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Dynamical processes in sequential-bipolar pulse sources supplying nonlinear loads

***Abstract**. A uniform basis for analysis of oscillations with essentially non-harmonic shapes, excited by sources of non-smooth or could be discontinuous in time signals is presented. The tool presented here employs non-smooth (impact) systems as a basis to describing not only impact, but also smooth or even linear dynamics. The focus is put on explicit links between impact dynamics and hyperbolic algebras analogously to the link between harmonic oscillations and conventional complex analyses. Illustrations and results of computer simulations are presented.*

***Streszczenie.** W artykule przedstawiono ujednolicone podstawy analizy drgań, głównie nieharmonicznych przebiegów, wzbudzanych źródłowymi sygnałami, niegładkimi lub nieciągłymi w czasie,. W analizie wykorzystano niegładkie (udarowe) systemy do wyznaczania nie tylko nieciągłej, lecz także ciągłej lub nawet liniowej dynamiki układu. Nacisk położony został na ustalenie powiązań między dynamiką uderzeniową i algebrą hiperboliczną, analogicznie do przypadku drgań harmonicznych i konwencjonalnych liczb zespolonych. Ilustracje i wyniki symulacji komputerowych są przedstawione. (**Dynamiczne procesy w źródłach sekwencyjno-bipolarnych pulsów zasilających nieliniowe odbiorniki**).*

Keywords: hyperbolic numbers, non-smooth time transformation, nonlinear circuits, periodic nonharmonic oscillations.
Słowa kluczowe: liczby hiperboliczne, niegładka transformacja czasu, obwody nieliniowe, drgania okresowe niesinusoidalne.

doi:10.12915/pe.2014.03.32

Introduction

Over the last decade, the topology and dynamical behavior of various nonlinear systems have been extensively studied by researchers. Nonlinear dynamical systems of various natures (electrical, chemical, biological, mechanical, astrophysical, etc.) quite often exhibit complex dynamical behavior and undergo bifurcations when one or more parameters change. New technologies of components and systems exhibiting various favorable properties for the practice and constantly increasing many applications have created needs for new mathematical tools that they will serve to describe the dynamic processes taking place in these parts and systems [1, 2, 3].

The main objective of this paper is to introduce a uniform basis for analyses of oscillations with essentially non-harmonic shapes, excited by non-smooth or could be discontinuous in time sources. It is known that possible transitions to non-smooth limits can make investigations especially difficult. This is due to the fact that the dynamic methods were originally developed within the paradigm of smooth oscillations based on the classical theory of differential equations, usually avoiding non-differentiable and discontinuous functions. Over the last three decades general interest in such a tool has experienced continuing growth. Presently, however, many theoretical and applied areas cover high-energy phenomena accompanied by strongly non-linear spatio-temporal behaviors making the classical smooth methods inefficient in many cases [4,5, 6].

Possible alternatives to such approaches can be built on generating models developing essentially nonlinear /nonharmonic behaviors as their inherent properties. Such models must be general and simple enough in order to play the role of physical basis.

The tool presented here employs nonsmooth (impact) systems as a basis to describing not only impact, but also smooth or even linear dynamics. This is built on the idea of non-smooth time transformations (NSTT) proposed originally for strongly nonlinear, but still smooth models [7, 8, 9].

The methodological role of NSTT is to reveal explicit links between impact dynamics and hyperbolic algebras analogously to the link between harmonic oscillations and conventional complex analyses. Currently, this is one of the principal challenges at the crossroad between mathematics, physics and computer science [10, 11,12, 13].

Non-smooth time transformation

The theory of nonlinear dynamical systems is technically difficult and includes complementary ideas and methods from many different fields of mathematics. With the advent of computers, starting in the 1960s, it became possible to study dynamical systems in real-time and to store data for analysis.

Generating models for strongly nonlinear analytical tools with a wide range of applicability must obviously: (*i*) capture the most common features of oscillating processes regardless their nonlinear specifics, (*ii*) possess simple enough solutions in order to provide efficiency of perturbation schemes, and (*iii*) describe essentially nonlinear phenomena out of the scope of the weakly nonlinear methods.

In the sequel we are focused on the analysis of a class of nonconstant solutions of differential equations which are next related to fixed points in the scale of complexity, namely periodic orbits. The latter are interesting in themselves as mathematical representation of periodicities in natural and social phenomena. These impose two principal features on the dynamical systems by generating specific algebraic structures and switching formulations to boundary-value problems. Further, dynamical systems with discontinuities can be simplified by means of appropriate non-smooth transformations of variables [13,14].

The present approach employs time histories of impact systems as new time arguments for strongly nonlinear, but smooth periodic oscillations with certain temporal symmetries. Despite the strong nonlinearity caused by impacts, the limit oscillator is also described by quite simple elementary functions such as triangular sine and rectangular cosine, say $p(t)$ and $\dot{p}(t) = e(t)$ which are presented in Fig.1. These two signals are associated with another subgroup of the oscillations namely translation and reflection.

The basic signals $p(t)$ and $\dot{p}(t)$ are expressed through the standard elementary functions in the closed form as

(1) $$p(t) = \frac{2}{\pi}\arcsin(\sin(\frac{2\pi}{T}t))$$

and

(2) $$e(t) = \dot{p}(t) = \frac{\cos(\frac{2\pi}{T}t)}{|\cos(\frac{2\pi}{T}t)|}$$

Any presence of signals $p(t)$ and $\dot{p}(t)$ in further developed analytical algorithms is not a simple match of different pieces of solutions. On the contrary, it has really itself physical basis and invokes specific mathematical tools [13]. These impose two principal features on the dynamical systems by generating specific algebraic structures and switching formulation to boundary-value problems. As a result, using discontinuities or distributions for modeling dynamical systems take the corresponding differential equations out of frames of the classic theory of differential equations.

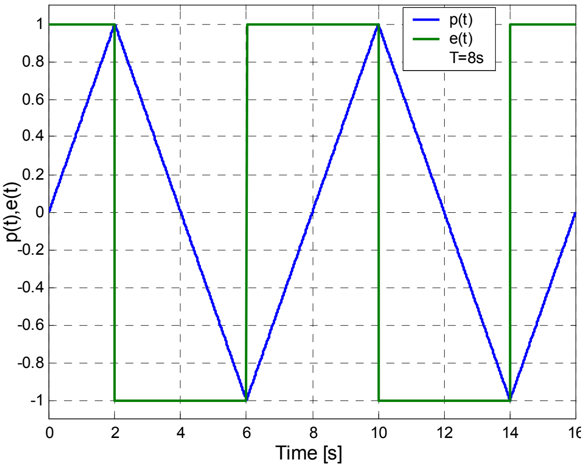


Fig.1. Triangular sine $p(t)$ and rectangular cosine $e(t) = \dot{p}(t)$ signals

Hyperbolic numbers

The hyperbolic numbers called also perplex numbers, or split-complex numbers, are a two-dimensional commutative algebra over the real numbers different from the complex numbers [14]. Every hyperbolic number has the form

(3) $$w = x + uy,$$

where x and y are real numbers. The number u is similar to the imaginary unit j, except that

(4) $$u^2 = +1.$$

Just as for complex numbers, one can define the notion of a hyperbolic conjugate number as

(5) $$w^* = x - uy,$$

The modulus of a hyperbolic number $w = x + uy$ is given by the isotropic quadratic form

(6) $$|w| = \sqrt{w \cdot w^*} = \sqrt{x^2 - y^2}$$

There are two nontrivial idempotents given by $q = (1 - u)/2$ and $q^* = (1 + u)/2$. Recall that idempotent means that $qq = q$ and $q^* \cdot q^* = q^*$. Moduli of both these elements are null:

(7) $$|q|=|q^*|=|q \cdot q^*|=0$$

Very often it is convenient to use q and q^* as an alternate basis for the hyperbolic plane. This basis is called the diagonal basis or null basis. The hyperbolic complex number w can be written in the null basis as

(6) $$w= x + uy = (x - y)q + (x + y)q^* .$$

Fig. 2 illustrates the differences between complex and hyperbolic planes. Note, that in contrast to the circle, each of the hyperbola branches is covered exactly once as the hyperbolic angle θ is varying in the infinite interval.

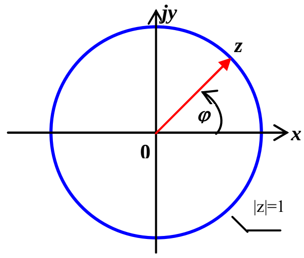
Respective portions of the hyperbolic plane show subsets with modulus zero (red), one (blue), and minus one green). The analogue of Euler's formula for the hyperbolic numbers is

(9) $$\exp(u\theta) = \cosh(\theta) + u \cdot \sinh(\theta)$$

where θ is standing for the hyperbolic angle.

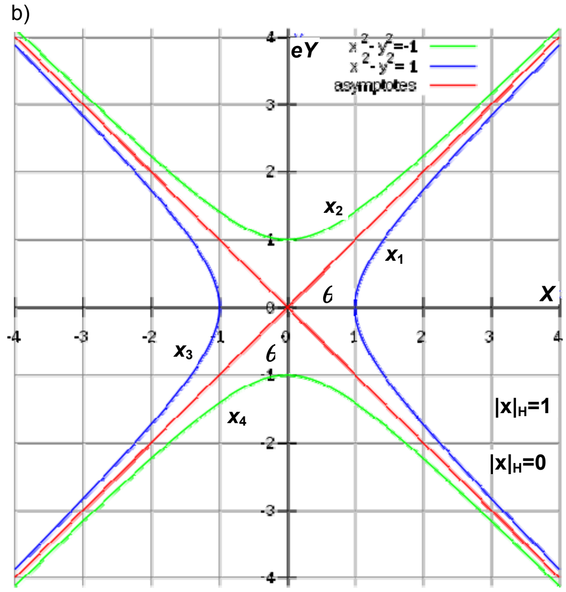
The above equality can be derived from a power series expansion using the fact that *cosh* has only even powers while *sinh* has odd powers only. The hyperbolic angle θ is twice the area of the sector $A0x_1$ in Fig.2. Also, to hyperbolic angles θ we can give the geometrical meaning of an area θ = 2*area*($A0x_1$) and this area has the same value measured in both "hyperbolic" or "Euclidean" way. By analogy with the circular angles φ defined on the unitary circle |z|=1, we can define *cosh*(θ) and *sinh*(θ) as the abscissa and the ordinate of the hyperbola point defined by θ, respectively. Then, such an approach still works for general cases by generating specific algebraic structures in terms of the coordinates.

a)



$z = \exp(j\varphi)$, $\varphi = \arctan(\frac{y}{x})$

b)



$x_{1,3} = \pm\exp(e\theta)$, $\theta = \operatorname{arctanh}(\frac{Y}{X})$;

$x_{2,4} = \pm e \cdot \exp(e\theta)$; $\theta = \operatorname{arctanh}(\frac{X}{Y})$; $-\infty < \theta < \infty$

Fig.2. a) complex plane, b) hyperbolic plane

In our case, the unipotent u is not a number but the discontinuous function of certain physical nature i.e. the rectangular cosine wave $e(t)$. Indeed since t is running then there is no unique choice for the magnitude of e, whereas always e^2 = 1.Therefore, identity (3) generates the hyperbolic structure from the very general properties of periodic processes.

Finally, let us mention that the hyperbolic plane has another natural basis associated with the two isotropic lines separating the hyperbolic quadrants as shown in Fig. 2. The transition from one basis to another is given by $e_{\pm}=(1\pm e)/2$ or, inversely, $1 = e_+ + e_-$ and $e = e_+ - e_-$. The elements e_+ and e_- are mutually annihilating (*idempotents*) so that $e_+e_-=0$, $e_-^2 = e_-$ and $e_+^2 = e_+$. It is clear also that $ee_+= e_+$ and $ee_-= -e_-$. Note that this basis usually couples the corresponding smoothness (boundary) conditions. Therefore, for any periodic function $x = x(t)$ whose period is T we can write

(10)
$$\begin{aligned} x &= X + Ye = X(e_+ + e_-) + Y(e_+ - e_-) \\ &= (X+Y)e_+ + (X-Y)e_- = X_+(p)e_+ + X_-(p)e_- \end{aligned}$$

where

(11) $$X_+(p) = X(p) + Y(p) \text{ and } X_-(p) = X(p) - Y(p)$$

This suggests possible recipes for effective dealing with the differential equations of oscillation on entire time intervals, despite discontinuity and/or non-smoothness points. Such approach is developed to satisfy the matching conditions automatically by means of specific coordinate transformations on preliminary stages of study.

Modelling methodology with applications of NSTT

At the beginning of the twentieth century, Albert Einstein developed his theory of special relativity, built upon Lorentzian geometry. The hyperbolic numbers, blood relatives of the popular complex numbers, serve not only to put Lorentzian geometry on an equal mathematical footing with Euclidean geometry; their study also helps researchers develop algebraic skills and concepts necessary for higher level methodology of modelling physical systems. The hyperbolic numbers also called the "perplex numbers", serve as coordinates in the Lorentzian plane in much the same way that the classic complex numbers serve as coordinates in the Euclidean plane [5, 14].

The present section is dealing with the differential equations of oscillations on entire time intervals, despite discontinuity and/or non-smoothness points. The method is developed to satisfy the matching conditions automatically by means of NSTT on preliminary stages of study. It will be shown that such an approach still works for general cases by generating specific algebraic structures in terms of the coordinates.

Major features induced by NSTT can be briefly listed as follows:

• Introducing non-smooth temporal variables, in particular triangular sine wave $p(t)$, brings the coordinates into the algebra of hyperbolic numbers;

• Under appropriate conditions, differentiation or integration of the coordinates keeps the result within the same algebra and therefore eases the corresponding manipulations with the dynamic systems;

• Explicit time argument can be used together with the non-smooth time in order to describe amplitude and/or frequency modulated processes.

The important point to note here is that the NSTT itself forms a preliminary stage of analysis finalized by specific boundary value problems on standard time intervals. In order to develop the method, the triangular sine wave $p(t)$ is introduced into dynamical systems as a new temporal argument

To introduce some preliminaries and fundamentals of the method based on NSTT let us consider a dynamical system described by first-order differential equation with respect to the vector-signal $\boldsymbol{x}(t) \in \boldsymbol{R}^n$

(12) $$\dot{x}(t) = \boldsymbol{f}(x)$$

where $\boldsymbol{f}(\boldsymbol{x})$ is a continuous vector-function, and the over dot indicates time derivative [15].

In what follows we consider the class of periodic oscillations of the period $T = 4a$, with $a > 0$ as a constant. Note, that the assumption of periodicity is imposed automatically by the form of representation for periodic solutions. Such formalism is based on the following statement:

Any periodic process $\boldsymbol{x}(t)$ *of the period T can be expressed through the dynamic state of the impact oscillator,* $\{p(t), \dot{p}(t)\}$ *in the form of 'hyperbolic number' as follows*

(13) $$\boldsymbol{x}(t) = \boldsymbol{X}(p) + \boldsymbol{Y}(p)e(p)$$

where $\boldsymbol{X}(p)$ *and* $\boldsymbol{Y}(p)$ *are unknowns to be determined*.

Equations for $\boldsymbol{X}$ and $\boldsymbol{Y}$ components are obtained by substituting (13) into the corresponding differential equation of oscillations (12). Then, either analytical or numerical procedures can be applied. For instance, one may seek solutions in the form of power series with respect to the 'oscillating time' $p(t)$. Therefore, expression (13) can be qualified as non-smooth time transformation, $t \to p(t)$, on the manifold of periodic oscillations and generates the hyperbolic structure from the very general properties of periodic processes.

It is important to note that, under some conditions on $\boldsymbol{X}$ and $\boldsymbol{Y}$, combination (13) can be of any class of smoothness even though the couple $\{p(t), \dot{p}(t)\}$ has singularities at such instances t where $p(t) = \pm 1$. Moreover, both terms on the right-hand side of (13) are essential as those responsible for components with different temporal symmetries. This follows from linear independence of the elements 1, $e(t)$ and $\dot{e}(t)$, as those from different classes of smoothness.

Substituting (13) in (12) yields

(14) $$(\boldsymbol{Y}' - a\boldsymbol{U}_f) + (\boldsymbol{X}' - a\boldsymbol{V}_f)e + \boldsymbol{Y}e' = \boldsymbol{0}$$

where

(15)
$$\begin{aligned} \boldsymbol{U}_f &= \boldsymbol{U}_f(\boldsymbol{X},\boldsymbol{Y}) = \frac{1}{2}[\boldsymbol{f}(\boldsymbol{X}+\boldsymbol{Y}) + \boldsymbol{f}(\boldsymbol{X}-\boldsymbol{Y})]; \\ \boldsymbol{V}_f &= \boldsymbol{V}_f(\boldsymbol{X},\boldsymbol{Y}) = \frac{1}{2}[\boldsymbol{f}(\boldsymbol{X}+\boldsymbol{Y}) - \boldsymbol{f}(\boldsymbol{X}-\boldsymbol{Y})]. \end{aligned}$$

with *'* as the symbol of differentiation with respect to $p(t)$ and provided that $\boldsymbol{f}(\boldsymbol{X} \pm \boldsymbol{Y})$ is defined.

By means of the boundary condition for $\mathbf{Y}(p)$ we can eliminate the periodic singular term $e' = de(t)/d(t)$, and obtain the non-linear boundary value problem on the standard interval $-1 \le p(t) \le 1$,

(16)
$$\begin{aligned} \boldsymbol{X}' &= a\boldsymbol{V}_f(\boldsymbol{X},\boldsymbol{Y}); \\ \boldsymbol{Y}' &= a\boldsymbol{U}_f(\boldsymbol{X},\boldsymbol{Y}); \\ \boldsymbol{Y}|_{p=\pm 1} &= \boldsymbol{0}. \end{aligned}$$

Note, that representation (13) unfolds the corresponding fragment on the entire time interval $-\infty < t < \infty$, and both terms on the right-hand side of (13) are essential as those responsible for components with different temporal symmetries. Since the $\boldsymbol{Y}$-component of the solution appeared in (16) to be even with respect to $p(t)$, then both of the conditions are satisfied even though one arbitrary constant only is available for $\boldsymbol{Y}$.

Numerical search procedure

Our aim in this section is to expose the effectiveness of the presented method for calculating periodic waveforms in nonlinear circuits with non-smooth input signals. For this purpose we present a mathematical model of two dual nonlinear circuits yielding periodic responses under excitations determined by sources with sequential-bipolar pulse signals. We show that relatively simple circuits with one nonlinear element, one independent source, and a linear dynamical element may yield quite complex periodic response diagrams with appropriate initial conditions impacting those diagrams. Secondly, this paper complements the analysis of the singularly perturbed ordinary differential equations model [12, 16] where a different perspective (analysis of concatenation solutions and periodizer) was used.

The nonlinear elements in the two dual circuits shown in Fig. 3 have their characteristics v_{nonl} = $f(i)$ (Fig. 3-left) and i_{nonl}=$f(v)$ (Fig. 3-right). If we assume that $x(t)=x(t+T)$ represent the state variable and f(t) = $f(t+T)$ corresponds to the source signals in particular circuits then both circuits can be described by the following nonlinear ordinary differential equation

(17) $$\dot{x}(t) + kg(x(t)) = kf(t)$$

where $k \equiv L^{-1}$ for the circuit in Fig. 3a) and $k \equiv C^{-1}$ for the circuit in Fig. 3b).

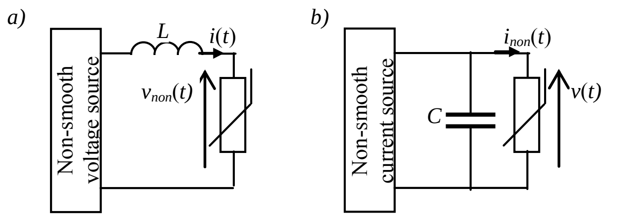


Fig. 3. Nonlinear circuits supplied by sequential-bipolar pulses: a) of voltage source, b) of current source

In the sequel we will continue to focus our attention on the issues raised by the practice [18]. It is well known that operating status of receivers such as LEDs, solar cells, electro-crystallization reactors, lightning arresters, semiconductor diodes, etc., can be reflected by the characteristics of the nonlinear element

(18) $$g(x) = hx^2$$

where x and h denote the state variable and appropriate parameter of the given nonlinear element, respectively. We also take into account the forcing term in the composed form of sequential-bipolar pulses represented by

(19) $$f(t) = F_0 + F_1 e(t)$$

where F_0 and F_1 represent DC component and magnitude of bipolar pulses, respectively.

Substituting (18) and (19) in (17) yields

(20) $$\dot{x}(t) + khx^2(t) = kF_0 + kF_1 e(t)$$

Following the methodology formulated in the previous section we present the solution of equation (20) in the form

(21) $$x(t) = X(p) + Y(p)e(p)$$

where $p(t)$ = $p(t+T)$ and $e(t)$ = e($t+T$) are triangular and rectangular waves, respectively, with the period T = 4a. Components X and Y can be determined on the base of relations (16).

Substituting (21) in (20) gives

(22)
$$a^{-1}(Y' + X'e + Ye') + kh(X^2 + Y^2 + 2XYe) = kF_0 + kF_1 e$$

where e' =de/dt, and therefore

(23)
$$Y' + akh(X^2 + Y^2) = akF_0,$$
$$X' + 2akhXY = akF_1,$$
$$Y(\pm 1) = 0$$

By introducing new unknowns $U = X + Y$ and $V = X - Y$, we can transform the boundary value problem (23) to the form

(24)
$$U' + akhU^2 = akF,$$
$$V' - akhV^2 = -akG,$$
$$U(\pm 1) = V(\pm 1)$$

where $F = F_0 + F_1$, and $G = F_0 - F_1$ are constant.

General solutions of both equations in (24), being separable, admit the forms

(25)
$$U(p, C_1) = \sqrt{\frac{F}{h}}[\operatorname{arctanh}(\sqrt{\frac{h}{F}}C_1) + \tanh(ak\sqrt{hF}\,p)],$$
$$V(p, C_2) = -\sqrt{\frac{G}{h}}[\operatorname{arctanh}(\sqrt{\frac{h}{G}}C_2) + \tanh(ak\sqrt{hG}\,p)],$$

where C_1 and C_2 are arbitrary constants of integration to be determined from the boundary conditions

(26)
$$U(1, C_1) = V(1, C_2),$$
$$U(-1, C_1) = V(-1, C_2),$$

Each real solution of (26) for the constants C_1 and C_2 substituted in (25) yields a periodic solution of differential equation (20). Thus, we have

(27) $$x(t) = \frac{1}{2}(U + V) + \frac{1}{2}(U - V)e$$

Fig. 4 shows what happens to the steady state variable profile as the period of excitation source signal becomes twice longer. The model parameters are k =1, h =1.5, F_0 = 2, F_1 =1.5. In cases T = 4s and T = 8s, the arbitrary constants are C_1 = 222.1473, C_2 = -0.0799 and C_1 = 17594.31, C_2 = -65.767, respectively. Diagrams presented below were obtained by using MATLAB computations [17].

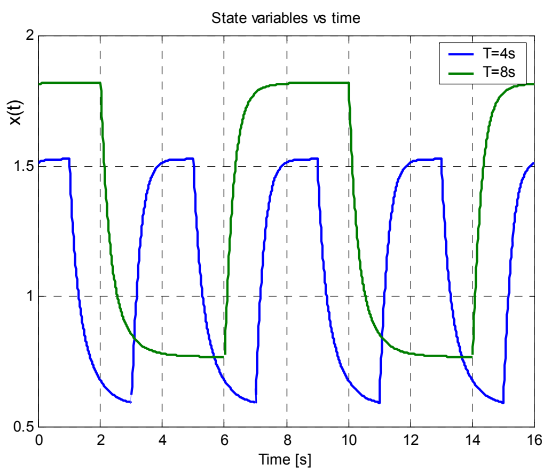


Fig.4. Profiles of steady state variables obtained for two different periods of the sequential –bipolar source signal

It is worth noticing that the longer period T = 8s gives somewhat smaller average of the state variables, comparing with the shorter period T = 4s. All details in this subject are beyond the scope of this paper.

Discussion, summary and conclusions

In this paper, a version of nonsmooth argument substitutions, specifically - nonsmooth time, is introduced with proofs of the related identities. Basic rules for algebraic and differential manipulations are described. In particular, final section shows how to implement nonsmooth argument substitutions in the differential equations of oscillations on entire time intervals, despite discontinuity and/or non-smoothness points. These impose two principal features on the behavior of dynamical nonlinear circuits by generating specific algebraic structures and switching formulation to boundary-value problems. However, the corresponding boundary value problems still remain coupled because the idempotent basis makes unfortunately boundary conditions coupled. The methodological role of non-smooth time transformation is revealed exhibiting explicit links between impact dynamics and hyperbolic algebras analogously to the link between harmonic vibrations and conventional complex analyses. It has to be noted that the transformation itself implies no constraints on dynamical systems and easily applies to both smooth and non-smooth systems [19].

In the present case, solution is approximated by the triangular sine wave *p*(*t*), and can be corrected by higher powers of the same triangular sine components. On the physical point of view, the model under consideration has to be close to the impact oscillator rather than the harmonic one. In terms of the new time variable *p*(*t*), such an assumption simply means that the right-hand side of the differential equation of oscillation (17) is small enough to justify the following generating system. Any further steps, however, should account for physical properties of the related systems.

Although the entire boundary value problem is still coupled through the boundary conditions, the problem caused by coupling is eased, however, since algorithms for solving equations are often more complicated than those applied to the boundary conditions. Therefore, the triangular wave time transformation *p*(*t*) possesses the unique property among all periodic time substitutions because it preserves the form of differential equations of conservative oscillators.

In order to deepen the discussion and exhibiting the efficiency of the presented approach for the analysis of nonsmooth systems let us consider the Duffing oscillator with the forcing function *Fp(t)*, where F = *const*, and p(*t*) is assumed to be the triangular sine wave of the period 4*a*. The oscillator is represented

(28) $$\ddot{x}(t) + b\dot{x}(t) + x^3(t) = Fp(t)$$

where the new temporal argument is the triangular sine.

Considering periodic solutions and taking into account (21) and then substituting it into equation (28) and considering the result as a two-component element of the hyperbolic algebra, one obtains the boundary value problem

(29) $$\begin{aligned} &a^{-2}X'' + a^{-1}bY' + XY^2 + X^3 = Fp(t), \\ &a^{-2}Y'' + a^{-1}bX' + 3X^2Y + Y^3 = 0, \\ &X'|_{p=\pm 1} = 0, \quad Y|_{p=\pm 1} = 0 \end{aligned}$$

where the boundary conditions for *X*' and *Y* stay for elimination of the periodic series of Dirac functions from the first and second derivatives of the state variable.

Although equations (29) have a more complicated form as compared to (23), their solutions can be determined in a numerical way applying appropriate procedures from the program package MATLAB. Note that in accord to the Cauchy's theorem for derivatives the corresponding solutions must be at least twice continuously differentiable functions of time.

The evolution of the state variable *x*(*t*) and the corresponding phase portrait exhibiting periodic steady state of the Duffing oscillator are presented in Fig.5. Observe, that under some conditions on *X* and *Y*, combination (21) and (29) can be of any class of smoothness even though the couple {*p*(*t*), $\dot{p}$ (*t*)} has singularities at such time instances *t* where *p*(*t*) = ±1.

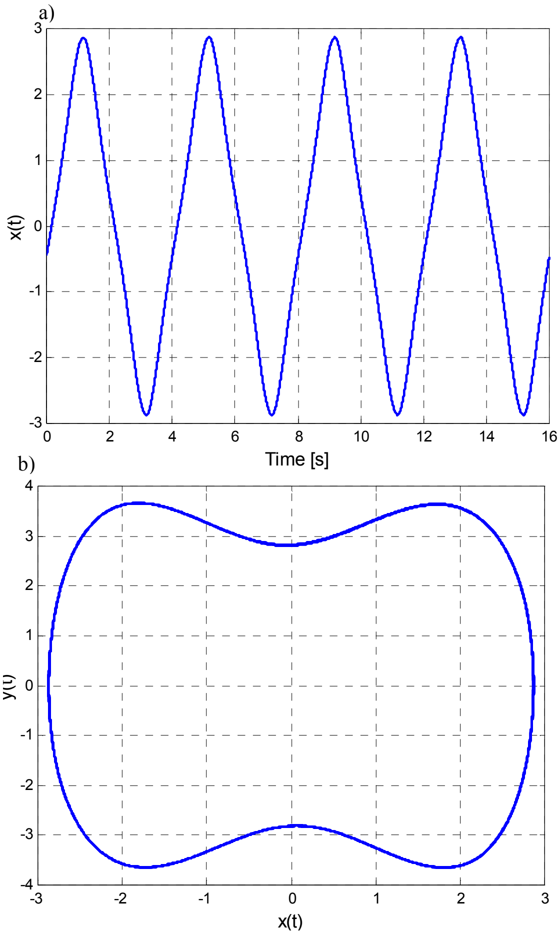


Fig.5. Periodic response of the Duffing oscillator (28) for *b*=0.5, *F*=10 and *T* =4s: a) state variable vs. time, b) phase portrait $y(t) = \dot{x}(t)$ vs. *x*(*t*)

As it is seen from the diagram in Fig. 5a the state variable oscillates with the same period as that of the forcing term. The phase portrait shown in Fig.5b exhibits typical form of the limit cycle for Duffing oscillators.

The association of hyperbolic numbers with the two-dimensional Lorentz's group of Special Relativity makes hyperbolic numbers relevant for physics and stimulate us to find their application in the same way as complex numbers are applied to Euclidean plane geometry.

The presented uniform physical basis for analyses of oscillations with essentially nonharmonic, non-smooth or may be discontinuous time shapes is efficient for identifying periodic steady state of nonlinear systems supplied by sequential-bipolar form of excitations. It is known that possible transitions to non-smooth limits can make investigations especially difficult but the method developed now that satisfy the matching conditions automatically by specific coordinate transformations on preliminary stages of study is very useful and promising. The occurrence of such algebraic structures seems to be essential feature of the approach since it justifies and simplifies analytical manipulations with non-invertible temporal substitutions such as NSTT. This point of view is illustrated by physical examples, problem formulations and solutions.

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