

Rotor Geometry Parameter Optimization of Synchronous Reluctance Motor Using Taguchi Method

Abstract. In this paper the effect of rotor geometry parameters on the d-axis and q-axis inductance of synchronous reluctance machines is investigated and an optimum design parameter combination is achieved to improve the saliency ratio as well as the performance of drive system. The 2D finite-element model of synchronous reluctance machine is used to construct the Taguchi design of experiments. The level of importance of each design parameter on the response surface is determined by using the Taguchi algorithm and analysis of variance results. The confirmation FEM simulation results indicated that the torque to current ratio and steady state performance of synchronous reluctance machines are improved appropriately in comparison with initial design, by using the proposed Taguchi optimization methodology.

Streszczenie. W artykule opisano wpływ parametrów geometrycznych wirnika maszyny reluktancyjnej synchronicznej, na wartość indukcyjności w osiach d i q. Opracowano optymalny sposób doboru tych parametrów w celu ograniczenia asymetrii magnetycznej maszyny. W budowie modelu metodą Taguchiego, wykorzystano model elementów skończonych w 2-D. Wykazano, że parametry maszyny, wyznaczone metodą optymalizacji Taguchiego pozwalają na osiągnięcie lepszych, w porównaniu z oryginalnymi, odpowiedzi prądowych oraz momentu mechanicznego. (Optymalizacja parametrów geometrycznych wirnika w synchronicznej maszynie reluktancyjnej – metoda Taguchiego).

Keywords: Synchronous reluctance machines, Saliency ratio, Taguchi method, Finite-element method, Orthogonal array.

Słowa kluczowe: synchroniczna maszyna reluktancyjna, asymetria magnetyczna maszyny, metoda Taguchi

Introduction

Generally, induction motors are the most common solution for many industrial applications thanks to their robust structure, line start capability, and simple manufacture process. However in recent years, synchronous reluctance machines have gained interest due to their several advantages, especially in variable speed and servo drive systems [1],[2]. The synchronous reluctance machines have high torque density, fault-tolerant capability, high efficiency, and simple controllability in comparison with induction machines [3]. Moreover the inverter fed SynRM machine has insignificant rotor losses which is lead to efficiency improvement and reduction in thermal problems related to rotor and bearing. Despite several advantage of SynRM, low operating power factor and high contents of torque ripple are the main drawbacks of such machines. Generally in SynRM, the power factor, the speed range at constant power and several aspects of the dynamic response are all directly related to saliency ratio. Hence this parameter can provides a more general guide for optimizing purposes [4],[5]. In SynRM, to improve the developed torque per ampere and power factor, high saliency ratio as well as high inductance difference is required. To achieve high saliency ratio different types of rotor structure such as; single- barrier rotor, multi-barrier rotors, rotor with axially laminated and transversally laminated can be employed [4],[6]. An axially laminated rotor (ALA) structure as a good choice from point of view of saliency ratio, but manufacturing of axially laminated rotor is very difficult and very expensive. On the other hand the multi-barrier rotors or transversally laminated synchronous reluctance machine (TLA), can be made by easier and cheaper method as same as induction machines[7]-[11]. Also it is possible to improve the saliency ratio of TLA machine using an optimum rotor shape. In order to optimize the rotor shape, it is necessary to study the different design parameter on saliency ratio as well as inductance difference of SynRM machine. Also there are so many rotor geometrical parameters that considering all of them in optimization procedure is impractical and may not be necessary. To reduce the number of runs as well as calculation time, Taguchi's method is used to determine the effect of each variable on response and select the list of most important design parameters.

Several approaches are presented to investigate the effects of different controlled variable in design process. Taguchi method is an engineering design optimization methodology that improves the quality of existing processes and simultaneously reduces their costs. In this method, by developing a set of standard orthogonal arrays (OAs) and a methodology for the analysis of results, information from experiment can be extracted more precisely and more efficiently; also, fewer tests are needed even when the number of parameters being investigated is quite large. In [7], the Taguchi's methodology in combination with finite element are used to optimizing a switched reluctance. Also the authors in [8] use the Taguchi's methodology and a limited number of finite element simulations to maximize the power to weight ratio of brushless DC motor. The shape optimization of SynRM rotor is studied in several papers with the aid of torque ripple reduction using response surface methodology. The effects of design parameters on the developed average electromagnetic torque and d-q inductances are studied in the rotor design of a low-cost permanent-magnet-assisted synchronous reluctance motor using finite-element approach is studied in [12]. The focus of this paper is achievement an optimum shape of rotor in consideration of saliency ratio of SynRM. In this paper the parametric analysis is performed to investigate the effect of several geometrical variables on saliency ratio of SynRM. Then the contribution of each geometry parameter on response is investigated by using the L36, nine parameter, three-level orthogonal array. The analysis of variance of results used to determine the optimum shape of rotor to maximize the saliency ratio. The interaction effects of the design variable are considered in optimization process. The FEM simulations verify the improvement of saliency ratio and consequently the performance characteristics of SynRM machine.

Principle of SynRM Machine and 2D Finite Element Model

Steady state phasor diagrams of SynRM is shown in fig.1. From the two-axes machine theory, the developed torque as function of current angle (θ) and load angle (δ) of SynRM machine can be expressed as follows [3]:

$$T_e = \frac{3P}{2} \frac{(L_{md} - L_{mq}) I_a^2 \sin(2\theta)}{2} \quad (1)$$

$$= \frac{3P}{2} \left(\frac{1}{L_{md}} - \frac{1}{L_{mq}} \right) \left(\frac{e_m}{\omega} \right)^2 \sin(2\delta) \quad (2)$$

$$I_a = \sqrt{i_d^2 + i_q^2} \quad (3)$$

$$\theta = \tan^{-1}(i_q / i_d)$$

where P is the pole pairs, L_{md} and L_{mq} are magnetization direct and quadrature axis inductance.

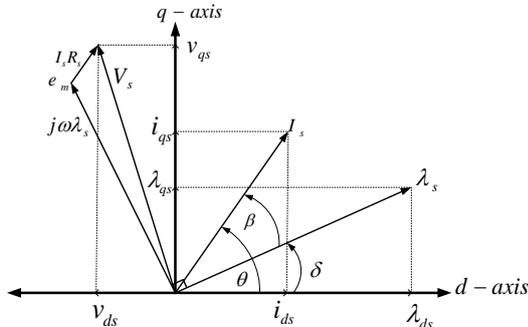


Figure.1. Phasor diagram of SynRM motor

This equation shows the high dependency of the machine torque to the direct and quadrature axis inductances of SynRM machine. Furthermore The maximum achievable power factor of a synchronous reluctance machine is given by[10]:

$$\cos \phi = \frac{\xi - 1}{\xi + 1} \quad (\text{for } \theta = \sqrt{\xi}) \quad (4)$$

where $\xi = L_{md} / L_{mq}$ is defined as saliency ratio of SynRM machine. As expected, higher saliency ratios yield higher power factors, which correspond to reduced winding losses and reduced volt-ampere ratings of the inverter driving the machine. In SynRM the saliency ratio and d-q axis inductance difference is restricted by finite q-axis reluctance and core saturation along the d-axis. Furthermore the direct and quadrature axis inductance can be affected by air gap length, stator slot geometry and winding leakage inductance. Typically the SynRM stator is the same like the stator of asynchronous motor. But for optimum design, it is necessary to consider the effect of the stator geometry on output characteristics. Since the q-axis reluctance is very large in comparison with d-axis, the effect of the stator slot geometry parameter on q-axis flux can be negligible. Whereas, the d-axis flux and air gap flux density harmonics are dependent on geometry parameters of stator slots.

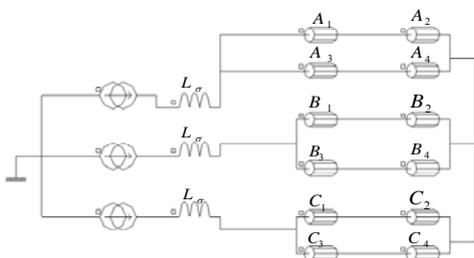


Figure.2. Coupled electrical circuit with 2D-FEM model

In this paper, an optimum design procedure of rotor shape is performed by using Taguchi algorithm. The goal of the optimization procedure is the large saliency ratio and d-q inductance difference value considering predefined design constraints. The 2D finite-element model of 2.5kw/400V/200hz/4pole/36slot/4 flux barrier SynRM machine is used to perform parametric analysis and construct Taguchi

design of experiments. The stator winding is configured as a two layer fractional Slot (1/9) distributed winding. As depicted in fig.2, the leakage inductance of each phase inserted in electrical coupled circuit in 2D FEM model. The leakage inductance calculated by using the finite element method is equal to $L_{\sigma} = 7.6$ mH. The 2D finite element model and calculated flux density obtained from magneto static analysis depicted in Fig. 2. Fig. 3 shows the rotor parameter identification to examine the effect of rotor shape on performance of SynRM machine. The rotor design parameters specification and initial values are summarized in Table 2.

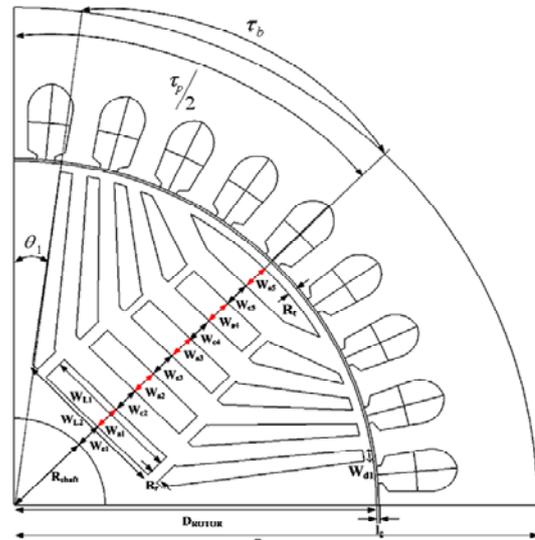


Figure.3. Rotor geometric parameter

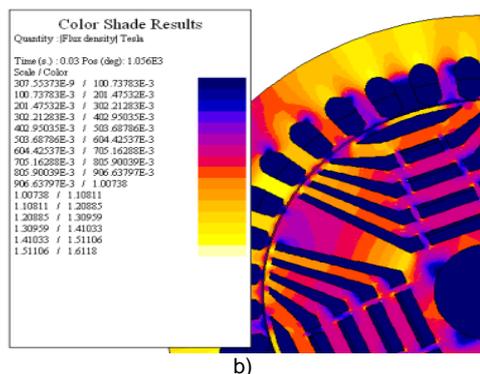
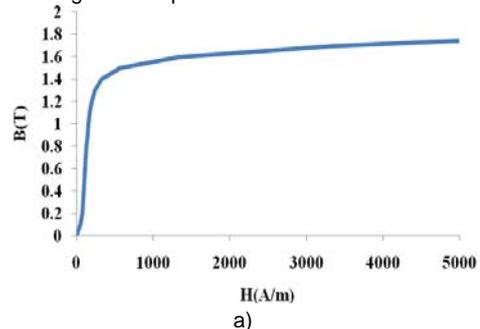


Figure.4. Magnetization curve (B-H) of M530-50A lamination and Stator and rotor flux density distribution

As shown in table.2, the rotor insulation ratio is divided in two separated parameters k_{w1} and k_{w2} for upper and lower flux barriers respectively:

$$KW_1 = \left(\sum_{i=1}^3 W_{ai} \right) / \left(\sum_{i=1}^3 W_{ci} \right) \quad (5)$$

$$KW_1 = \left(\sum_{i=4}^5 W_{ai} \right) / \left(\sum_{i=4}^5 W_{ci} \right)$$

Taguchi Optimization Design Procedure

Taguchi method is a statistical method developed by Genichi Taguchi during the 1950s as a systematic and efficient method for determining optimum design parameters [13]. The proposed Taguchi method provides a new experimental strategy based on a modified and standardized form of design of experiment. Basically the Taguchi method utilizes orthogonal arrays from design of experiments theory to investigate several design variables with a small number of run [14]. Using the orthogonal arrays, makes the design of experiment procedure easy and it requires only few number of runs to study the entire design parameter space. Also, the conclusions drawn from the Taguchi method are valid over the entire experimental region. Taguchi method established based on the signal to noise (S/N) ratio to measure the quality characteristics deviating from the desired values [15]. The S/N measures the level of performance and is an evaluation of the stability of performance of an output characteristic. For analyzing of the S/N ratio, three type of quality characteristic are defined, i.e. The-lower-the-better, the-higher-the-better, and the nominal-the-better [16]. Figure.5 shows the detail procedure of Taguchi optimum design methodology.

Table 1. Initial rotor geometry parameter and their levels

Parameter	Var.	level			Sy.	Val.
		Lev.1	Lev.2	Lev.3		
Rotor barrier span	d1	28	32	-	τ_b	35 ⁰
Width of the end of the barrier	d2	3	3.4	3.6	W_{d1}	0.8
Length of the rectangular segment	d3	14.8	16.72	18.53	W_{L2}	20
Insulation ratio(1)	d4	0.55	0.6	0.65	KW_i	.5
Insulation ratio(2)	d5	0.6	0.7	0.8	KW_i	.5
Tangential rib width	d6	0.4	0.7	1	R_i	1.2
Radial rib width	d7	0.625	0.937	1.875	R_r	2.5
Radial ribs interval	d8	9.474	10.90	12.14	W_{L1}	12
Rotor barrier angle	d9	1	3	5	θ_1	0

In this paper the Larger is better quality characteristic is chosen to maximize the objective function. The S/N ratio is used to measure the sensitivity of the quality characteristic being investigated in an optimum procedure. To determine the effect of each design parameter on the output, the signal-to-noise ratio should be calculated for each experimental data. The signal-to-noise ratio for each experimental data can be calculated based on Eq.(6) [17]:

$$(6) \quad SN_i = 10 \log\left(\frac{\bar{y}_i^2}{s_i^2}\right)$$

where, \bar{y}_i and s_i^2 are mean value and variance for given experiment respectively. Also, \bar{y}_i and s_i^2 can be calculated as[17]:

$$(7) \quad \bar{y}_i = \frac{1}{N_i} \sum_{u=1}^{N_i} \bar{y}_{i,u}$$

$$(8) \quad s_i^2 = \frac{1}{N_i - 1} \sum_{u=1}^{N_i} (y_{i,u} - \bar{y}_i)^2$$

where, i and u are the experiment number and trial number of experiments, respectively. N_i is the number of trials for experiment i. For the case of maximizing the objective function, the signal-to-noise ratio calculated by:

$$(9) \quad SN_i = -10 \log \left[\frac{1}{N_i} \sum_{u=1}^{N_i} \frac{1}{y_u^2} \right]$$

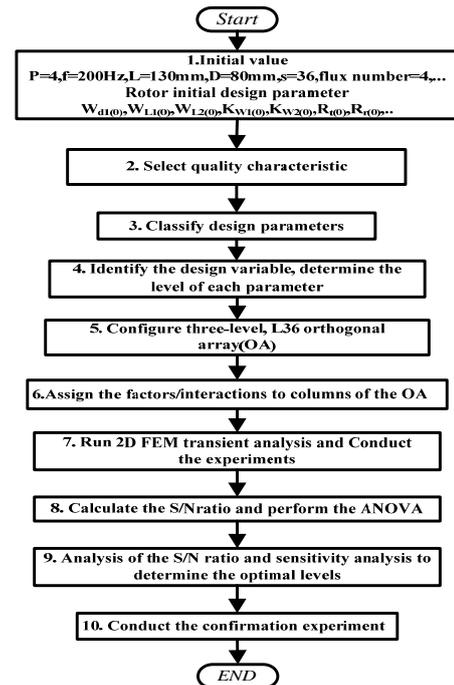


Fig.5. Taguchi method algorithm

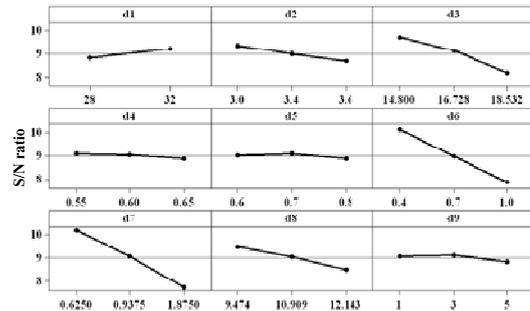


Figure.6. Main effects plot for S/N ratios

The experimental layout for the design parameters using the three-level L36-OA is performed. Since the large saliency ratio improve the overall performance of SynRM, the direct to quadrature axis inductance ratio is considered as objective function. The experimental results are transformed into a signal-to-noise ratio to measure the quality characteristics deviating from the desired values. After calculating the S/N ratio for each experiment, the average S/N value is calculated for each factor and level. Regardless of the category of the quality characteristic, a greater S/N ratio corresponds to better quality characteristics and implies that the signal is much higher than the random effects of the noise factors or minimum variance. The corresponding main effects plots for S/N are shown in fig.6. The mean S/N ratio for each level of the other parameters can be computed in the similar manner. The effect of each rotor geometry parameter on the S/N ratio at different levels can be separated out because of the orthogonality of this experimental design. To analyse the signal to noise ratio, the analysis of variance (ANOVA) is performed to find that which parameter significantly affect the response[16]. Analysis of variance is similar to regression which is used to study and determine the relationship between a response variable and design parameters. The technique does not directly analyze the data, but rather determines the variance of the data. To

conduct ANOVA, the sum of squares (SS) is calculated first. It is a measure of the deviation of the experimental data from the mean value of the data[18]. The analysis of the results carried out using MINITAB 15 software. Table.2 shows the calculation result of analysis of variance. Larger F-value implies that that the variation of the design parameter has large effect on the performance. The effect of each parameter is calculated by determining the range of average S/N value for each factor. The range of average for main effect of S/N ratio is calculated by:

$$(10) \quad \Delta = \max(SN_{average}) - \min(SN_{average})$$

The analysis of variance result shows that $R_r, R_t, W_{L2}, W_{L1}, W_{d1}$ were found to be the major factors affecting the objective function, whereas τ_b, θ_1, KW_i were found to be have relatively insignificant effect on response.

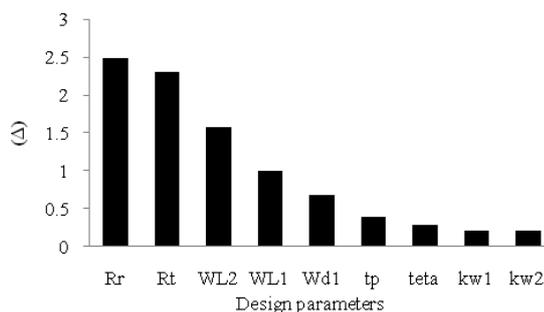


Fig.7. Contribution of each design parameter on response

Table 2. Analysis of variance result

Source	DF	Seq SS	Adj SS	Adj MS	F	P
d1	1	1.399	1.3989	1.3989	30.6	0
d2	2	2.8109	0.1361	0.0680	1.49	0.271
d3	2	15.055	0.0157	0.0078	0.17	0.844
d4	2	0.327	0.6767	0.3383	7.42	0.011
d5	2	0.273	0.4452	0.2226	4.88	0.033
d6	2	31.707	0.4044	0.2022	4.44	0.042
d7	2	37.048	6.5905	3.295	72.2	0
d8	2	6.0825	1.8899	0.945	20.7	0
d9	2	0.5773	0.6911	0.345	7.58	0.01
d1*d2	2	0.5278	0.5894	0.294	6.47	0.016
d1*d3	2	0.1978	0.2797	0.139	3.07	0.091
d1*d4	2	0.0694	0.1085	0.054	1.19	0.344
d1*d5	2	0.4684	0.4683	0.2341	5.14	0.029
Residual Error	10	0.4559	0.4558	0.045		
Total	35	97.000				

S = 0.2135 R-Sq = 99.5% R-Sq(adj) = 98.4%

Optimum Design Selection

According to the result of S/N ratio and ANOVA, the main effect of design variable indicates the trend of each factor. As shown in fig.6 and 7 by considering the influences of individual design variable on the desired response it can be said that the response increases sharply as the radial or tangential rib width decreased. It should be noted that the effect of each design parameter on response only valid within the experimental parameter setting level ranges. The parametric analysis and Taguchi method results gives a clear overall picture about the effect of the rotor slot geometry parameter on the objective function and an optimum combination of these parameters can be selected.

Considering the S/N ratio and ANOVA analyses the initial optimal combination of parameters and their levels, for achieving maximum saliency ratio, is (2,1,1,1,2,1,1,1,2). In previous section only the main effect of each design parameter on the response is considered and the

interaction effect of the design parameter is not be taken into optimization process. The orthogonal arrays do not examine all design variable combinations. Furthermore in complex multivariable process the interaction effects of the variable become significant and should be considered in optimization process. As shown in table.2 the analysis of variance results confirm that the interaction effects between $d_1 \times d_2, d_1 \times d_3, d_1 \times d_4$ and $d_1 \times d_5$ terms are considerable while interaction between other factors is insignificant. The interaction for S/N ratio is shown in fig.8. By considering the interaction effect, the new optimum levels of design variable is (2,1,1,2,2,1,1,1,2).

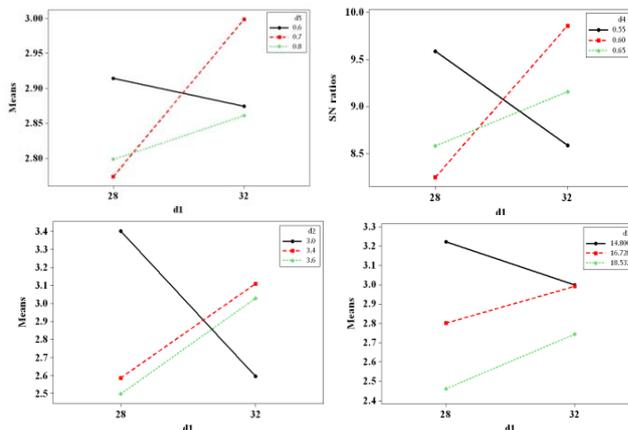


Fig.8. Interaction plot for S/N ratio



Figure.9. Configurations of optimized and initial design model for rotor design

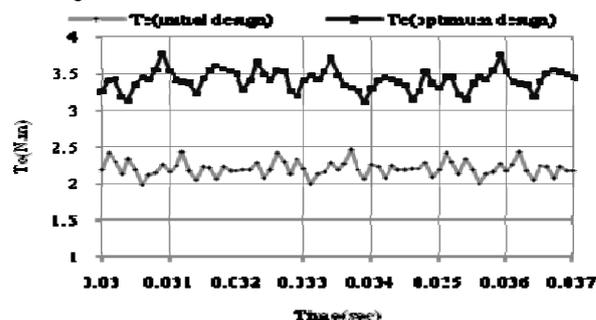


Fig.10. Maximum developed torque for initial and optimum design

Fig.9 shows the optimized rotor geometry in comparison with the initial design. A confirmation experiment is conducted to verify the optimal process parameters obtained from the Taguchi method. The torque ripple calculated by:

$$(10) \quad \% \Delta T_e = \frac{T_{e-MAX} - T_{e-MIN}}{T_{av}}$$

where T_{av} , denote the developed average torque. Using the rotor shape optimization, the saliency ratio as well as inductance difference increase from 2.0791 to 3.78 and 15.71e-3 to 22.25mH respectively. The confirmation

transient FEM simulation results under steady state condition verify the performance of optimization procedure. As shown in fig.10 the maximum developed torque(for maximum load angle) increases from 2.21N.m to 3.4 N.m. The torque ripple increase from %17.377 to %17.47, slightly. Under optimum design condition the torque to current ratio increase from 0.305 to 0.4008. So high torque to current ratio cause to increase the torque capability of SynRM and consequently the efficiency and power factor improved acceptably. fig.11 and fig12 show the air gap flux density and percentage of harmonic content of air gap flux density of initial design and optimum design respectively. Stator and rotor core loss calculation results indicate that although the stator core loss increase slightly but the rotor core loss decrease %22 because of decrease the air gap flux density harmonic content .As result a cooler rotor is obtained and the over load capability of SynRM machine is increased.

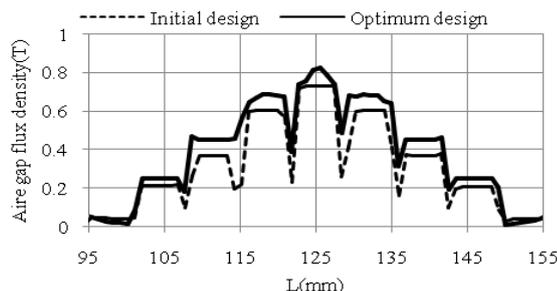


Fig.11. Air gap flux density after optimization

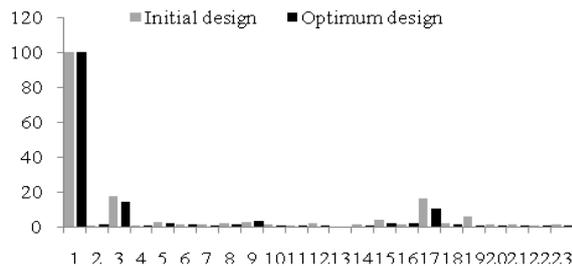


Fig.12. Air gap flux density harmonic content after optimization (percentage of fundamental component: 0.51T and 0.61T for initial and optimum design respectively)

Conclusion

In this paper the parametric analysis carried out to investigate the effect of rotor geometry parameters on direct and quadrature axis inductance ,saliency ratio and inductance difference of SynRM machine. The Taguchi design of experiment method is used to determine the contribution of each design parameter on desired objective function. Several transient 2D finite element calculation is conducted to perform sensitivity analysis using Taguchi design of experiment methodology. An optimal combination of design parameters is determined base on the S/N ratio and ANOVA analyses results. To achieve an appropriate predictor for the measure of performance the possible interaction effects between design variable is taken into optimization process. Finally the confirmation FEM simulation results indicated that the proposed optimization procedure increases the performance of SynRM machine in comparison with to initial design, effectively.

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