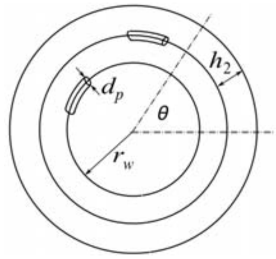


Fig. 6. Top view on a flat coil consisting of round circular wires

Given there formula looks as follows

$$L = \mu_0\sum_{k=1}^{zw-1}\sum_{l=1}^{zw-1}\int_0^{\pi}\frac{\left(r_w + \frac{dp}{2} + h_2 l\right)\left(r_w + h_2 k\right)\cos\theta}{\sqrt{\left(r_w + \frac{dp}{2} + h_2 l\right)^2 + \left(r_w + h_2 k\right)^2}} \rightarrow \tag{11}$$

$$\rightarrow \frac{\mathrm{d}\theta}{-2\left(r_w + \frac{dp}{2} + h_2 l\right)\left(r_w + h_2 k\right)\cos\theta}$$

Assuming that $r_w = D_w/2 = 0,1475$; $z_w = 30$ (number of spiral turns); $d_p = 0,0005$; insulation thickness $h_2 = 1,67\cdot 10^{-4}$ and using MathCad program for integral calculations, we get $L = 506,4$ μH.

Inductance of flat spiral coil

The next method used to determine the inductance of spiral coil is taken from the monograph [1] (1926). The formulas and references to the tables contained there will be cited here.

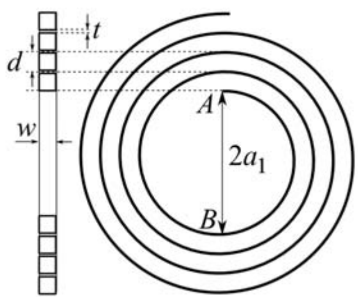


Fig. 7. Top and side view on spiral coil and its geometry

Further calculations were performed with a coil of rectangular cross section and parameters $b = w$, $c = nd$, $a = a_1 + 0,5\ (n\text{-}1)d$, as shown in Figure 7.

According to [1], the following algorithm was applied:

a) calculate the inductance of a rectangular coil (in μH):

$$L = 0{,}002(\pi n)^2 \left(\frac{2a}{b}\right) \cdot a \cdot (K - k)$$

with the values of parameters for the test coil $a = 0{,}15725$; $b = 0{,}02$; $d = 6{,}(6)\cdot10^{-4}$; $n = 30$ and estimated parameters $K = 0{,}215$ and $k = 0{,}0336$ found in the Table 4 of [1].

b) calculate the adjustment $\Delta L = 0{,}01257 \cdot n \cdot a \cdot (A_1 + B_1)$ respecting the winding shape and presence of insulation, where $d = 6{,}(6)\cdot10^{-4}$; $t = 0{,}0005$:

$$A_1 = \ln\frac{b+d}{b+t} = 8{,}097 \cdot 10^{-3}$$

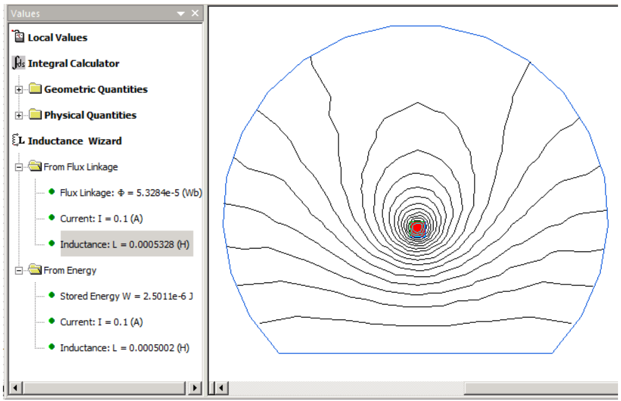
$$B_1 = -2\left(\frac{n-1}{n}\delta_2 + \frac{n-2}{n}\delta_3 + \cdots + \frac{1}{n}\delta_n\right) = -3 \cdot 10^{-3}$$

coefficients $\delta_{2\text{-}9} = 0{,}001$ and 0 for all others, what gives the adjustment $\Delta L = 3{,}021\cdot10^{-4}$ μH.

c) calculate corrected value of $L{:}=L+\Delta L = 510{,}9$ μH.

Fields method

Computer programs being currently in wide use for determining of fields distributions are equipped with tools supporting post-processing calculations. The Quickfield software [7] belongs to them and in the case of magnetic fields, among other things, can determine the inductance of a coil, using two different methods - on the basis of field energy or directly from the definition of the linkage flux. Due to the limitations of the program (student's version with a limited number of finite elements), the analyzed coil was modeled only as a solid body, without considering the inter-turn insulation.



Rys. 8. Magnetic field ditribution image together with inductance wizzard window and results of calculations.

Nevertheless, the program allows to use the number of turns forming a specific coil thickness, although the geometric model does not show this explicitly, so it is a purely software operation. Then multiplier z^2 appears, instead of dividing the entire current into $n = z$ wires. The field type was selected as magnetostatic (I = const), the number of turns $n = 30$ and then the "Inductance Wizzard" was chosen. As we can see (Fig. 8) the inductances are as follows (in μH): $L = \Psi/I = 532{,}8$ – from the linkage flux calculations and $L = 0{,}5W_m/I^2 = 500{,}2$ – from the energy of magnetic field.

Measurement by technical method

As a criterion for assessing the accuracy of the methods used, a direct measurement of inductance by technical metod was performed [8]. With DC supply, one can measure current I, hence the resistance $R = U/I$, while with the AC supply of frequency f the measurement will show the value of impedance:

$$(12)\quad Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2} \quad \text{so} \quad L = \frac{\sqrt{Z^2 - R^2}}{2\pi f}$$

The measurements gives following results: $R = 0{,}059$ Ω; $Z = 0{,}18$ Ω, what, for $f = 50$ Hz, leads to $L = 549{,}8$ μH.

Conclusions

The problem of determining the inductance of the spiral coil gave a chance to review various methods and formulas for coils geometrically similar to original one in shape. The results of calculations and measurements are given in Table 2.

Table 2. Comparison of obtained inductances and relative error

The variant/method	Value in μH	ε %
Solid ring of rectangular cross-section	525,5	4,4
Solid ring of circular cross-section	505,8	8,0
Coil of a square cross-section	525,7	4,4
Coil of a circular cross-section	550,2	0,1
Spiral coil with the wire of circular cross-section	506,7	7,8
Flat spiral coil with tape winding of rect. shape	510,9	7,1
Fields method		
on the base of linkage flux	532,8	3,1
on the base of magnetic field energy	500,2	9,0
Technical method – direct measurement	549,8	0,0

Assuming that the self-inductance measurement obtained by the technical method will be referred to, it is concluded that all other methods did not exceed the error of more than 10%. The field method, despite quite a thick division into FEM elements, in the variant with the linkage flux - gives correct results (error of 3.1%), while on the basis of field energy - the biggest error, of 9%.

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