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Performance Evaluation of Two Simplified Algorithms of WLS Power System State Estimation

***Abstract**. This paper presents a performance evaluation of two simplified versions of the well-known basic weighted least squares algorithm for power system state estimation. Different simulations are tested on IEEE 14, 30 and 118 bus systems: a comparative study was carried out in order to identify the advantages and disadvantages of each method. The results bring out the interest of WLS2 Algorithm which reduces by half the computation time compared to the basic algorithm with the same reliability and precision .*

Streszczenie.** W artykule zaprezentowano badanie właściwości dwóch uproszczonych wersji algorytmu najmnijeszych kwadratów stosowanego do oceny systemu energetycznego. Przeprowadzono symulacje systemu IEEE z 134, 30 I 118 szynami. W algorytmie WLS2 udaje się zredukować o połowę czas obliczeń. **Badanie właściwości dwóch uproszczonych wersji algorytmu najmnijeszych kwadratów stosowanego do oceny systemu energetycznego

Keywords: Performance evaluation, Weighted least squares Algorithm , Power system state estimation, Simulations.
Słowa kluczowe: metoda najmniejszych kwadratów, system energetyczny, symulacja.

Introduction

State estimation represents an essential tool for monitoring the power system. In energy control centers, power system state estimation is carried out in order to provide best estimates of what is happening in the system based on real-time measurement and a predetermined system model. It is required in the critical operational functions of a power grid such as real-time security monitoring, load forecasting, economic dispatch, and load frequency control. Therefore, an optimal performance of state estimation output is the ultimate concern for the system operator. This need is particularly more in focus today due to deregulated and congested systems and smart grid initiatives[1].

Most state estimation programs in practical use are formulated as over determined systems of non-linear equations and solved as weighted least-squares(WLS) problems [2]. In fact, WLS state estimation algorithm provides the best estimation quality and good convergence rate. However The gain and Jacobian matrices need to be recalculated each iteration which needs a large amount of calculation , a big memory requirement and long computing time[3].

To face this problem , many researches are carried out on the development of new state estimation techniques, great progress has been made and new algorithms have emerged including simplified Algorithms (SWLS1 and SWLS2) based on constant matrices , active/reactive decoupling (Fast decoupled WLS) [4,5,6] and simplified Direct Current (DC) approximation [5,7,8] to reduce the computational burden associated with the traditional algorithm. However no paper has already studied carefully the characteristics of the simplified algorithms based on constant matrices which makes the particularity of this paper. In fact, this paper is the first one presenting an evaluation of the effectiveness of the simplified algorithms (SWLS) compared to the basic one in terms of convergence rate and computing time. Section 1 presents a description of the traditional weighted least squares algorithm and its modified alternatives. Section 2 ,discuss the simulation results tested on IEEE 14, 30 and 118 bus systems.

Weighted Least Squares Algorithm

Basic Algorithm

The Network model employed is the single phase model with N buses and m measurements gathered from remote meters.

Most commonly used measurements are the line power flows, bus power injections and bus voltage magnitudes.

The aim of state estimator is to provide the best possible values of the bus voltage magnitudes and angles by processing the available network data recognizing that there are errors in the measured quantities.

The starting equation for the WLS state estimation algorithm is:

(1) $$z = h(x) + e$$

where: z is the (mx1) measurement vector; x is an (nx1) state vector to be estimated: The number of estimated states is n=2*N-1 , since the balance phase's is already known θ=0. e is an (mx1) measurement error vector.

h is the vector of nonlinear functions that relate the states to the measurements defined below :

- Real and Reactive power injection at bus i:

(2) $$P_i=V_i\Sigma_{i\neq j} V_j(G_{ij}cos\theta_{ij}+ B_{ij}sin\theta_{ij})$$

(3) $$Q_i=V_i\Sigma_{i\neq j} V_j(G_{ijsin}\theta_{ij}+ B_{ij}cos\theta_{ij})$$

- Real and reactive power flow from bus i to bus j:

(4) $$P_{ij}=V_i^2(g_{si}+g_{ij})- V_jV_j(g_{ij}cos\theta_{ij}+ b_{ij}sin\theta_{ij})$$

(5) $$Q_{ij}=-V_i^2(b_{si}+b_{ij})- V_jV_j(g_{ij}cos\theta_{ij} - b_{ij}sin\theta_{ij})$$

where : Vi is the voltage magnitude at bus I; θi is the phase angle at bus I; $\theta_{ij}= \theta i- \theta j$; $G_{ij} + jB_{ij}$ is the ijth element of the Y-bus matrix; $g_{ij} + jb_{ij}$ is the admittance of the series branch between bus i and bus j; $g_{si} + jb_{sj}$ is the admittance of the shunt branch at bus i[5].

In practice , it is required to have the number of measurements larger than the number of states , this is called redundancy [9]. So state estimator can take into account the various operation layouts used and to cover for the unavailability of transmission and telemetering equipment failures [10]. A measure of the redundancy may be denoted by the redundancy factor η, which is defined as [11]:

(6) $$\eta=Dimension\ of\ z\ /Dimension\ of\ x = m/ n$$

The measurement errors ei are assumed to satisfy the following statistical properties:

First, the errors have zero mean: E(ei) = 0, i = 1, ..., m. Second, the errors are assumed to be independent, such that the covariance matrix is diagonal:

(7) $$Cov(e) = E (e, eT) = R = diag\{\sigma_1^2 , \sigma_2^2 , ...,\sigma_m^2\}$$

The solution to the state estimation problem can be formulated as a minimization of the following objective function [12]:

(8) $$J(x)=\sum_{i=1}^{m}\frac{(z_i-h_i(x))^2}{R_{ii}}=[z-h(x)]^T R^{-1}[z-h(x)]$$

To find the minimization of this objective function the derivative should be set to zero. The derivative of the objective function is denoted by g(x):

(9) $$g(x)=\frac{\partial J(x)}{\partial x}=-H^T(x)R^{-1}[z-h(x)]$$

where: H(x)= ∂h(x)/∂x called the measurement Jacobian matrix.

Ignoring the higher order terms of the Taylor series expansion of the derivative of the objective functions yields an iterative solution as shown below:

(10) $$x^{k+1}=x^k+\left[G(x^k)\right]^{-1}\left[\left[H(x^k)\right]^T\left[R^{-1}\right]\left[z-h(x^k)\right]\right]$$

Where the gain matrix, G, is defined as:

(11) $$G(x^k)=\frac{\partial g(x)}{\partial x}=H^T R^{-1}H$$

WLS Algorithm steps' [5]

1. Set k = 0
2. Initialize the state vector x^k, typical a flat start(all of the voltage magnitudes are 1.0 per unit and all of the voltage angles are 0 degrees)
3. Calculate the measurement function $h(x^k)$: Pi , Qi , Pij , Qij et V .
4. Build the measurement Jacobian $H(x^k)$
5. Calculate the gain matrix of $G(x^k)$
6. Calculate the RHS of the normal equation $HT(x^k)R^{-1}(z-h(x^k))$
7. Solve $\Delta x^k=x^{k+1}-x^k$
8. Check for convergence $|\Delta x^k|\le \varepsilon$
9. If not converged, update $x^{k+1}=x^k+\Delta x^k$ and go to 3. Otherwise stop.

Simplified Algorithms

Mostly, it is observed that the estimated state is not far from the initial state. Consequently the elements of the Jacobian and gain matrices vary very little from iteration to another. Based on this principle, two methods have emerged : SWLS1 and SWLS2.

The first simplified method SWLS1

The first simplified method (SWLS1) calculates the Jacobian matrix at every iteration but preserves the gain matrix constant after an iteration k chosen.

The obvious advantage of this method is to reduce the number of calculations of the gain matrix which represents the main disadvantage of the basic algorithm.

The second simplified method SWLS2

The second method SWLS2 admits that the gain and the Jacobian matrices remain constant after an iteration k chosen [6].

Simulation Results

To investigate the performance of the proposed methods presented above, different simulations cases were tested on three IEEE systems : 14 , 30 and 118 bus; The network data files can be downloaded from Power Systems Test Case Archive [13] .

To compare the state estimate accuracy of the following simulations, mean absolute percentage error (MAPE) is introduced as follow [14]:

(12) $$MAPEV=\frac{1}{n}\sum_{t=1}^{n}\left|\frac{V_t-V_e}{V_t}\right|*100\%$$

where, Vt is the true value of voltage magnitude obtained from Newton Raphson load flow results and Ve is the estimated value. A smaller value of MAPE indicates a more accurate state estimation result.

The measurement data is obtained from the true values with a normally distributed random noise added.

For all test cases, measurements are set to ensure a redundancy factor >1 . They should be of different types and uniformly distributed through the network in order to have a good estimation [15].

The simplified algorithms were tested for different value of the iteration k from which just the gain matrix is considered constant (SWLS1) or the gain and Jacobian matrices are constant (SWLS2).

The results will be evaluated in terms of convergence , computational time and number of iterations.

Simulation Results for IEEE 14 bus System

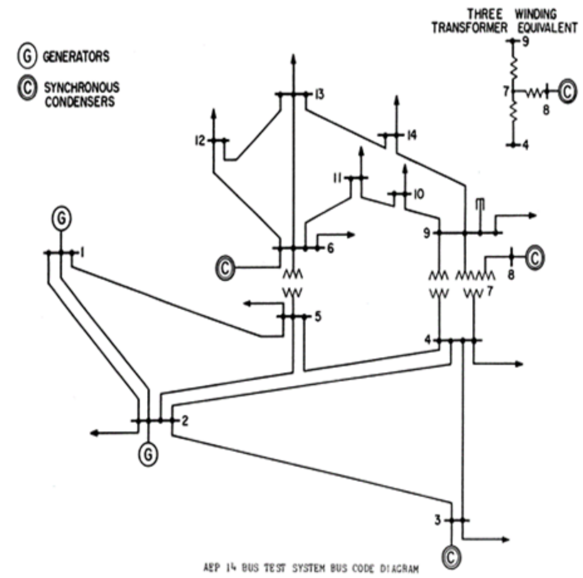

Fig.1. IEEE 14 Bus System

For IEEE 14 bus system test case , a set of 41 measurements (η =1,5) is chosen as below:

- 1 voltage magnitude at bus 1.
- 8 real power injections and 8 reactive power injections at buses : 2 ,3,7,8,10,11,12,14.
- 12 real power flow and 12 reactive power flow on branches : 1-2,2-3,4-2,4-7,4-9,5-2,5-4,5-6,6-13,7-9,11-6,12-13.

Table1. Performance evaluation of Simplified Algorithms as a function of k (IEEE 14 bus System)

Algorithm	Computation time (seconds)	Iterations number	MAPEV (%)
Basic WLS	0,004541	4	1,28
SWLS1 (k=1)	Program doesn't converge		
SWLS1 (k=2)	0,005751	7	1,28
SWLS1 (k=3)	0,004102	4	1,28
SWLS2 (k=1)	0,002113	8	0,57
SWLS2 (k=2)	0,002550	4	1,33
SWLS2 (k=3)	0,003545	4	1,28

Table 1 shows that SWLS1 algorithm doesn't converge when gain matrix G is constant after just the first iteration (k=1). For k=2 , the SWLS1 algorithm converge but number of iterations is increased and computation time is higher compared to the basic WLS ; for k=3 , the results matches those obtained in the classic WLS , that's obvious since the WLS algorithm converges at the next iteration . It is noted that SLWS1 solution is the same as obtained through the basic algorithm (MAPEV% constant) and doesn't change whatever the k iteration value . Indeed , the gain matrix G influences only the convergence rate and has no effect on the solution value .

Regarding the second algorithm SWLS2 , It is observed in table 1 , that the solution changes slightly as function of k value (MAPEV% variable) but the precision remains still good. So, we deduce that the Jacobian matrix H influences the value of the estimated solution. In addition , we note that the computation time is reduced: the difference is more perceptible when gain and Jacobian matrices are considered constant after the first iterations (k=1 and k=2). However, more iterations are required for k=1.

Simulation Results for IEEE 30 bus System

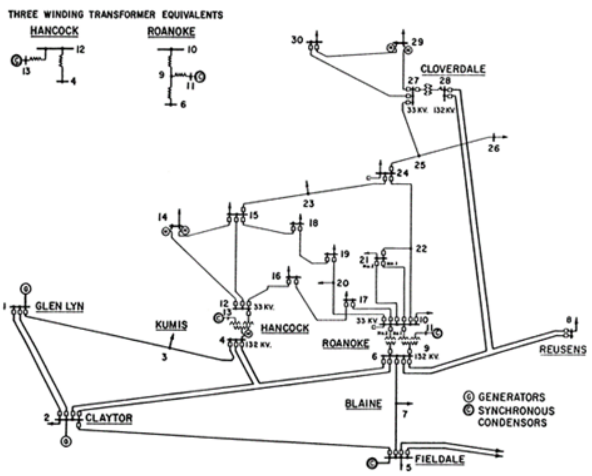

Fig.2. IEEE 30 Bus System

A set of 93 measurements (η=1,5) distributed as follow :

- 1 voltage magnitude at bus 1 .
- 18 real power injections and 18 reactive power injections at buses : 4, 5, 6, 8, 10,11,13,14,15,16,18,20,21,24,25,26,28 et 29.
- 28 real power flow and 28 reactive power flow on branches: 2-4, 2-5,3-1,4-3,4-6,5-7,6-2,7-6,9-6,10-6,10-9,12-14,15-12,15-18,16-17,17-10,19-20,20-10,21-10,21-22,22-10,23-24,24-2,25-27,27-28,28-6,29-30,30-27 .

Table2. Performance evaluation of Simplified Algorithms as a function of k (IEEE 30 bus System)

Algorithm	Computation time (seconds)	Iterations number	MAPEV (%)
Basic WLS	0,017011	4	1,74
SWLS1 (k=1)	Program doesn't converge		
SWLS1 (k=2)	0,013582	4	1,74
SWLS1 (k=3)	0,015297	4	1,74
SWLS2 (k=1)	0,006209	6	1,76
SWLS2 (k=2)	0,009288	4	1,74
SWLS2 (k=3)	0,013150	4	1,74

Table 2 shows a similar results to those obtained for the IEEE 14 bus system. Indeed, the algorithm SWLS1 does not converge for k = 1, for the other values of k the execution time decreases slightly compared to the basic form. Also , it's observed that algorithm SWLS2 converges in all situations and the computation time is significantly reduced for (k = 1 and k = 2). However, we note in Figure 3 that the solution of the SWLS2 algorithm applied to k = 1 diverges at the beginning of execution which increases iterations number.

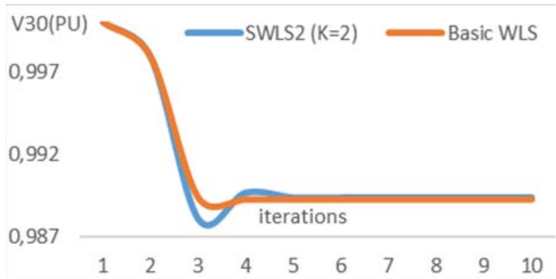

Fig.3. Evolution of estimated voltage magnitude of bus30 V30 per iteration (IEEE 30 bus system)

Simulation Results for IEEE 118 bus System

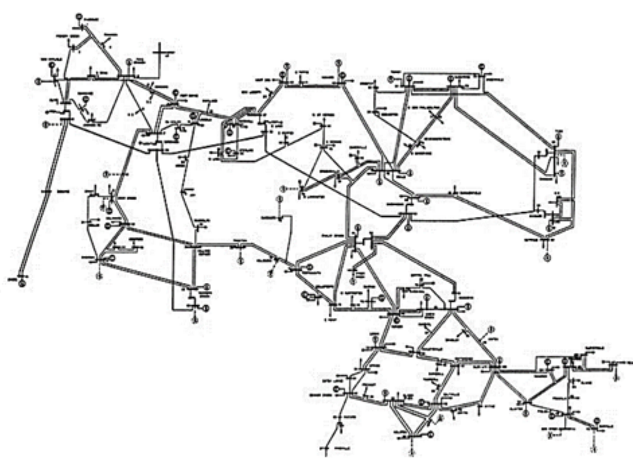

Fig.4. IEEE 118 Bus System

A set of 726 measurements is chosen a below :

- 118 voltage magnitudes for all buses.
- 118 real power injections and 118 reactive power injections for all buses.
- 186 real power flow and 186 reactive power flow for all network branches.

Table 3. Performance evaluation of Simplified Algorithms as a function of k (IEEE 118 bus System)

Algorithm	Computation time (seconds)	Iterations number	MAPEV (%)
Basic WLS	0,664319	5	3,53
SWLS1 (k=1)	Program doesn't converge		
SWLS1 (k=2)	Program doesn't converge		
SWLS1 (k=3)	0,625026	6	3,53
SWLS1 (k=4)	0,606933	5	3,53
SWLS2 (k=1)	0,285715	19	3,14
SWLS2 (k=2)	0,316678	8	3,47
SWLS2 (k=3)	0,415575	5	3,53
SWLS2 (k=4)	0,539947	5	3,53

In this case , the SWLS2 does not converge only for k=1 like the previous cases but also for k=2 as seen in Table 3. It reveals that considering gain matrix constant at first iterations associated to variable Jacobian matrix may lead to convergence problems especially for large networks which requires a big number of iterations. There are no noticeable difference in computation time for SWLS1 (k=3 and k=4) compared to the basic Algorithm. For SWLS2 results shown in table 3, the computation time is reduced by half for (k = 1 and k = 2) however the number of iterations is very high especially for k = 1 (19 iterations instead of 5 for the basic algorithm), which indicates that SWLS2 stability is not guaranteed in this case as shown in figure 2, in addition it is observed that the solution obtained doses not correspond to the basic WLS solution .

Fig.5. Evolution of estimated voltage magnitude of bus1 V1 per iteration (IEEE 118 bus system)

To sum up, simulation results highlights the interest of SWLS2 algorithm which presents more advantages compared to SWLS1. Firstly, the algorithm converges in all cases. Secondly, the computation time is reduced (2 times less than the Basic WLS): the difference is more perceptible when gain and Jacobian matrices are considered constant after the first iterations (k=1 and k=2), even if the convergence requires a higher iterations number.

To explain those results, the computing time of each intermediate steps was analyzed :

- Calculation of h the vector of nonlinear functions that relates the states to the measurements.
- Calculation of the Jacobian matrix H.
- Calculation of the gain matrix G.
- Calculation of the final estimated state S.

Table 4. Computation time average per iteration for each one of intermediate steps

Computation time average per iteration (seconds)	14 bus	30bus	118 bus
Measurements number	41	93	726
h	0,000105	0,000337	0,007900
H	0,000556	0,002147	0,066987
G	0,000439	0,001715	0,057385
S	0,000035	0,000054	0,000591

As seen in table 4 and figures (6 ,7 and 8), the mean time to calculate the gain and jacobian matrices is more important compared to the other parameters h and S. So, even if the SWLS2 applied at the first iterations (k=1 and k=2) requires more iterations number ,the computation time is lower thanks to the reduction of the repetitive calculation of the important parameters (G and H matrices).

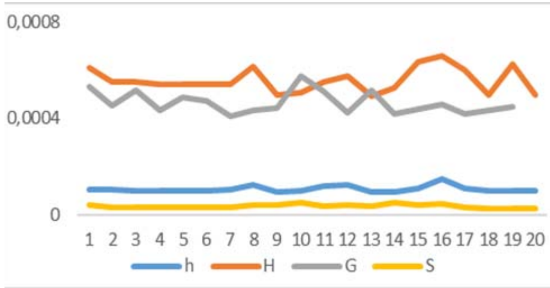

Fig.6. Computation time per iteration of intermediate steps in seconds (IEEE 14 bus system)

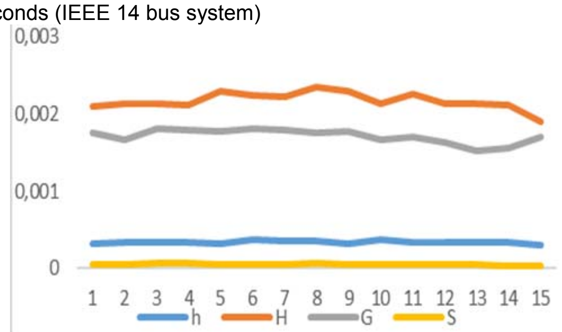

Fig.7. Computation time per iteration of intermediate steps in seconds (IEEE 30 bus system)

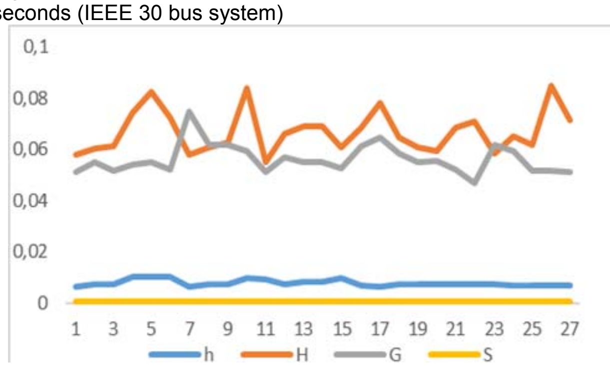

Fig.8. Computation time per iteration of intermediate steps in seconds (IEEE 118 bus system)

The next figures present details of computation time of intermediate steps for each algorithm.

As seen in figures 9, 10 and 11 , SWLS1 algorithm reduces just the computation time of the gain matrix while the other parameters remain recalculated at each iteration , in particular the Jacobian matrix which have a big impact . As a result, the application of this algorithm is not very interesting, especially for small value of k which usually increases iterations number.

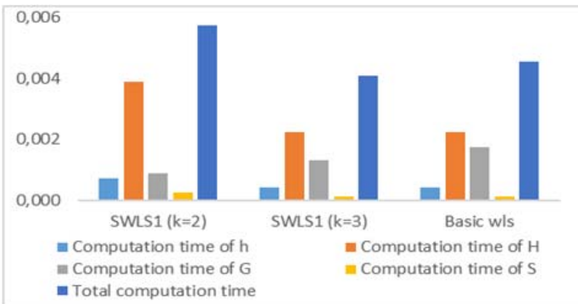

Fig.9. Computation time details in seconds for SWLS1 algorithm applied to IEEE14 bus system

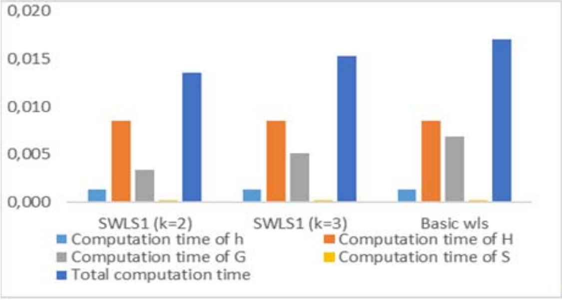

Fig.10. Computation time details in seconds for SWLS1 algorithm applied to IEEE30 bus system

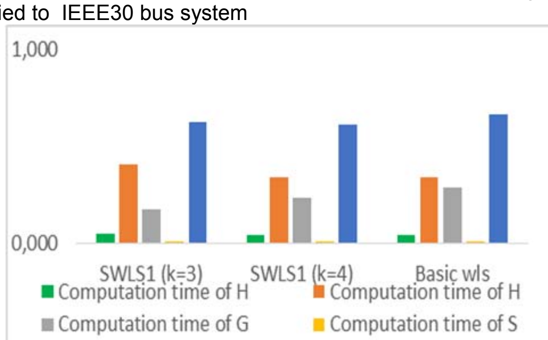

Fig.11. Computation time details in seconds for SWLS1 algorithm applied to IEEE118 bus system

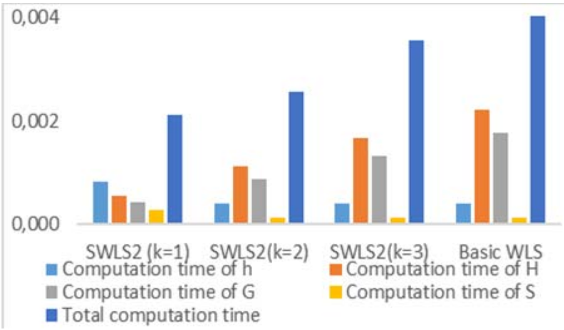

Fig.12. Computation time details in seconds for SWLS2 algorithm applied to IEEE14 bus system

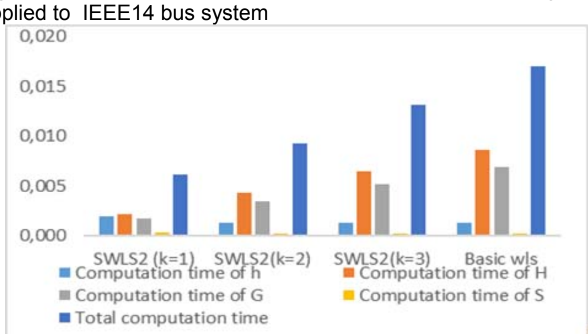

Fig.13. Computation time details in seconds for SWLS2 algorithm applied to IEEE30 bus system

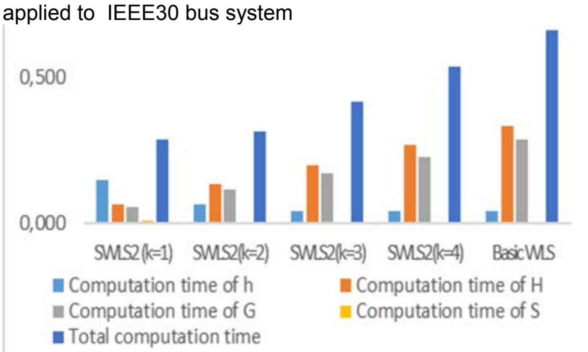

Fig.14. Computation time details in seconds for SWLS2 algorithm applied to IEEE118 bus system

Figures 11, 12, and 13 confirm the previous results, and we can say that the computation time of SWLS2 algorithm is proportional to k iteration value . It's interesting to apply this algorithm at the second iteration (k=2) since computation time is reduced without altering the algorithm stability and the solution obtained is closer to the traditional WLS solution compared to SWLS2 solution applied at the flat start (k=1).

Conclusion

This paper has presented an evaluation study of two simplified versions of the traditional weighted least squares algorithm for power system state estimation. Different simulation cases were tested on IEEE 14 , 30 and 118 bus systems to check and generalize the results.

This article reveals the interest of SWLS2 algorithm , which is the most efficient in terms of computation time, accuracy and convergence rate. Indeed , SWLS2 decrease by half the calculation time compared to the conventional algorithm , with the same reliability and accuracy .

The application of this modified algorithm is beneficial for large systems which need a large computation time and when the decoupling active/ reactive is not possible.

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