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The object behaviour analysis, control and prognosis in a state space.

Abstract. *The paper deals with possibility of analyzing the behaviour of complex processes, controlling them and their prognosis in a state space. We express the coefficients of the state equations with increases in terms of the coefficients of the partial regression models based on the Kolmogorov-Gabor equations with sensitivity functions [3]. Then we examine what derived increases' values can mean. And whether they can be used in order to predict future values of the studied process. The results are promising for the processes characterized by certain regularities (without noises).*

Streszczenie. *Poniższy artykuł omawia możliwość analizowania zachowania się złożonych procesów, sterowania nimi oraz ich prognostyki w przestrzeni stanów. Próbujemy wyrazić współczynniki równań stanów w przyrostach poprzez współczynniki cząstkowych modeli regresyjnych opartych na równaniach Kołmogorowa-Gabora zawierających funkcje wrażliwości [3]. Badamy, jaki sens mogą mieć uzyskane wartości przyrostów oraz czy można ich wartości wykorzystać w celu predykcji przyszłych wartości badanego procesu. Dla funkcji charakteryzujących się pewnymi regularnościami (brak szumów) wyniki są obiecujące. (Analiza zachowania się obiektów, ich kontrola i prognoza w przestrzeni stanów)*

Keywords: control systems, system identification, prognosis, state equations, regression models.
Słowa kluczowe: systemy sterowania, identyfikacja systemów, prognoza, równania stanów, modele regresyjne.

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Introduction

An object formally treated as a system is known through modelling and identification and can be understood by analysis. Modelling and identification techniques help to develop knowledge about a system. Modelling by itself is a vast area rich in a host of well-established methods. The field of system identification grew both in size and diversity over the last several decades and it was surveyed at different stages. One of the highly valuable engineering tool in this area is black-box modelling, whether linear or nonlinear. We propose a matrix identification method which can be used in economic forecasting.

General features of the object state equations

The analyzed object (process) can be formally considered as a *black box* and variables which characterize the behaviour of the object can be divided into two groups – input and output variables. In the system identification theory [1, 2] a way of description of object's behaviour is called *input-output model*. A connection between inputs and outputs can be represented by a regression dependence of the process variables $x_1(t_{N+1}), x_2(t_{N+1}), ..., x_n(t_{N+1})$ at the time point t_{N+1} [3] (output variables of the black box) on variables (output variables of the *black box*) on variables $x_1(t_N), x_2(t_N), ...,$ $x_n(t_N)$ at previous time point t_N (input variables).

In some cases, while studying the behaviour of the objects, it is useful to introduce a mathematical model with additional variables characterizing 'internal' features of studied objects like unmeasured disturbances or some other. The equations with input, output and state variables are called the *input-states-output equations* in the system identification theory, or simply object (model) state equations [1, 2].

As is known, [1, 2], in the system identification theory, the methods based on that mathematical tool are traditionally called state space identification methods. An important advantage of these methods is their flexibility in testing both the linear and nonlinear object models. As is known, [1] the idea of states is the fundamental concept which cannot be determined by more 'basic' category like the concept of 'a set' in mathematics. This gives a foundation for the selection of the state variables at the discretion of the investigator provided however, that selected state variables and equations meet certain conditions. In the system identification theory these conditions are divided into two categories:

- conditions of determinism of objects behaviour which come under mathematical simulation,
- conditions concerning directly state equations.

Whatever the physical meaning of the variables is, input, output and state variables are formally considered to be 'signals' in the system identification theory. Conditions of determinism of the object's behaviour expect that this behavior can be generally mathematically described. Namely, the object can be referenced in the category of deterministic objects if its behavior satisfies the following conditions [1] (bold letters stand for the corresponding vectors):

1. For a given object there is a class of time functions $\mathbf{v}(t)$, called the permissible input functions that describe the object's input signals (this may be not only the signals but the external inductions or the impact of different origin, which, according to the tasks should be taken into account in the analysis of the object).
2. At any time point *t* we can use a set of variables X_t, where the $\mathbf{x}(t)$ describes the object's states at time (these variables, as has been mentioned above, belong to the category of object's state variables).
3. Each pair of $\mathbf{v}(t)$, $\mathbf{x}(t)$ corresponds to at least one time function, called the output function $\mathbf{y}(t)$, and at each time point $t' > t$ of $X_{t'}$ there is a single component $\mathbf{x}(t')$.

Equations with input, output and state variables (if we want them to be regarded as a state model) should meet the following requirements [1]:

1. If $v_1(t)$ and $v_2(t)$ are permissible input functions, then:
   $v_3(t) = v_1(t), t \leq t_0,$
   $v_3(t) = v_2(t), t > t_0,$
   should also be an input function.
2. The future values of output variables do not depend on the nature of reaching the current state by the object. The object state at the current time and the current and future values of the inputs clearly determine the current and future values of their outputs. Therefore for every $\mathbf{x}(t_0)$ of X_{t_0} and for permissible input functions $\mathbf{v}_1(t)$, $\mathbf{v}_2(t)$ (where $\mathbf{v}_1(t) = \mathbf{v}_2(t)$ for $t > t_0$ any output function corresponding to $\mathbf{x}(t_0)$ and $\mathbf{v}_1(t)$ is identical to any output function corresponding to $\mathbf{x}(t_0)$ and $\mathbf{v}_2(t)$ for $t > t_0$.
3. If the initial object states and the input $\mathbf{v}(t)$, $t \geq t_0$ are specified, then the output $\mathbf{y}(t)$, $t \geq t_0$ is uniquely determined.

The above three conditions can be written in the form of two vector equations, called state equations:

(1) $$\mathbf{x}(t) = f[\mathbf{x}(t_0); \mathbf{v}(t_0, t)],$$

(2) $$\mathbf{y}(t_0, t) = g[\mathbf{x}(t_0); \mathbf{v}(t_0, t)],$$

where f and g are the unique vector functions. The equation (2) shows that the output signal **y** on a time interval (t_0, t) is the unique function of input signal **v** on that time interval and the state at the beginning of this interval. Moreover, following the (1) equation the state at the end of the interval is the unique consequence of the state at the beginning of the interval. Thus, the selection of input, output and state variables is conditioned by the properties of the state equations and their variables. Due to the presence of the state variables, the process subject to the simulation can be formally presented in the form of an object whose behaviour is described by three groups of variables: input vector $\mathbf{v}(t_0, t) = [v_1(t_0, t), v_2(t_0, t), ..., v_n(t_0, t)]$, output vector $\mathbf{y}(t_0, t) = [y_1(t_0, t), y_2(t_0, t), ..., y_m(t_0, t)]$ and state vector $\mathbf{x}(t_0, t) = [x_1(t_0, t), x_2(t_0, t), ..., x_s(t_0, t)]$, wherein the size of these vectors, in principle, may be different. Namely, using the terminology of systems theory, relations (1)-(2) can be viewed as the equations describing the object's behaviour represented in the form of a 'black box' of n input signals, m output signals and s state variables.

From the analysis presented above, in particular, we can draw two conclusions. First, the fulfillment of the conditions of determinism is a sufficient condition that for a given object, in principle, we can select a state variables which satisfy the conditions of equations in the state space (eg, in the particular case the input variables can be identified as a state variables of the object). Secondly, if the conditions of determinism are not met, the behaviour of this object, in principle, cannot be described by any mathematical model, because the determinism conditions comply with the conditions of immanence of the model [2]. Therefore, without loss of generality of the obtained results, we believe that the processes to be tested, always comply with the conditions of determinism, and the state equations, which we will use further satisfy the above state models conditions.

Object state variables can be intuitively defined as the minimum amount of information about the object which is necessary to determine both its future outputs and states of the system if the input function is known [1]. States models can be represented in the form of structural diagrams reflecting the interrelationships between variables entering into these models. Depending on the state equations, there are different structural diagrams. Let's consider three models: 1) for the system of linear state equations, 2) for the system of nonlinear state equations, 3) for the system of nonlinear discrete-time state equations with increases:

1. *The linear state equations*. If the behaviour of an object can be presented in the form of linear relationships between variables, then the system of equations (1)-(2) can be represented as a set of linear differential and algebraic equations [1]:

   (3) $$d\mathbf{x}/dt = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{v}(t),$$

   (4) $$\mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t) + \mathbf{D}(t)\mathbf{v}(t),$$

   where **A**(t), **B**(t), **C**(t), **D**(t)– matrixes with fixed or time-varying components. The components of the matrix **A**(t) are the parameters that define the interrelationships between the states variables. The components of the matrix **B**(t) determine the impact of input signals on the values of the state variables. The components of the matrix **C**(t) determine the impact of the state variables on the output variables, and the components of the matrix **D**(t) reflects a direct impact of the value of the input signals on the output signals while there is a direct relationship between inputs and outputs in the studied object.
2. *The nonlinear state equations*. The state equations (1)-(2) in general case, may reflect both linear and nonlinear relationships between variables. In addition, the time dependences can be presented both as a continuous-time variable and discrete-time variable function (for example, during the experimental measures of these variables at t_0, t_1, t_2, ..., t_N, t_{N+1}, ...,). Sets of parameters which are defined at the discrete time points are called samples, and we assign them appropriate numbers: $0, 1, 2, \ldots, N, N+1, \ldots$. The multivariable discrete time systems take the form of *the discrete state equations* [1, 2]:

   (5.a) $$\mathbf{x}(NT + T) = f[\mathbf{x}(NT); \mathbf{v}(NT)],$$

   (6.a) $$\mathbf{y}(NT) = g[\mathbf{x}(NT); \mathbf{v}(NT)],$$

   or in the simplified form:

   (5.b) $$\mathbf{x}(N + 1) = f[\mathbf{x}(N); \mathbf{v}(N)],$$

   (6.b) $$\mathbf{y}(N) = g[\mathbf{x}(N); \mathbf{v}(N)],$$

   we assume functions f and g are unique, continuous, in general case nonlinear time functions, T-a interval, N-the number of discrete time points (ie, number of samples). Let the state variables of the studied process be evaluated on samples N and $N + 1$, then the state vectors contain n variables defined on these samples: $\mathbf{x}(N) = \{x_1(t_N), x_2(t_N), ..., x_n(t_N)\}$ and $\mathbf{x}(N+1) = \{x_1(t_{N+1}), x_2(t_{N+1}), ..., x_n(t_{N+1})\}$. Moreover, let the linear combinations of the states variables be the output variables $\mathbf{y}(N)$, then the state equation takes the form [4]:

   (7) $$\mathbf{x}(N + 1) = f[\mathbf{x}(N); \mathbf{v}(N)],$$

   (8) $$\mathbf{y}(N) = \mathbf{C}\mathbf{x}(N),$$
3. *The set of nonlinear discrete-time state equations with increases*. If the right side vector function f (7) is differentiable with respect to its arguments in a neighborhood of the point t_N, then we can obtain a linear approximation of the system in the neighborhood of the point t_N based on the nonlinear set of equations using the Taylor formula; this approximation will result in the linearized system of states equations with increases in the neighborhood of the point t_N [2]:

   (9) $$\delta\mathbf{x}(N + 1) = f'_x(N)\delta\mathbf{x}(N) + f'_v(N)\delta\mathbf{v}(N)$$

   (10) $$\delta\mathbf{y}(N) = \mathbf{C}\delta\mathbf{x}(N)$$

We can find a correspondence between the set of state equations (3)-(4) and the set of equations with increases (9)-(10). Indeed, the vector $\mathbf{x}(t)$ of the set (3) - (4) corresponds to the vector $\delta\mathbf{x}(N)$ of set (9)–(10), vector $\mathbf{v}(t)$ corresponds to vector $\delta\mathbf{v}(N)$, vector $d\mathbf{x}/dt$ corresponds to vector $\delta\mathbf{x}(N + 1)$, vector $\mathbf{y}(t)$ corresponds to vector $\delta\mathbf{y}(N)$, matrix $\mathbf{A}(t)$ corresponds to matrix $f'_x(N)$, and matrix $\mathbf{B}(t)$ corresponds to matrix $f'_v(N)$.

Connection between the state equations with increases and the partial regression models

Let's formulate the problem: to express the coefficients of state equations with increases (9)-(10) by the coefficients of partial regression models examined in [3]. For this purpose, we analyse, for example two partial regression models of the form (11)-(12), using denotation like in [3]:

$$X = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 y^2 + a_5 xy, \tag{11}$$

$$Y = b_0 + b_1 y + b_2 x + b_3 y^2 + b_4 x^2 + b_5 xy, \tag{12}$$

where X and Y – future values of x and y variables (at time point t_{N+1}), x and y - values of x and y variables at (previous) time point t_N.

The equations with increases take form:

$$\begin{aligned}\delta X = a_0 + a_1(x+\delta x) + a_2(y+\delta y) + a_3(x+\delta x)^2 + \\ +a_4(y+\delta y)^2 + a_5(x+\delta x)(y+\delta y) - \\ -[a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 y^2 + a_5 xy] = \\ = (a_1 + 2a_3 x + a_5 y)\delta x + (a_2 + 2a_4 y + a_5 x)\delta y + \\ +a_3(\delta x)^2 + a_4(\delta y)^2 + a_5 \delta x \delta y,\end{aligned} \tag{13}$$

$$\begin{aligned}\delta Y = (b_1 + 2b_3 y + b_5 x)\delta y + (b_2 + 2b_4 x + b_5 y)\delta x + \\ +b_3(\delta y)^2 + b_4(\delta x)^2 + b_5 \delta x \delta y,\end{aligned} \tag{14}$$

Then (after comparing the coefficients in equations with increases δx and δy from (13)–(14) with formulas of sensitivity functions obtained in [3]), we use the definition of differential sensitivity functions ($S^f_x = \frac{\partial f}{\partial x}$) and obtain:

$$\begin{aligned}\delta X = S^X_x \delta x + S^X_y \delta y + \frac{1}{2} S^X_{x^2} (\delta x)^2 + \\ +\frac{1}{2} S^X_{y^2} (\delta y)^2 + S^X_{xy} (\delta x)(\delta y)\end{aligned} \tag{15}$$

$$\begin{aligned}\delta Y = S^Y_y \delta y + S^Y_x \delta y + \frac{1}{2} S^Y_{y^2} (\delta y)^2 + \\ +\frac{1}{2} S^Y_{x^2} (\delta x)^2 + S^Y_{xy} (\delta x)(\delta y)\end{aligned} \tag{16}$$

or using matrix notation:

$$\begin{aligned}\begin{bmatrix} \delta X \\ \delta Y \end{bmatrix} = \begin{bmatrix} S^X_x & S^X_y \\ S^Y_x & S^Y_y \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \end{bmatrix} + \\ + \begin{bmatrix} \frac{1}{2} S^X_{x^2} & \frac{1}{2} S^X_{y^2} & S^X_{xy} \\ \frac{1}{2} S^Y_{x^2} & \frac{1}{2} S^Y_{y^2} & S^Y_{xy} \end{bmatrix} \begin{bmatrix} (\delta x)^2 \\ (\delta y)^2 \\ (\delta x)(\delta y) \end{bmatrix}\end{aligned} \tag{17}$$

Thus, it can be seen that there is a certain correspondence between equations (9) and (17), namely:

$$\begin{aligned}\delta\mathbf{x}(N+1) = \begin{bmatrix} \delta X \\ \delta Y \end{bmatrix}; \delta\mathbf{x}(N) = \begin{bmatrix} \delta x \\ \delta y \end{bmatrix}; \\ f'_x(N) = \begin{bmatrix} S^X_x & S^X_y \\ S^Y_x & S^Y_y \end{bmatrix};\end{aligned}$$

(18)

$$f'_v(N) = \begin{bmatrix} \frac{1}{2} S^X_{x^2} & \frac{1}{2} S^X_{y^2} & S^X_{xy} \\ \frac{1}{2} S^Y_{x^2} & \frac{1}{2} S^Y_{y^2} & S^Y_{xy} \end{bmatrix}; \delta\mathbf{x}(N) = \begin{bmatrix} (\delta x)^2 \\ (\delta y)^2 \\ (\delta x)(\delta y) \end{bmatrix}.$$

From equation (17) we can draw an important conclusion that knowing the values of the partial increases δx and δy at the previous sample (time point t_N), we can calculate the (partial) increases δX and δY of these variables at the next sample, so we have the prognosis in terms of the increases.

The results of the experiments on dataset with regularities

What exactly are δx and δy? Are they partial increases ddx and ddy of simple partial models(19)-(20) or expanded models (21)-(22) known from [3] at previous time sample N, then δX and δY - partial increases at next sample $(N+1)$? They may not be partial increases but relative or absolute increases of these variables. It does not matter what sense we put in that components of the model, since it is a purely formal model to determine the values of the increases.

$$X = x + S^X_x ddx + S^X_y ddy \tag{19}$$

$$Y = y + S^Y_y ddy + S^Y_x ddx \tag{20}$$

$$\begin{aligned}X = x + S^X_x ddx + S^X_y ddy + \frac{1}{2} S^X_{x^2} (ddx)^2 + \\ +\frac{1}{2} S^X_{y^2} (ddy)^2 + S^X_{xy} (ddx)(ddy)\end{aligned} \tag{21}$$

$$\begin{aligned}Y = y + S^Y_y ddy + S^Y_x ddx + \frac{1}{2} S^Y_{y^2} (ddy)^2 + \\ +\frac{1}{2} S^Y_{x^2} (ddx)^2 + S^Y_{xy} (ddx)(ddy)\end{aligned} \tag{22}$$

.

Let's consider a time series which could simulate a 5-variable complex process without noises. The assumption of regularity is obligate in view of necessity of using regular datasets in methods described in [3] in order to find future values of the studied process, Table. 1. Then we calculate relative and absolute increases of every variable of the process at first few samples and partial increases for every partial model [3] using (19)-(20) or (21)-(22). Assuming δX and δY be one of these increases we calculate the set of equations (13)-(14) in order to find values of δx and δy. Then we compare our results. And indicate the most accurate one. It turns out that values of δx and δy treated as the absolute increases at time point t_N are significantly close to values of δX and δY at next time point t_{N+1}. So we could use them in order to predict (calculate) future values of studied process variables. The procedure requires 6-7 first values of all variables of the studied process in order to find the coefficients $a_0, a_1, ..., a_5$. Then we use them for calculating values of the sensitivity functions which occur in (15)-(16).

Time points	x	y	z	q	r
P(0)	1.21	0.7935	0.3192	0.5992	0.9057
P(1)	1.331	0.9125	0.1544	0.355	0.5427
P(2)	1.4641	1.0494	0.0742	0.2232	0.337
P(3)	1.6105	1.2068	0.0354	0.1518	0.2205
P(4)	1.7716	1.3878	0.0167	0.1146	0.1557
P(5)	1.9487	1.596	0.0078	0.0993	0.1235
P(6)	2.1436	1.8354	0.0036	0.1034	0.1174
P(7)	2.3579	2.1107	0.0016	0.137	0.1449
P(8)	2.5937	2.4273	0.0007	0.2489	0.2533

Table 1. The few first values of the 5-variable complex process

Increases	$\triangle x$	$\triangle y$	$\triangle z$	$\triangle q$	$\triangle r$
$\triangle t_1$	0.121	0.119	-0.165	-0.244	-0.363
$\triangle t_2$	0.331	0.137	-0.080	-0.132	-0.205
$\triangle t_3$	0.4641	0.157	-0.039	-0.071	-0.117
$\triangle t_4$	0.6105	0.181	-0.019	-0.037	-0.065
$\triangle t_5$	0.7716	0.208	-0.009	-0.015	-0.032
$\triangle t_6$	0.9487	0.239	-0.004	0.004	-0.006
$\triangle t_7$	0.1436	0.275	-0.002	0.034	0.027
$\triangle t_8$	0.3579	0.317	-0.0009	0.112	0.108
$\triangle t_9$	0.5937	0.364	-0.00041	0.433	0.431

Table 2. The absolute increases of the complex process values of variables

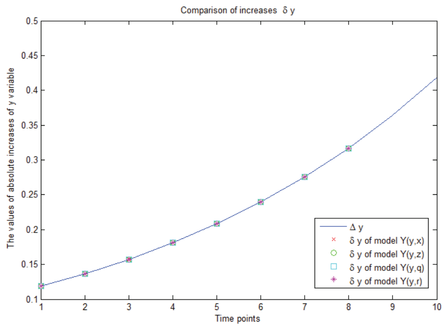

Fig. 1. The plot of the values of increases of y variable.

The comparison of the values δx and δy at t_N and δX and δY at t_{N+1} for chosen variables is shown in Figures 1-3. Using partial models [3] we have four possibilities of finding the values of every variable. We can use different combination of two variables for every variable. For example, we use models $X(x, y)$, $X(x, z)$, $X(x, q)$ and $X(x, r)$ to calculate the future values of variable x, where X is partial two-variable function of form (19)-(20) or (21)-(22).

We also calculate the case of the every partial increases ddx and ddy from 19)-(20) or (21)-(22) and relative increases and try to find a dependence. The results are not so promising (Figure 4). Values of δx and δy at t_N are not the values of ddx and ddy at t_{N+1}. Maybe there are different connections between them.

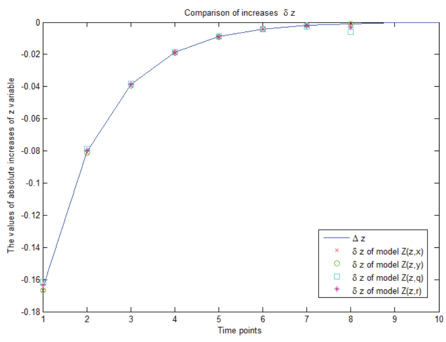

Fig. 2. The plot of the values of increases of z variable.

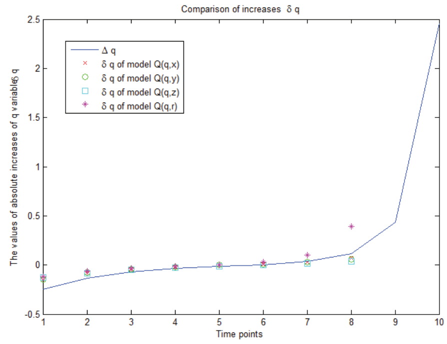

Fig. 3. The plot of the values of increases of q variable.

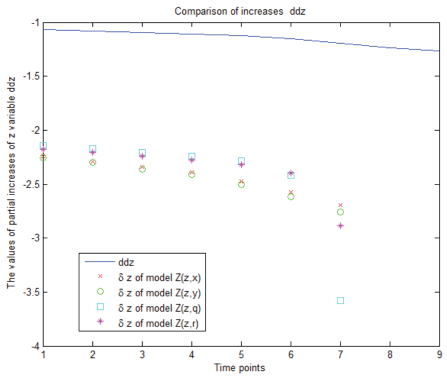

Fig. 4. The plot of the values of partial increases of z variable.

Conclusion

This paper shows the possible way of analysing a complex process in order to find its future values (at next time points). It is required to have only a few measurements (at 6-7 time points) [3]. Finding dependence between the variables of the model we can provide a prediction procedure for the given object using the notation of the state equation with increases. Using a identification method based on the observation of increases gives the chance to adapt a less complicated mathematical models.

These results suggest that the model estimation may be adequate in some situations, but, nothing is known in general. It is time now to take appropriate steps towards integration of that model with those of [3].

However, when it comes to process control we should establish causal relationships between group of variables which are considered as controlled variables, and another group considered as controlling variables. The formula (17) is useless in establishing these relations, because in this formula we cannot divide variables into the indicated group.

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