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Otimization of μ-law companding Quantizer for Laplacian source using Muller's method

***Abstract**. The motivation of this paper is based on the fact that a straightforward solution to optimization of the widely used μ-law companding quantizer has not been proposed so far. We deal with this problem for the case of a Laplacian source and we apply Muller's method for the optimization of the quantizer in question. Particularly, we use minimal distortion criteria and we apply Muller's method in order to determine the key parameter of the μ-law companding quantizer. The optimization method we propose is very general and it is easily modified for non-Laplacian sources. It can be applied for speech compession, because speech signals are well modeled by Laplacian sources.*

***Streszczenie.** W artykule opisano sposób optymalizacji kwantyzatora kompansji u-law. Do tego celu zastosowano metodę Muller'a oraz kryterium minimalnych zakłóceń. Proponowana optymalizacja jest ogólna i może być zmodyfikowana dla źródeł nielaplasjanowych. (**Wykorzystanie metody Muller'a do optymalizacji kwantyzatora kompansji u-law dla źródła laplasjanowego**).*

Keywords: μ-law companding quantizer, Muller's method, optimization, speech compession
Słowa kluczowe: kwantyzator kompansji u-law, metoda Muller'a, optymalizacja, kompresja mowy.

Introduction

Although the two modified logarithmic compressor characteristics obtained by the piecewise linear approximation to the A-law and the μ-law characteristics have become widely used as a design guideline for nonuniform quantization of speech signals in digital telephony [1], the fundamental question of how to provide a simple manner to optimize parameters of these two quantizers has long been open for signals with Laplacian and Gaussian probability density function (PDF). More specifically, it has remained undefined how to provide the straightforward approach to solving the complex system of nonlinear equations, i.e., how to determine the parameters of the quantizers in question that minimize the mean-squared error (MSE) distortion.

In this paper, as in [2, 3, 4], we assume Laplacian PDF of the input signal and we focus on the robust μ-law companding quantization which gives an almost constant signal to quantization noise ratio (SQNR) in a wide range of variances. μ-law companding quantizer is preferable for use when the input signal's variance changes with time in a wide range, as it is the case with speech signals [4]. As reported in [5], one of the reasons of often considering the Laplacian source is that the first approximation to the long-time-averaged PDF of speech amplitudes is provided by the Laplacian PDF.

The optimization problem observed in [3, 4] has been solved by numerical optimization of the compression factor μ and the support region of the μ-law companding quantizer under the constraint that compression factor μ has an integer value. In this paper, we go one step further in the optimization. Namely, in order to reduce the search time of an optimal solution to the system of two nonlinear equations, we do not set the constraint on the integer values of the compression factor μ, but instead we apply Muller's method that provide simple and fast determining the optimal compression factor from the range of real values.

The rest of this paper is organized as follows. Section 2 provides a detailed description of the proposed simple solution to the problem of optimizing the μ-law companding quantizer designed for the Laplacian source of unit variance. The achieved numerical results are the topics addressed in Section 3. Finally, Section 4 is devoted to the conclusions which summarize the contribution achieved in the paper.

Optimization of μ-law companding quantizer for Laplacian PDF

An N-level scalar quantizer Q is defined by mapping $Q: R \rightarrow Y$ [6, 7], where R is a set of real numbers, and $Y \equiv (y_1, y_2, y_3, \ldots, y_N) \subset R$ is a set of representation levels that makes the code book of size $|Y| = N$. Every N-level scalar quantizer partitions the set of real numbers into N cells $R_i = (t_{i-1}, t_i]$, $i = 1, \ldots, N$, where t_i, $i = 0, 1, \ldots, N$ are decision thresholds and where it holds that $Q(x) = y_i$, $x \in R_i$.

Companding technique, we consider in this paper, defines the following steps: compress the input signal x by applying the compressor function $c(x)$; apply the uniform quantizer on the compressed signal $Q_u(c(x))$; expand the quantized version of the compressed signal using an inverse compressor function $c^{-1}(Q_u(c(x)))$. For a μ-law companding quantizer, denoted by Q_μ, compression is done using the μ-law compressor function $c_\mu(x)$: $[-x_{max}, x_{max}] \rightarrow [-x_{max}, x_{max}]$ [6, 7]:

$$c_\mu(x) = \frac{x_{max}}{\ln(1+\mu)} \ln\left(1 + \mu \frac{|x|}{x_{max}}\right) \operatorname{sgn}(x), \quad |x| \le x_{max}, \tag{1}$$

where the parameter μ is the compression factor and x_{max} is the μ-law companding quantizer's support region threshold. For the assumed Laplacian PDF $p(x)$ [6]:

$$p(x) = \frac{1}{\sqrt{2}\sigma} \exp\left(-\frac{|x|\sqrt{2}}{\sigma}\right), \tag{2}$$

the expression for the total distortion of the μ-law companding quantizer is given by [8]:

$$D(Q_\mu) = \frac{\ln^2(\mu+1)\sigma^2}{3N^2}\left[\frac{1}{\mu^2}\frac{x_{max}^2}{\sigma^2} + \frac{x_{max}}{\sigma}\frac{\sqrt{2}}{\mu} + 1\right] + \sigma^2 \exp\left(-\frac{\sqrt{2}x_{max}}{\sigma}\right). \tag{3}$$

Let us aasume that the μ-law companding quantizer is designed for the unit variance, $\sigma^2 = 1$. Then, the expression for the total distortion becomes:

$$D(Q_\mu) = \frac{\ln^2(\mu+1)}{3N^2}\left[\frac{x_{max}^2}{\mu^2} + \frac{\sqrt{2}x_{max}}{\mu} + 1\right] + \exp\left(-\sqrt{2}x_{max}\right). \tag{4}$$

By setting the first derivate of the distortion given by (4) to zero with respect to μ, we obtain:

(5) $$\frac{\partial D(Q_\mu)}{\partial \mu} = f(\mu) = 2\mu^3 + \sqrt{2}x_{\max}\mu^2(2 - \ln(\mu+1)) +$$

$$+ \mu\sqrt{2}x_{\max}\left[\sqrt{2}x_{\max}(1 - \ln(\mu+1)) - \ln(\mu+1)\right] - 2x_{\max}^2 \ln(\mu+1) = 0$$

.

In order to provide the solution of this equation, i.e. to determine the optimal compression factor of the considered μ-law companding quantizer, we use the following closed-form formula from [2], which defines the optimal support region threshold of the μ-law companding quantizer:

(6) $$x_{\max} = \frac{1}{\sqrt{2}} \ln\left(\frac{3\mu N^2}{\ln^2(\mu+1)}\right),$$

and we apply Muller's method.

Most of the root-finding methods approximate the function in the neighborhood of the root by a straight line [9, 10]. Muller's method is based on approximating the function in the neighborhood of the root by a quadratic polynomial. In fact, Muller's method generalizes the secant method of root finding by using quadratic 3-point interpolation [9, 10]. Particularly, Muller's method uses three points, constructs the parabola through these three points, and takes the intersection of the *x*-axis with the parabola to be the next approximation. The rate of convergence of Muller's method is faster than the secant method but slower than Newton's method [9, 10]. However, in contrast to Newton's method, Muller's method requires only function values and the derivative need not be calculated. In addition, it is well known that Muller's method is more efficient than Newton's method when the range of values for root finding is wide, as it is the case we consider here [9, 10]. The optimization method we consider in this paper is very general and it is easily modified for non-Laplacian sources.

Muller's method is an iterative method that requires three starting points $(x_1, f(x_1))$, $(x_2, f(x_2))$, and $(x_3, f(x_3))$. The parabola passing through these three points can be written as [9, 10]:

(7) $$f = f_3 + c_2(x - x_3) + d_1(x - x_3)(x - x_2),$$

where the coefficients are determined by:

(8) $$c_1 = \frac{f_2 - f_1}{x_2 - x_1},\ c_2 = \frac{f_3 - f_2}{x_3 - x_2},\ d_1 = \frac{c_2 - c_1}{x_3 - x_1},$$

and where it holds $f_1 = f(x_1)$, $f_2 = f(x_2)$, and $f_3 = f(x_3)$. The intersection of the *x*-axis with the parabola gives:

(9) $$x_4 = x_3 - \frac{2f_3}{s + \mathrm{sgn}(s)\sqrt{s^2 - 4f_3 d_1}},$$

(10) $$s = c_2 + d_1(x_3 - x_2),$$

which is better approximation of the root than any of x_1, x_2, or x_3.

Numerical Results

In this section, Muller's method of root-finding is described. By using Muller's method the optimal compression factor of the μ-law companding quantizer designed for the Laplacian PDF of unit variance has been determined. In addition, in this section, the performances that we have ascertained by applying the proposed quantizer in quantization of signals having Laplacian PDF and a wide variance range are discussed and compared with the one obtained in [2, 3].

By using Muller's method we have determined the optimal compression factor of the μ-law companding quantizer with $N = 256$ quantization levels where, specifically, we have applied the following algorithm:

1. Make an initial guess of μ by setting $(x_1, x_2, x_3) = (1, 255, 128)$, where (x_1, x_2) are border values. Then calculate the root value x_4 according to (9);
2. Update boundary points (x_1, x_2) by using criteria that boundary value further to the root x_4 takes x_3 value;
3. Set the root value x_4 as a new value for x_3 and rearrange (x_1, x_2, x_3) for the next iterative step.

Starting with real numbers, iterates will remain real [9, 10]. By making global initializations of the algorithm with boundary points $(x_1, x_2) = (1, 255)$, we have assured that the algorithm converges to the solution, which is thus certainly to be the real value. As already mentioned, Muller's method uses three initial evaluations of a function f_1, f_2 and f_3, but does not require the derivative of the function, which is a convenience of Muller's method and one of the reasons we have decided to utilize this method.

The relative error of estimating the root value:

(11) $$\delta[\%] = \left|\frac{x_4 - x_3}{x_4}\right| \cdot 100,$$

is usually appointed criteria for stopping the iterative algorithm [9, 10]. Accepting the accuracy in the first decimal place, Muller's method, in our case, stops at the nineth step with the optimized value for the compression factor of $\mu = 16.9227$, and according to Table 1 with the relative error of estimating the root value of $\delta = 0.1$ %. Further, the optimal support region threshold determined from (6) for the corresponding compression factor $\mu = 16.9227$ is $x_{\max} = 9.12$. Similar results has been obtained in [3] in the case of μ-law companding quantizer designed for the same Laplacian PDF of unit variance and for the same number of quantization levels ($N = 256$), where the numerical optimization procedure of the compression factor and the support region threshold have been performed simultaneously, with the restriction that μ has an integer value. As a result of such optimization the optimum value for compression factor of $\mu = 17$ has been determined along with the optimal value of the support region threshold of $x_{\max} = 9.9$.

Table 1. Parametres for analysis of the accuracy of the Muller's method

Step	x_1	x_2	x_3	x_4	δ [%]
1	1	255	128	89.5645	42.9137
2	1	128	89.5645	57.1664	56.6733
3	1	89.5645	57.1664	39.2620	45.6023
4	1	57.1664	39.2620	27.9580	40.4320
5	1	39.2620	27.9580	21.5898	29.4963
6	1	27.9580	21.5898	18.3749	17.4961
7	1	21.5898	18.3749	17.1830	6.9365
8	1	18.3749	17.1830	16.9400	1.4344
9	1	17.1830	16.9400	16.9227	0.1022

Quality of a quantized signal is along with distortion commonly specified by signal to quantization noise ratio [6]:

(12) $$\mathrm{SQNR}(Q_\mu)=10\log\left(\frac{\sigma^2}{D(Q_\mu)}\right)[dB].$$

Since one of the main goals when designing quantizers is to provide as high as possible quality of the quantized signal, i.e. to achieve as high as possible SQNR, Fig. 1 verifies that the considered Muller's method is capable of significantly increasing the maximum of SQNR of a μ-law companding quantizer. We can observe that, for the unit variance, the μ-law companding quantizer, designed in accordance with Muller's method, gives the highest SQNR, precisely 2.72 dB higher than the μ-law companding quantizer designed in [2] for the case when the values of compression factor and the support region threshold amount to μ = 255 and x_{max} = 10.1147. As shown in Fig. 1, it is obvious that when the μ-law companding quantizer is designed for Laplacian PDF of unit variance, with the support region threshold x_{max} = 9.12 and with the compression factor μ = 16.9227, the SQNR characteristic does not exceed characteristic of the G.712 Recommendation [11] in the whole variance range. Namely, employing forward adaptive technique as in [4], the quality of the quantized signal could be substantially balanced. However, the emphasis in this paper is on the fixed μ-law companding quantizer's optimization, which can certainly be easily adapted using well-known adaptation techniques.

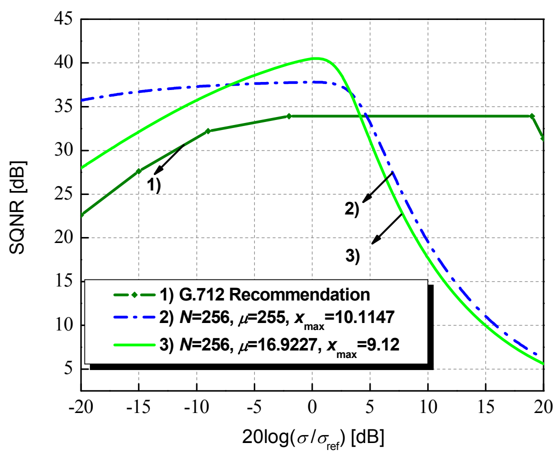

Fig. 1. SQNR characteristics in the wide variance range for N = 256, μ = 255, x_{max} = 10.1147 and μ = 16.9227, x_{max} = 9.12

Conclusion

In this paper, we have proposed an algorithm for the optimization of μ-law companding quantizer designed for the Laplacian source of unit variance. Particularly, we have proposed an algorithm that provides the optimization of compression factor μ and accordingly, the optimization of the corresponding support region threshold. We have utilized Muller's method of root-finding and we have determined the optimal compression factor μ from the range of real values. Globaly initializing the algorithm with boundary points of 1 and 255, we have assured that the algorithm converges to the solution, which is thus certainly to be the real value. We have calculated optimal value for compression factor μ in the nineth iteration step with relative error of 0.1 %. We have shown that iteratively calculated results compare nicely with the ones obtained in [3]. In such a manner we have confirmed the correctness of the proposed algorithm and of the resulting optimized value for the compression factor μ. Eventually, we have pointed out that Muller's method is fast and no so demanding root-finding method, which generally can be easily modified for non-Laplacian sources.

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