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A Comparison of Bistatic Bearings-Only Tracking Methods

Abstract*. Target tracking using bistatic bearings-only measurements has obtained distinct interest recently. It is a nonlinear problem that traditional Kalman filter (KF) can not be applied directly. In this paper, the triangular ranging formula has been derived first for bistatic bearings-only tracking. The ranging error is then proved to be Gaussian noises, which enable the traditional KF applicable. The recently developed unscented Kalman filter (UKF) is also applied to the nonlinear measuring equation directly. To further improve the tracking accuracy especially in case of maneuvering target tracking, interactive multiple model (IMM) is adopted. Simulation results for both constant velocity moving target and maneuvering target are included to compare the performance of the aforementioned methods. The triangular ranging method, triangular-ranging-based Kalman filtering (TRKF), UKF, TR-IMMKF, and IMM-UKF are compared extensively using the criterion of root of the mean squared error (RMSE) and computational burden, as well as the robustness.*

Streszczenie. *W artykule zaproponowano formułę o zasięgu trójkątnym w zastosowaniu do bistatycznego wyznaczania namiaru (pelengu). W rozwiązaniu wykorzystano m. In. filtr Kalman'a do pomiaru wielkości nieliniowych. Wyniki badań symulacyjnych, dla obiektów w ruchu jednostajnym lub zmiennym, pozwalają na porównanie działania metod. Porównano także metody TRKF, UKF, TR-IMMKF, IMM-UKF, pod względem błędów średniokwadratowych, odporności, złożoności obliczeń. (**Porównanie bistatycznych metod wyznaczania namiaru kątowego**).*

Keywords: bearings-only, target tracking, interactive multiple model, unscented Kalman filter.
Słowa kluczowe: śledzenie celu, namiar, wieloskładnikowy model, filtr Kalmana.

Introduction

In many applications, such as submarine tracking and aircraft surveillance, bearings-only sensors are commonly used to collect observations about target trajectory [1-2]. The aim of bearings-only tracking is to determine the target trajectory data using noise-corrupted bearings-only measurements from one or more platforms. Using bearings-only sensors, the platforms can be concealed very well, and the enemy target can be disturbed and attacked effectively [3]. Therefore, target tracking using bearings-only sensors have obtained distinct interest in last decades. Single bearings-only tracking is prone to be unobservable at one hand, optimizing the maneuvering trajectories of the platform is an open problem at the other hand; More importantly, the robustness from more than one platforms based tracking system can be improved. If one platform does not work because of the reasons such as sudden attack, the tracking mission can be still carried on. Hence two or more bearings-only platforms based cooperative target tracking has received more and more attentions these years [4-5].

Unfortunately, this particular estimation problem is not amenable to simple solution because of the intrinsic nonlinearities in the observing equation. This also makes the traditional Kalman filter not applicable. For years, the problem has served as a typical example to which many methods for nonlinear filtering have been applied. There are usually two kinds of solutions to handle the problem. The first one consists of using the extended Kalman filter (EKF) in a Cartesian coordinate system to solve the problem. This solution does not account for the spread of the random variables and uses only the first-order Taylor expansion of the nonlinear functions. Therefore, it often leads to the divergence of the filter. The second one, proposed by Lindgren and Gong in [3], consists of deriving a pseudo-linear measurement equation. Then a Kalman filter can be used to solve the problem. The stochastic stability analysis of the estimates had been addressed by Song and Speyer in [6]. However, it always contains bias by pseudo-linearization and debias techniques bring more computational burden.

To avoid the flaws of the EKF, several recent works [see e.g. 7-8] (referred as unscented Kalman filters, UKF) have used deterministic sampling techniques and the unscented transform to propagate the mean and variance. In addition, the particle filter (PF) has been applied to this problem [9]. It approximates the posterior density by a large number of random (Monte Carlo) samples. However, even the PF did not show any obvious advantage over other traditional methods [10].

Other notable references in this field include [11-12]. In [11] use of multiple sensors in bearings only tracking is studied, where sensors are relatively close to each other (multiple sonar arrays towed by the same ship), and receive the same signal from the target. In [12] dynamic programming is used to track maneuvering targets using bearings only measurements, where the track state space is discretized in two models, nonmaneuvering and maneuvering.

In this paper, the three dimensional (3-D) triangular ranging formula is derived first for bistatic bearings-only tracking; then the ranging error is proved to be Gaussian noises with zero-mean, which enables both traditional Kalman filter and unscented Kalman filter applicable. To further improve the tracking accuracy, interactive multiple model (IMM) is adopted. Simulation results for both constant velocity moving target and maneuvering target are included to compare the performance of the aforementioned methods. The triangular ranging method, triangular-ranging-based Kalman filtering (TRKF), UKF, TR-IMMKF, and IMM-UKF are compared extensively using the criterion of root of the mean squared error (RMSE) and computational burden, as well as the robustness.

Problem formulation

In this paper, we consider the scenario of 3-D tracking by two airborne platforms, as shown in Fig. 1. The i-th (i=1,2) bearing-only sensor observes the target in terms of

$$\begin{bmatrix} \varphi_i(k) \\ \theta_i(k) \end{bmatrix} = \begin{bmatrix} \arctan\dfrac{y_p(k)-y_i(k)}{x_p(k)-x_i(k)} \\ \arctan\dfrac{z_p(k)-z_i(k)}{\sqrt{\left(x_p(k)-x_i(k)\right)^2+\left(y_p(k)-y_i(k)\right)^2}} \end{bmatrix} + \begin{bmatrix} d_{\varphi_i}(k) \\ d_{\theta_i}(k) \end{bmatrix} \tag{1}$$

where $T(x_p(k), y_p(k), z_p(k))$ and $P_i(x_i(k), y_i(k), z_i(k))$, i=1,2 are the target and the i-th platform location in the Cartesian coordination at the k-th time-slot, respectively. $\varphi_i(k)$ and $\theta_i(k)$ are the noise corrupted azimuth and elevation measurement for i-th sensor respectively. $d_{\varphi_i}(k)$ and $d_{\theta_i}(k)$ the noise. For easy notation, the sampling time k is omitted thereafter without illegibility. Furthermore, the measuring noises are supposed to be independent Gaussian noises with zero mean and

$$E\left\{\begin{bmatrix} d_{\varphi i} \\ d_{\theta i} \end{bmatrix}\begin{bmatrix} d_{\varphi i}, d_{\theta i} \end{bmatrix}\right\} = \begin{bmatrix} \sigma^2_{\varphi i} & 0 \\ 0 & \sigma^2_{\theta i} \end{bmatrix} \tag{2}$$

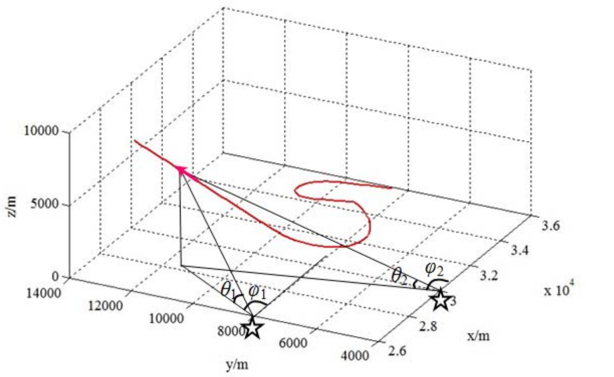

Fig.1. Bistatic bearings-only tracking in 3-D. The platforms are denoted as a pentagon respectively, a single target denoted as the red real line moves along a constant velocity or a maneuvering model

The purpose of bearings-only tracking is to estimate state of the target including position, velocity and acceleration etc. As can be seen from Fig. 1, the target location can be determined by triangular ranging given the bearings-only measuring. However, the bearings-only measures unavoidably contain noises that affect the tracking accuracy. Hence, a novel triangular ranging based Kalman filtering algorithm will be proposed in section 3.2. Considering the nonlinearity in measuring equation (1), recently developed nonlinear filter techniques such unscented Kalman filter is also included in section 3.3. To attack the target maneuvering, interactive multiple model using either triangular-ranging-based Kalman filters or unscented Kalman filters is adopted in section 3.4. All these methods are compared in section 4 including the tracking accuracy and the computational burden, as well as the robustness.

Target tracking methods

Triangular ranging error analysis

In the absence of measuring noise, we can obtain the following passive raging formula through triangulation

$$\begin{cases} x_p = \dfrac{(y_1 - y_2) - (x_1 \tan\varphi_1 - x_2 \tan\varphi_2)}{\tan\varphi_2 - \tan\varphi_1} \\ y_p = y_1 + \dfrac{(y_1 - y_2) + (x_2 - x_1)\tan\varphi_2}{\tan\varphi_2 - \tan\varphi_1}\tan\varphi_1 \\ z_p = z_1 + \sqrt{(x_p - x_1)^2 + (y_p - y_1)^2}\,\tan\theta_1 \end{cases} \tag{3}$$

However, it is unavoidable to have noises in practical application. Then the ranging formulation in (3) presents location error. In order to derive the error, we give a differential to each element in (1) and obtain the following location error equation

$$\begin{bmatrix} d\varphi_1 \\ d\varphi_2 \\ d\theta_1 \end{bmatrix} = \begin{bmatrix} dK_1 \\ dK_2 \\ dK_3 \end{bmatrix} + \begin{bmatrix} -\dfrac{\sin\varphi_1}{r_1} & \dfrac{\cos\varphi_1}{r_1} & 0 \\ -\dfrac{\sin\varphi_2}{r_2} & \dfrac{\cos\varphi_2}{r_2} & 0 \\ -\dfrac{r\cos\varphi_1\cos\theta_1}{r_1} & -\dfrac{r\sin\varphi_1\cos\theta_1}{r_1} & \dfrac{1}{r_1} \end{bmatrix}\begin{bmatrix} dx_p \\ dy_p \\ dz_p \end{bmatrix} \tag{4}$$

where

$$r_1 = \sqrt{(x_p - x_1)^2 + (y_p - y_1)^2}\ ,$$

$$r_2 = \sqrt{(x_p - x_2)^2 + (y_p - y_2)^2}\ ,$$

$$r = \sqrt{(x_p - x_1)^2 + (y_p - y_1)^2 + (z_p - z_1)^2}\ ,$$

$$dK_1 = \frac{\sin\varphi_1}{r_1}dx_1 - \frac{\cos\varphi_1}{r_1}dy_1\ ,$$

$$dK_2 = \frac{\sin\varphi_2}{r_2}dx_2 - \frac{\cos\varphi_2}{r_2}dy_2\ ,$$

$$dK_3 = \frac{r\cos\varphi_1\cos\theta_1}{r_1}dx_1 + \frac{r\sin\varphi_1\cos\theta_1}{r_1}dy_1 - \frac{1}{r_1}dz_1\ .$$

In a more compact form, it can be rewritten as

$$dV = CdX + dK \tag{5}$$

where

$$dV = \begin{bmatrix} d\varphi_1 \\ d\varphi_2 \\ d\theta_1 \end{bmatrix},\ dX = \begin{bmatrix} dx_p \\ dy_p \\ dz_p \end{bmatrix},\ dK = \begin{bmatrix} dK_1 \\ dK_2 \\ dK_3 \end{bmatrix},$$

$$C = \begin{bmatrix} -\dfrac{\sin\varphi_1}{r_1} & \dfrac{\cos\varphi_1}{r_1} & 0 \\ -\dfrac{\sin\varphi_2}{r_2} & \dfrac{\cos\varphi_2}{r_2} & 0 \\ -\dfrac{r\cos\varphi_1\cos\theta_1}{r_1} & -\dfrac{r\sin\varphi_1\cos\theta_1}{r_1} & \dfrac{1}{r_1} \end{bmatrix}$$

It is obvious that matrix C is reversible. Hence the location error can be derived as

$$dX = C^{-1}(dV - dK) \tag{6}$$

It is worth mentioning that the location error for (3) in the Cartesian coordination (dx_p, dy_p, dz_p) is a linear mapping of ($d\varphi_1$, $d\varphi_2$, $d\theta_1$) and (dx_i, dy_i, dz_i) (i=1,2), the positioning error of both platforms. Recalling that the measuring noise is Gaussian with zero mean and the platform location contains no error (i.e. dK=0), we can conclude that the location error (dx_p, dy_p, dz_p) is still a zero mean Gaussian random variable. Moreover, the covariance matrix can be easily derived from (6) and formulated as follows

$$R = E\left[dXdX^T\right] = C^{-1}E\left[dVdV^T\right]C^{-T}$$

$$(7) \qquad = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_z^2 \end{bmatrix}$$

where

$$\sigma_x^2 = \frac{1}{\sin^2(\varphi_1-\varphi_2)}\left(\cos^2\varphi_2 r_1^2\sigma_{\varphi_1}^2 + \cos^2\varphi_1 r_2^2\sigma_{\varphi_2}^2\right)$$

$$\sigma_{xy} = \frac{1}{\sin^2(\varphi_1-\varphi_2)}\left(\frac{\sin^2\varphi_2 r_1^2}{2}\sigma_{\varphi_1}^2 + \frac{\sin^2\varphi_1 r_2^2}{2}\sigma_{\varphi_2}^2\right)$$

$$\sigma_y^2 = \frac{1}{\sin^2(\varphi_1-\varphi_2)}\left(\sin^2\varphi_2 r_1^2\sigma_{\varphi_1}^2 + \sin^2\varphi_1 r_2^2\sigma_{\varphi_2}^2\right)$$

$$\sigma_{xz} = \frac{\tan\theta_1\cos\varphi_2\cos(\varphi_2-\varphi_1)r_1^2\sigma_{\varphi_1}^2 + \tan\theta_1\cos\varphi_1 r_2^2\sigma_{\varphi_2}^2}{\sin^2(\varphi_1-\varphi_2)}$$

$$\sigma_{yz} = \frac{\tan\theta_1\sin\varphi_2\cos(\varphi_2-\varphi_1)r_1^2\sigma_{\varphi_1}^2 + \tan\theta_1\sin\varphi_1 r_2^2\sigma_{\varphi_2}^2}{\sin^2(\varphi_1-\varphi_2)}$$

$$\sigma_z^2 = \frac{\sin^2\theta_1\cos^2(\varphi_2-\varphi_1)r_1^2\sigma_{\varphi_1}^2 + \sin^2\theta_1 r_2^2\sigma_{\varphi_2}^2 + \sin^2(\varphi_1-\varphi_2)r^2\sigma_{\theta_1}^2}{\sin^2(\varphi_1-\varphi_2)\cos^2\theta_1}$$

Triangular-ranging-based Kalman filtering (TRKF)

Based on the passive ranging formula in (3), the bearings-only measurements are converted to target position in Cartesian coordination ($x_p(k)$, $y_p(k)$, $z_p(k)$), where the location covariance matrix is given by (7). Then, based on traditional linear filtering techniques such as Kalman filter, the target state can be obtained recursively [13]. It has been proven that the location error (dx, dy, dz) is a zero mean Gaussian random variable. Hence the conversion is unbiased and the new measuring equation can be formulated as

$$(8) \qquad z(k) = Hx(k) + v(k)$$

where $x(k) = \left[x_p, \dot{x}_p, \ddot{x}_p, y_p, \dot{y}_p, \ddot{y}_p, z_p, \dot{z}_p, \ddot{z}_p\right]^T$ is the state vector of the target. H is the corresponding measurement matrix and

$$(9) \qquad H = \begin{bmatrix} 1,0,0,0,0,0,0,0,0 \\ 0,0,0,1,0,0,0,0,0 \\ 0,0,0,0,0,0,1,0,0 \end{bmatrix}$$

If the constant acceleration (CA) kinetic model is used, the state evolves as

$$(10) \qquad x(k+1) = F(k)x(k) + G(k)w(k)$$

where

$$F = diag\left\{\begin{bmatrix} 1 & T & \frac{1}{2}T^2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & T & \frac{1}{2}T^2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & T & \frac{1}{2}T^2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}\right\}$$

$$G = diag\left\{\begin{bmatrix} \frac{1}{2}T^2 \\ T \\ 1 \end{bmatrix}, \begin{bmatrix} \frac{1}{2}T^2 \\ T \\ 1 \end{bmatrix}, \begin{bmatrix} \frac{1}{2}T^2 \\ T \\ 1 \end{bmatrix}\right\}$$

whereas T stands for the sampling period. Based on the kinetic model (10) and measuring equation (8), the traditional Kalman filter can be used to get the target state estimate.

Unscented Kalman filtering (UKF)

Using the dynamic equation (10) and the measuring equation (1), the state of the target can be directly estimated through some traditional nonlinear filtering scheme. Noticing the dynamic equation (10) is linear, the main difficult lies in the nonlinearity in (1). The celebrated extended Kalman filtering (EKF) linearizes (1) around a single point ($\hat{x}(k \mid k-1)$ for the observation update). This solution does not account for the spread of the random variables and uses only the first-order Taylor expansion of the nonlinear functions. Therefore, it often leads to the divergence of the filter. To avoid the flaws of the EKF, unscented Kalman filters using deterministic sampling techniques and the unscented transform to propagate the mean and variance is proposed.

Let $f(\cdot)$ be some nonlinear function and x be some random variable with mean $\hat{x}$ and variance P_{xx}. We wish to estimate the mean and variance of u=$g(x)$. The sigma-point transform uses a set of weighted points $\{\chi_j, \omega_j\}_{j=0,1,\dots,2n}$ called sigma-points to represent the p.d.f. of x. It is chosen such that the mean and covariance ar e consistent with the prior information $\bar{x} = \hat{x}$ and $\bar{P}_{xx} = P_{xx}$, with $\bar{x} = \sum_{j=0}^{2n}\omega_j\chi_j$, $\bar{P}_{xx} = \sum_{j=0}^{2n}\omega_j(\chi_j-\bar{x})(\chi_j-\bar{x})^T$, and $\sum_{j=0}^{2n}\omega_j = 1$. The sigma-points are then propagated through the nonlinear function $\mu_j = f(\chi_j)$ to approximate the mean and covariance of u:

$$(11) \qquad E(u) \approx \bar{u} = \sum_{j=0}^{2n}\omega_j\mu_j$$

$$(12) \qquad \mathrm{Var}(u) \approx \bar{P}_{uu} = \sum_{j=0}^{2n}\omega_j\left(\mu_j-\bar{u}\right)\left(\mu_j-\bar{u}\right)^T$$

$$(13) \qquad \mathrm{Cov}(x,u) \approx \bar{P}_{xu} = \sum_{j=0}^{2n}\omega_j\left(\chi_j-\bar{x}\right)\left(\mu_j-\bar{u}\right)^T$$

Within the UKF framework, the different available algorithms differ in the way they specify the initial set of weighted sigma-points so as to capture the most important information about the random variable of interest. We will only summarize the UKF for illustration purpose. In this algorithm, the n-dimensional random variable x is approximated by 2n+1 sigma-points given by

$$\chi_0 = \hat{x}\,,\ \omega_0 = \frac{\kappa}{n+\kappa}$$

$$(14) \qquad \chi_j = \hat{x} + \left(\sqrt{(n+\kappa)P_{xx}}\right)_i,\ \omega_j = \frac{1}{2(n+\kappa)},\ j=1,2,\dots,n$$

$$\chi_{j+n} = \hat{x} - \left(\sqrt{(n+\kappa)P_{xx}}\right)_i,\ \omega_{j+n} = \frac{1}{2(n+\kappa)},\ j=1,2,\dots,n$$

where κ is a scaling parameter usually chosen as 0 or 3-n and $\left(\sqrt{P}\right)_i$ denotes the i-th row of the Cholesky decomposition of P. It is shown that this procedure produces accurate results for the predicted mean and covariance up to the third order of the Taylor series for Gaussian noises and at least up to the second order for other types of noises.

IMM approach to naneuvering target tracking

The aforementioned TRKF and UKF are both single-model-based. However, a single-model-based tracking approach is not adequate to handle complex maneuvering scenarios. One way to treat this problem is the interacting multiple model (IMM) filter [14]. For the IMM approach, the single-model-based filters interact each other in a highly cost-effective fashion and thus lead to significantly improved performance. It also consists of a bank of single-model-based filters running in parallel at each cycle. The initial estimate at the beginning of each cycle for each filter is a mixture of all most recent estimates from the single-model-based filters. It is this mixing that enables the IMM to effectively take into account the history of the modes (and, therefore, to yield a more fast and accurate estimate for the changed system states) without the exponentially growing requirements in computation and storage as required by the optimal estimator.

The following procedures should be performed in the application of the IMM estimation technique for target tracking: (i) filter reinitialization; (ii) model-conditional filtering; (iii) model probability updating; (iv) estimate fusion.

Step 1 Interaction and mixing of the estimates: filter reinitialization (interacting the estimates) obtained by mixing the estimates of all the filters from the previous time (this is accomplished under the assumption that a particular mode is in effect at the present time).

1) Compute the predicted model probability from instant k to k+1:

$$\mu_j(k+1|k)=\sum_{i=1}^{N}\pi_{ij}\mu_i(k) \tag{13}$$

2) Compute the mixing probability:

$$\mu_{i|j}(k)=\pi_{ij}\mu_i(k)\big/\mu_j(k+1|k) \tag{14}$$

3) Compute the mixing estimates and covariance:

$$\hat{x}_j^0(k|k)=\sum_{i=1}^{N}\hat{x}_i(k|k)\mu_{i|j}(k) \tag{15}$$

$$\begin{aligned}P_j^0(k|k)=\sum_{i=1}^{N}\{&P_i(k|k)+[\hat{x}_j^0(k|k)-\hat{x}_i(k|k)]\cdot\\&[\hat{x}_j^0(k|k)-\hat{x}_i(k|k)]^T\}\mu_{i|j}(k)\end{aligned} \tag{16}$$

where the superscript 0 denotes the initial value for the next step.

Step 2 Model-conditional filtering

The filtering techniques such as (E)KF and UKF can be applied for model-conditioning filtering. It contains the prediction step and correction step.

A. Prediction step:

1) Compute the predicted state and covariance:

$$\hat{x}_j(k+1|k)=F_j(k)\hat{x}_j^0(k|k)+F_j(k)w_j(k) \tag{17}$$

$$P_j(k+1|k)=F_j(k)P_j^0(k|k)F_j^T(k)+G_j(k)Q_j(k)G_j^T(k) \tag{18}$$

where the subscribe j denotes the j-mode in effect at the present time-slot k. $Q_j(k)$ is the process noise covariance matrix for j-model.

2) Compute the measurement residual and correct the state prediction according to different filtering techniques such as Kalman filter and UKF.

Step 3 Updating the model probability

The model probability is an important parameter for the system fault detection and diagnosis. For this, a likelihood function should be defined in advance, and then the model probability be updated based on the likelihood function.

1) Compute the likelihood function:

$$L_j(k+1)=\frac{1}{\sqrt{2\pi|S_j|}}\exp\left[-\frac{1}{2}r_j^TS_j^{-1}r_j\right] \tag{19}$$

2) Update the model probability:

$$\mu_j(k+1)=\frac{\mu_j(k+1|k)L_j(k+1)}{\sum_{j=1}^{N}\mu_j(k+1|k)L_j(k+1)} \tag{20}$$

Step 4 Estimate fusion and combination that yields the overall state estimate as the probabilistically weighted sum of the updated state estimates of all the filters. The estimates and covariance matrices can be obtained as:

$$\hat{x}(k+1|k+1)=\sum_{j=1}^{N}\hat{x}_j(k+1|k+1)\mu_j(k+1) \tag{21}$$

$$\begin{aligned}P(k+1|k+1)=\sum_{j=1}^{N}[&P_j(k|k)+(\hat{x}(k+1|k+1)-\\&\hat{x}_j(k+1|k+1))(\hat{x}(k+1|k+1)-\\&\hat{x}_j(k+1|k+1))^T]\mu_j(k+1)\end{aligned} \tag{22}$$

More details for the IMM algorithm can be found at [14-15] and the references therein.

Simulation and comparison

In this section, the aforementioned triangular ranging, TRKF, and UKF are implemented to Monte Carlo simulation for tracking in two different scenarios: target moving with constant velocity and target maneuvering scenario. In order to attack the maneuvering of the target, both TRKF and UKF based interactive multiple model method are also included. The performance criterions include root of the mean squared error (RMSE) and the computational burden.

Scenario 1: target moving with constant velocity

The trajectories of the target and platform 1 (P1) and platform 2 (P2) are illustrated in Fig. 2. The target moves from (35, 15, 0.1) Km with a constant velocity 200m/s; P1 and P2 are also moving along the line with constant velocity 200m/s.

The sampling period is 1s and the standard deviation of the measuring noise is 6mrad. Three different cases are tested: (i) model matched case; (ii) model mismatched case; and (iii) IMM case.

(i) Model matched case

In this case, the constant velocity model is used for the TRKF and UKF, which are compared with triangular ranging (TriRang) technique. After 500 Monte Carlo runs, the mean RMSE on position are compared in Fig. 3. From the figure, it is obvious that the triangular ranging has the worst tracking accuracy as the angle measurements contain noises. The TRKF, on the contrary, has much better accuracy than triangular ranging method. The UKF obtains even better accuracy than the TRKF since the deterministic sampling techniques are adopted by UKF. Similar results have been obtained for the velocity RMSE, which is omitted for the space reason. As far as the computational burden is concerned, we use the mean running time as the criterion. The mean running time over 500 Monte Carlo simulations for the TriRang, TRKF and UKF are 3.1, 7.2 and 11.2 ms, respectively.

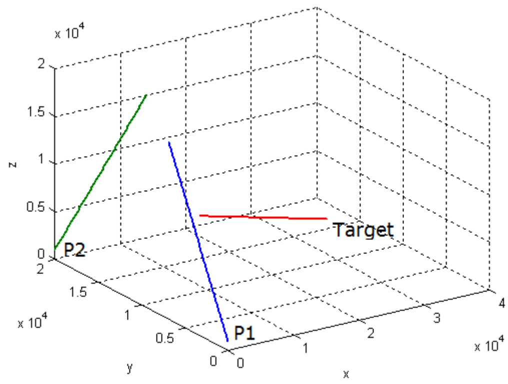

Fig. 2. Trajectories for the first scenario/m

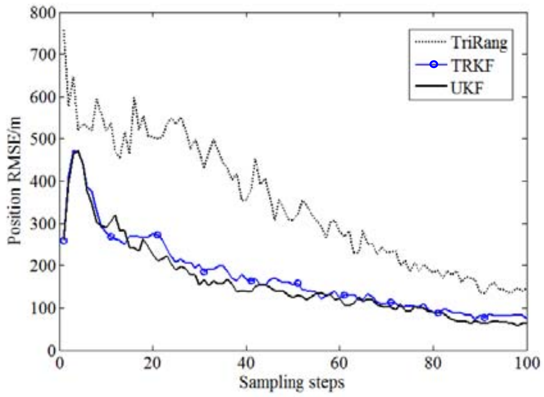

Fig. 3. Position RMSE comparison among TriRang, TRKF and UKF in case of matched model

(ii) Model mismatched case

The purpose is to check the robustness of TRKF and UKF. In this case, we use the constant acceleration model for the TRKF and UKF. After 500 Monte Carlo runs, the performance degradation on RMSE for TRKF and UKF are respectively 43.7% and 31.5% compared with the model matched case. This means that the UKF is a little more robust against model mismatch than the TRKF method.

(iii) IMM method

We use a constant velocity model and two constant acceleration models with different process noises to approximate the target trajectory and set the transition probability matrix

$$\pi = \begin{bmatrix} 0.92 & 0.05 & 0.03 \\ 0.1 & 0.8 & 0.1 \\ 0.1 & 0.2 & 0.7 \end{bmatrix}$$

for both the proposed and triangular ranging based IMM-KF (TR-IMMKF) and IMM-UKF approach. Both initial mode probabilities are set as $\mu_0 = [1/3, 1/3, 1/3]$. After 500 Monte Carlo runs, the RMSE for the three approaches are given in Fig. 4. From this figure, the triangular ranging approach has the lowest tracking accuracy while the proposed TR-IMMKF approach performs not better, at least very closely to the IMM-UKF approach. The mean computational burden for the three approaches is respectively 16.3，29.7, and 83.2ms, respectively. In other words, the proposed TR-IMMKF approach has much smaller computational cost since the interacting process is based on triangular ranging using Kalman filters directly. On the contrary, for the IMM-UKF, each filtering bank uses deterministic sampling techniques.

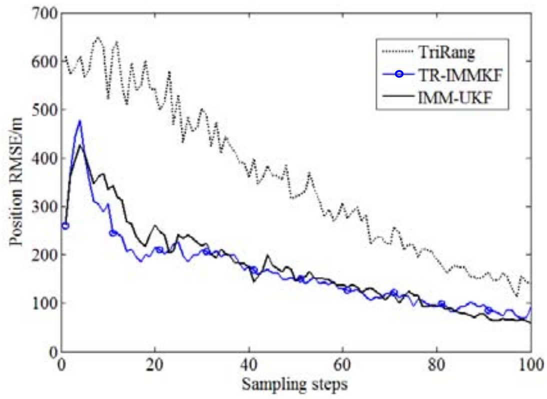

Fig. 4. Position RMSE comparison among TriRang, TR-IMMKF and IMM-UKF

Scenario 2: target maneuvering

The trajectories for the target and the platforms (P1 and P2) are given in Fig. 5. The target first moves for 10s from (35,8,1)km with the velocity of 200m/s; then turns left for 20s with acceleration centripetal 20m/s2; then moves along the straight line for 10s and turns right for 30s with acceleration centripetal 20m/s2; finally moves 30s with the constant velocity. Other parameters are set same as in the 1st scenario. The mean RMSE and mean computational cost after 200 Monte Carlo runs are given in Table 1.

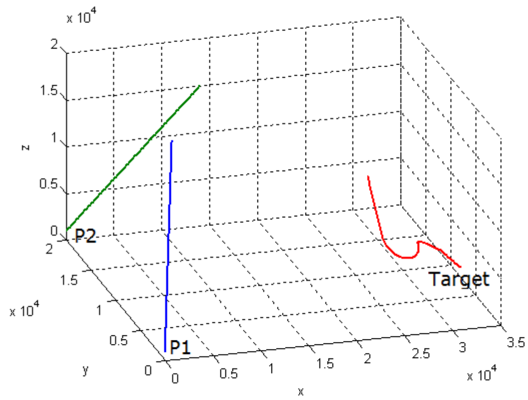

Fig. 5. Trajectories for the maneuvering scenario/m

Table 1 Performance comparison in the maneuvering scenario

Tracking Method	Position RMSE [m]	Velocity RMSE [m/s]	Acceleration RMSE [m/s^2]	Computational Burden [ms]
Triangular Ranging	444.3	430.2	152.8	15.9
TR-IMMKF	291.2	84.4	21.9	29.4
IMM-UKF	248.9	91.6	19.8	67.9

Conclusion

Target tracking by two airborne platforms with bearings-only measurements is a nonlinear problem, in which Kalman filter can not be applied directly. In this paper, the triangular ranging formula has been derived for bistatic bearings-only tracking. The location error has been proved to be Gaussian noises, which enable the traditional Kalman filter and unscented Klman filter applicable. To further improve the tracking accuracy, interactive multiple model (IMM) has been adopted. Simulation results for both constant velocity moving target and maneuvering target have been included to compare the performance of the aforementioned methods. The triangular ranging method, triangular-ranging-based Kalman filtering, unscented Kalman filter, and interactive multiple model are compared extensively using the criterion of RMSE and computational burden, as well as the robustness.

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REFERENCES

[1] Musicki D., Bearings only single-sensor target tracking using Gaussian mixtures, *Automatica*, 45 (2009), No. 9, 2088-2092

[2] Jauffret C., Pillon D., Pignol A.C., Bearings-Only Maneuvering Target Motion Analysis from a Nonmaneuvering Platform, *IEEE Trans. Aeros. & Electr. Syst.*, 46 (2010), No. 4, 1934-1949

[3] Lindgren A.G., Gong K. F., Position and velocity estimation via bearing observations, *IEEE Trans. Aerosp. Electron. Syst.*, 14 (1978), No. 4, 564–577

[4] Gharehshiran O.N., Krishnamurthy V., Coalition Formation for Bearings-Only Localization in Sensor Networks-A Cooperative Game Approach, *IEEE Trans. Signal Process.*, 58 (1010), No. 8, 4322-4338

[5] Musicki D., Bearings only multi-sensor maneuvering target tracking, *Systems & Cotrol Lett.*, 57 (2008), No. 3, 216-221

[6] Song T. L., Speyer J. L., A stochastic analysis of a modified gain extended Kalman filter with applications to estimation with bearings only measurements, *IEEE Trans. Automatic Control*, 30 (1985), No. 10, 940-949

[7] Julier S.J., Uhlmann J.K., Durrant-Whyte H.F., A new method for the nonlinear transformations of means and covariances in filters and estimators, *IEEE Trans. Autom. Control*, vol. 45 (2000), No. 3, 477-482

[8] Ito K., Xiong K., Gaussian filters for nonlinear filtering problems, *IEEE Trans. Autom. Control*, 45 (2000), No. 5, 910-927

[9] Ristic B., Arulampalam S., Gordon N., *Beyond the Kalman Filter: Particle Filters for Tracking Applications*. Norwood, MA: Artech House, 2004

[10] Lin X., Kirubaranjan T., Bar-Shalom Y., Comparison of EKF, pseudo-measurement, and particle filer for a bearing-only target tracking problem, *in Proc. SPIE Conf. Signal Data Processing of Small Targets*, Orlando, FL, 2002

[11] Le Cadre J. P., Bearings-only tracking for maneuvering sources, *IEEE Trans. Aerospace & Elect. System*, 34 (1998), No. 1, 179- 191

[12] Trémois O., Le Cadre J.P., Target motion analysis with multiple arrays: performance analysis, *IEEE Trans. Aerospace Electron. Systems*, 32 (1996), No. 3, 1030–1046

[13] Kalman R. E., A new approach to linear filtering and prediction problems, *J. basic Eng. - T. ASME*, 82 (1960), 35-45

[14] Zhang Y., Li X. R., Detection and diagnosis of sensor and actuator failures using IMM estimator, *IEEE Trans. Aeros. & Elect. System*, 34 (1998), No. 4, 1293-1311

[15] Johnstone A. L., Krishnamurthy V., An improvement to the interacting multiple model (IMM) algorithm, IEEE Trans. on Signal Proces., 49 (2011), No. 12, 2909-2923

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