

# A Comprehensive Study of Voltage Balancing Problem of Cascaded H-bridge Inverter for Power Quality Conditioner

**Abstract.** In recent years, multilevel technology has become an effective and practical solution for moderate and high voltage application field. In all multilevel technology, cascade multilevel inverters have become the major circuits used in power quality conditioners. Evidently, the voltage balancing problem of DC capacitors of cascade inverters is a key problem to the power quality conditioners. This paper mainly focuses on exploring the hypostasis of various voltage balancing control methods. A comprehensive theoretical analysis of voltage balancing control fundament is presented. The issue of voltage balance is discussed from output pulses of H-bridge inverters point of view, and a model for analyzing the charge and discharge process of H-bridge DC capacitor is established. Furthermore, all regulation means of output pulses of H-bridge inverters for redistributing active power is reported and quantified. The simulation results illustrate that the research is satisfactory.

**Streszczenie.** Zaprezentowano teoretyczną analizę sterowania jakością napięcia w kaskadowo połączonych przekształtnikach mostkowych typu H. Przeanalizowano proces ładowania i rozładowania kondensatora. Przedstawiono badania i symulacje na przykładzie kondycjonera jakości energii. (Analiza problemu równoważenia napięcia w kaskadowo połączonych przekształtnikach typu H)

**Keywords:** Cascade H-bridge inverter, DC capacitor voltage, voltage balancing, power quality conditioner

**Słowa kluczowe:** przekształtniki typu H, jakość energii, równoważenie napięcia

## Introduction

Power quality problems have been hot topics because of the widespread use of nonlinear electronic equipments and the stricter power quality requirements of sensitive loads. To meet the objective of power quality at the point of common coupling (PCC) of a distribution system, the implementation of advanced power electronic equipments that can improve power quality will be in demand. Power quality conditioner, including voltage regulation, reactive power and harmonic compensation is widely used and researched in the power engineering field now[1-6]. But due to the limitation of the voltage capability of the power devices, it is very difficult to handle the nonlinear loads for the traditional power quality conditioner with two-level inverter in high voltage grid.

In recent years, multilevel technology has become an effective and practical solution for high-voltage high-power application field. As described in many literatures, using multilevel technology, the voltage stress on switches will be reduced, the shape of output waveform will be improved and the rate of voltage and power can be increased too. It is believed to have wider application for harmonic restriction and reactive power compensation especially[7-9].

In 1996, cascade multilevel inverters were introduced to be an alternative circuit of reactive power compensation [10]. Then lots of researches have been done towards the cascade multilevel inverter STATCOM. Moreover, the APFs with cascade multilevel inverters are widely researched at present too. In all multilevel technology, cascade multilevel inverters have become the major circuits used in power quality conditioners at present.

When the cascade multilevel inverter is applied to the power quality conditioner, each of the cascaded H-bridge inverters is equipped with an isolated DC capacitor without any power source. To make the equipment work properly, each DC capacitor voltage must be maintained high enough and balance. Otherwise the proper function of the equipment will not be performed. However, tolerances of passive components, unequal conducting and switching losses produced by power switching devices, and parameter imbalance in the control circuit will cause voltage imbalance to DC capacitors in an actual system. Voltage balancing problem, which is an inherent problem of cascade multilevel topology, restricts the development and application to power quality conditioners.

The DC capacitor voltage balancing problem has been widely recognized in literatures. Various strategies have been presented, and successful operation has been demonstrated with DC capacitor balance maintained[11-18]. In [11], a voltage-balancing control algorithm including "clustered balancing control" between the three clusters and "individual balancing control" between the three cascaded H-bridge converters in each cluster is presented. The control algorithm based on a phase-shifted carrier modulation strategy is prominent in having no restriction on the cascade number. This literature [12] presents a similar method to the literature [11]. This method addresses a voltage-balancing control for the energy storage system based on the cascade multilevel PWM converter. The literature makes a detailed description of a design procedure for control gains and parameters. The literature [13] presents a capacitor voltage balance control scheme which is achieved by shifting the switching patterns. This controller on every capacitor is used to prolong or shorten the current conduction of every capacitor, which can maintain the capacitor voltage balance. This literature [14] presents a balancing scheme based on biasing of modulation indexes at each cell. The scheme utilizes a simple controller to track each cell dc link capacitor voltage magnitude and accordingly biases the power flowing into each cell to ensure that the voltages across each cell capacitor converge. This literature [15] presents a balancing scheme that based on the feedback voltage of each DC capacitor and the phase current, duty cycle of the switching states are modified to vary the charged and discharged periods of capacitor selectively so as to implement the energy exchange between cascade cells. In order to control each DC-link voltage, this literature [16] presents that the proportion of inverter voltages should be changed according to the DC-link voltages, and then the integral of the current flowing through the capacitor can be controlled. Based on this method, DC-link voltages can be kept at the rated value using PI controllers. In this literature [17], DC voltages of each phase arc compared with its average values, and their errors are regulated by voltage regulators. Then the outputs of voltage regulators, which are the corrected phase-shifts, are added to the calculated switching angles. The modified switching angles are fed to the SPWM signal generator so as to control DC voltages. In this literature [18], capacitor voltages can be controlled by slightly shifting the switching angle of each H-bridge inverters.

At the present time, various strategies of balancing each DC bus voltage have been presented by researchers. But the model for analyzing DC capacitor voltage balance and consistency of various strategies hasn't be found due to the difficulty in understanding the absence of the charge and discharge process of each H-bridge DC capacitor. Nothing in the literature has been published on a comprehensive legible theoretical analysis of the voltage balancing control fundament of cascaded H-bridge inverters for power quality conditioner. This paper explores the hypostasis of voltage balancing control of cascaded H-bridge inverters for power quality conditioner. A model based on output pulses of H-bridge inverters utilizing voltage switching functions is developed as a means to investigate the control law and to offer a more intuitive insight into the problem. All regulation means of output pulses of H-bridge inverters for redistributing active power is reported and quantified.

### Principle and Mathematic Model of Cascaded H-Bridge Structure of Power Quality Conditioner

The structure of the power quality conditioner with cascade multilevel inverter is shown in Fig.1. Desired current of the power quality conditioner is injected into the power system by controlling the cascade PWM inverters to achieve voltage regulation, reactive power and harmonic compensation. Fig.1 shows that each phase of this equipment consists of two H-bridge inverter units.  $u_{dc\_a1}$ ,  $u_{dc\_a2}$  are DC flying capacitor voltages of the two H-bridge units of A phase respectively. The H-bridge inverter is built-up of four switches, each one with its freewheeling diode. The states of  $T_1$  and  $T_3$ ,  $T_2$  and  $T_4$  are complementary. The switch  $T_1$  is closed, whilst the other  $T_3$  is opened at every time instant. The allowed logic configuration of switches of the H-bridge inverter can produce four switching states. The H-bridge inverter is able to provide the three different output voltage values. The general function of this cascade multilevel inverter is to synthesize a desired voltage from output voltage of each H-bridge inverter of each phase.

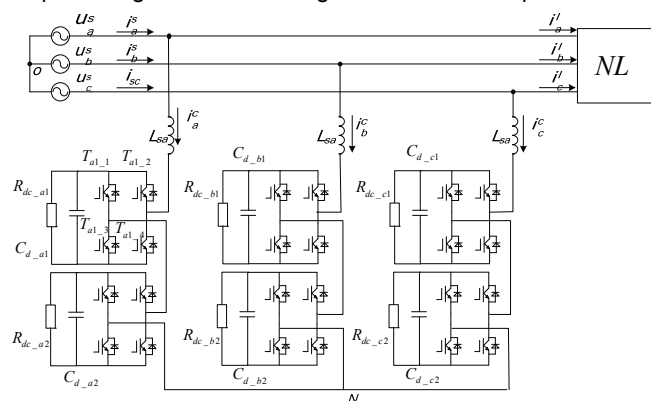


Fig.1 Schematic diagram of power quality conditioner with cascade multilevel inverter

To analyze DC capacitor voltage balancing problem, we use an equivalent resistance in parallel with dc flying capacitor and an equivalent resistance in series with H-bridge unit to represent all the losses of each H-bridge unit. The losses include switching losses produced by power switching devices and losses of DC capacitors. In Fig.1,  $R_{dc\_j1}$  and  $R_{dc\_j2}$  ( $j=a,b,c$ ) are equivalent resistances of the two H-bridge units in parallel with dc flying capacitor respectively,  $C_{d\_j1}$  and  $C_{d\_j2}$  ( $j=a,b,c$ ) are capacitors of the two H-bridge units respectively. The power losses on the inductances  $L_{si}$  ( $i=a,b,c$ ) are represented with equivalent resistances in series with the inductances. In Fig.1, equivalent resistances  $R_{sa}$  ( $i=a,b,c$ ) represent series losses of H-bridge units and power losses on the inductances.

The function of switching state of the first H-bridge of phase A,  $S_{a1}$  of power devices  $T_{a1\_1}$ – $T_{a1\_4}$ , is defined as:  $S_{a1}=1$  when power devices  $T_{a1\_1}$  and  $T_{a1\_4}$  are switched on AND  $T_{a1\_3}$  and  $T_{a1\_2}$  are switched off.  $S_{a1}=-1$  when power devices  $T_{a1\_2}$  and  $T_{a1\_3}$  are switched on AND  $T_{a1\_1}$  and  $T_{a1\_4}$  are switched off.  $S_{a1}=0$  when power devices  $T_{a1\_1}$  and  $T_{a1\_2}$  are switched on AND  $T_{a1\_3}$  and  $T_{a1\_4}$  are switched off, or when power devices  $T_{a1\_3}$  and  $T_{a1\_4}$  are switched on AND  $T_{a1\_1}$  and  $T_{a1\_2}$  are switched off, which is shown as follows:

$$S_{a1} = \begin{cases} 1 & , T_{a1\_1}, T_{a1\_4} \text{ ON and } T_{a1\_2}, T_{a1\_3} \text{ OFF} \\ 0 & , T_{a1\_1}, T_{a1\_2} \text{ ON and } T_{a1\_3}, T_{a1\_4} \text{ OFF or } T_{a1\_3}, T_{a1\_4} \text{ ON and } T_{a1\_1}, T_{a1\_2} \text{ OFF} \\ -1 & , T_{a1\_2}, T_{a1\_3} \text{ ON and } T_{a1\_1}, T_{a1\_4} \text{ OFF} \end{cases}$$

The switch states are coded by symbols -1, 0 and 1 identifying the three voltage levels on each H-bridge inverter. According to explanations above, switching functions of the first and second H-bridge unit in phase A can be demonstrated by  $S_{a1}$  and  $S_{a2}$  respectively. Using the same method,  $S_{b1}$ ,  $S_{b2}$ ,  $S_{c1}$  and  $S_{c2}$  are obtained too. From the equation above, the following equation of the current and voltage relationship between dc part and ac part of the first H-bridge is easily got:

$$\begin{cases} u_{a1} = S_{a1} \cdot u_{dc\_a1} \\ i_{dc\_a1} = S_{a1} \cdot i_{ca} \end{cases}$$

$u_{a1}$  is the output voltage of the first H-bridge,  $i_{a1}$  is the output current of the first H-bridge;  $i_{dc\_a1}$  is the current flying through the DC flying capacitor. The comprehensive mathematic model of the power quality conditioner with cascade two H-bridge inverter in stator coordinates is established as follow:

$$Z\dot{X} = A \cdot X + B \cdot e$$

where:

$$A = \begin{bmatrix} -R_{sa} & 0 & 0 & \frac{2}{3}S_{a1} & \frac{2}{3}S_{a2} & -\frac{1}{3}S_{b1} & -\frac{1}{3}S_{b2} & -\frac{1}{3}S_{c1} & -\frac{1}{3}S_{c2} \\ 0 & -R_{sb} & 0 & -\frac{1}{3}S_{a1} & -\frac{1}{3}S_{a2} & \frac{2}{3}S_{b1} & \frac{2}{3}S_{b2} & -\frac{1}{3}S_{c1} & -\frac{1}{3}S_{c2} \\ 0 & 0 & -R_{sc} & -\frac{1}{3}S_{a1} & -\frac{1}{3}S_{a2} & -\frac{1}{3}S_{b1} & -\frac{1}{3}S_{b2} & \frac{2}{3}S_{c1} & \frac{2}{3}S_{c2} \\ S_{a1} & 0 & 0 & -\frac{1}{R_{dc\_a1}} & 0 & 0 & 0 & 0 & 0 \\ S_{a2} & 0 & 0 & 0 & -\frac{1}{R_{dc\_a2}} & 0 & 0 & 0 & 0 \\ 0 & S_{b1} & 0 & 0 & 0 & -\frac{1}{R_{dc\_b1}} & 0 & 0 & 0 \\ 0 & S_{b2} & 0 & 0 & 0 & 0 & -\frac{1}{R_{dc\_b2}} & 0 & 0 \\ 0 & 0 & S_{c1} & 0 & 0 & 0 & 0 & -\frac{1}{R_{dc\_c1}} & 0 \\ 0 & 0 & S_{c2} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{R_{dc\_c2}} \end{bmatrix}$$

$$Z = \text{diag}[L_{sa} \quad L_{sb} \quad L_{sc} \quad C_{d\_a1} \quad C_{d\_a2} \quad C_{d\_b1} \quad C_{d\_b2} \quad C_{d\_c1} \quad C_{d\_c2}]$$

$$X = [i_{ca} \quad i_{cb} \quad i_{cc} \quad u_{dc\_a1} \quad u_{dc\_a2} \quad u_{dc\_b1} \quad u_{dc\_b2} \quad u_{dc\_c1} \quad u_{dc\_c2}]^T$$

$$B = \text{diag}[1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

$$e = [u_{sa} \quad u_{sb} \quad u_{sc} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$$

### Theory Analysis of Voltage Balancing Control Fundament

We assume that the total active power absorbed from the grid has been controlled based on this equipment's total losses. If various parameters and conducting of each H-bridge of each phase unit are absolutely the same, the actual active power assigned to each H-bridge unit will be the same. So DC capacitor voltage of each H-bridge is the same. But because of the dispersion of DC capacitor parameters, unequal conducting, switching losses produced

by power switching devices and losses of DC capacitors, different unit needs different amount of active power to maintain DC capacitor voltage. In order to guarantee DC capacitor voltage equal to the reference value, additional control loop must be added to redistribute the total active power.

$i_{ca}(t)$  is output current of phase A, suppose:

$$(4) \quad i_{ca}(t) = \sqrt{2}I \cos \omega t$$

From the equation (2), (3) and (4), the following equation is easily got:

$$(5) \quad C_{d\_a1} \frac{du_{dc\_a1}}{dt} + \frac{u_{dc\_a1}}{R_{dc\_a1}} = S_{a1} \cdot \sqrt{2}I \cos \omega t$$

Based on the equation (5), we find that the parameter value of  $C_{d\_a1}$  and  $R_{dc\_a1}$ , output current and the switching state of each H-bridge inverter will influence the voltage of DC capacitor. Evidently, in the three factors, the parameter value of  $C_{d\_a1}$ ,  $R_{dc\_a1}$  and output current is uncontrollable, only the variation of the function of switching state of each H-bridge inverter is controllable for balancing DC capacitor voltage.

We use Fig.2 to analyze the process of the active power absorbed by DC capacitor of the H-bridge in detail[17,19]. In order to simplify analysis process, we assume that switching frequency is power frequency. That is only one time switching per fundamental cycle.  $\varphi_{a1}$  is phase of the output pulse,  $W_{a1}$  is width of it. In a fundamental cycle, the active power absorbed by the H-bridge is:

$$(6) \quad P_{a1} = 2 \int_{\pi/2 - \varphi_{a1} - W_{a1}/2}^{\pi/2 - \varphi_{a1} + W_{a1}/2} u_{dc} \cdot \sqrt{2}I \cos \omega t d\omega t = 4\sqrt{2}u_{dc}I \sin \varphi_{a1} \cdot \sin \frac{W_{a1}}{2}$$

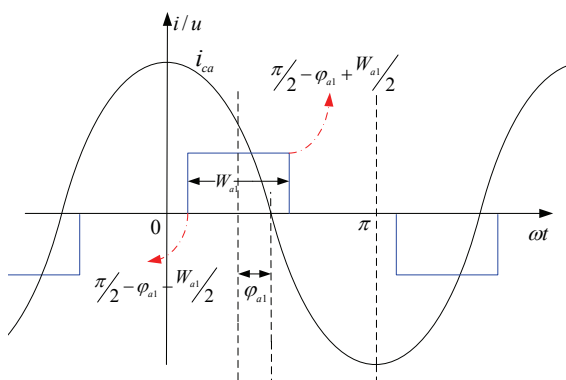


Fig.2 The output current and pulse of inverter

So from the equation (6), it is clear that, if we want to change the active power absorbed by the H-bridge, we can't do other than to change output pulse phase  $\varphi_{a1}$  or width  $W_{a1}$ , or both. Evidently, the active power absorbed by the H-bridge is larger, DC capacitor voltage is higher too. So regulating output pulse of each H-bridge is the only one method for balancing DC capacitor voltage, when parameters of H-bridge units are unequal. The output pulse is decided by the function of switching state of each unit. Based on the above paragraphs mentioned, to redistribute active power absorbed from grid and guarantee dc capacitor voltage of each H-bridge equal to the reference value, we only can use method of fine-tuning the switching states of each unit. There are four regulation means for each H-bridge unit, they are:

- (1) Regulate width of output pulses;
- (2) Regulate phases of output pulses;
- (3) Regulate both width and phases of output pulses;

- (4) Add DC value to output pulses;

Whether the four situations can achieve the desired results or not, which will be proved in the following part. In the situations (1), (2) and (3), output pulses may be marked by the vectors to analyze the balancing control fundament. The output of the first H-bridge unit that hasn't been fine-tuned is marked with  $A_1$ , while the second  $A_2$ , and they are just the same. The summation of  $A_1$  and  $A_2$  is  $A_0$ , as shown in Fig.3. Red line represents the second unit, blue line represents the first one and black line represents the composed final results.

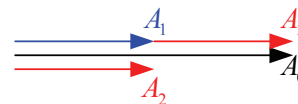


Fig.3 Vector of output pulses before regulated

- (1) Regulate width of output pulses: change length of  $A_1$  and  $A_2$  on the premise of fixing the direction of them, as shown in Fig.4.

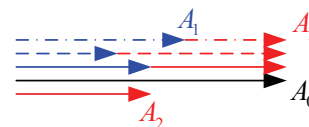


Fig.4 Vector of output pulses after regulating width

- (2) Regulate output pulse angles: change directions of  $A_1$  and  $A_2$  on the premise of fixing length of them, as shown in Fig.5.

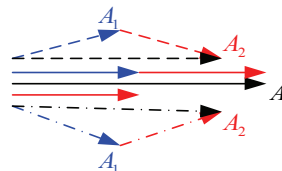


Fig.5 Vector of output pulse after regulating angles

- (3) Regulate both the width and angles of output pulses: both the length and the directions of  $A_1$  and  $A_2$  are changed, as shown in Fig.6.

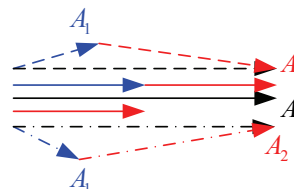


Fig.6 Vector of output pulse after regulating both width and angles

Evidently, to keep the invariability of this equipment's total output voltage, the changed final results must be the same as the precious one. It is easily found that in the situations (1) and (3), the changed final results may be the same as the precious one, while in situation (2) the changed one is different from the precious one. That is to say, regulating width and both the width and the angles of output pulses won't influence the final output of the equipment and can be used as effective methods, but only regulating angles of output pulses cannot. In the next part, from the point of output pulses, their quantitative relationships will be discussed in details.

The output pulses of the cascade H-bridge inverters, no additional balance regulation loop being added, are shown in Fig.7. The system is supposed to be sinusoidal modulation. The output pulses of the two H-bridges are the same. Each device is switched only once per line cycle. Each H-bridge inverter unit generates a quasi-square wave. The output pulse voltage of the cascade multilevel inverter is the sum of the two H-bridge inverter units' outputs.

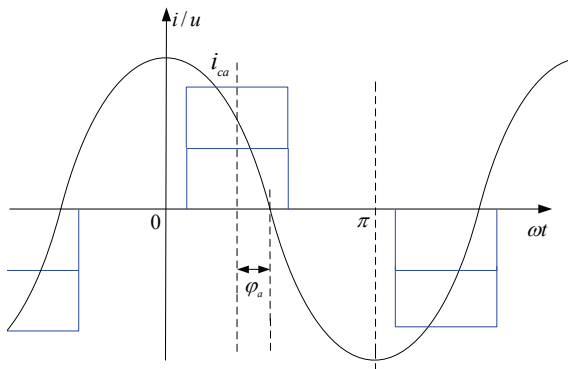


Fig.7 Output pulses of the inverter without regulation

$W_{a1}, W_{a2}$  are width of the first and second H-bridge unit.  $W_{a1} = W_{a2} = W_a$ .  $\varphi_a$  is phase of output pulses of the first and second H-bridge unit. Mathematical expression of the first H-bridge output pulses is shown as follows.

(7)

$$f_1(\omega t) = \begin{cases} 1 & 2m\pi + \pi/2 - \varphi_a - W_a/2 \leq \omega t \leq 2m\pi + \pi/2 - \varphi_a + W_a/2 \\ -1 & 2m\pi + 3\pi/2 - \varphi_a - W_a/2 \leq \omega t \leq 2m\pi + 3\pi/2 - \varphi_a + W_a/2 \\ 0 & \text{others} \end{cases}$$

$$m = \dots, -1, 0, 1, \dots$$

$f_1(\omega t)$  is a periodic function and its period is  $2\pi$ . According to terms that periodic function can be expanded in Fourier series,  $f_1(\omega t)$  can be written in Fourier series. In this Fourier series, the fundamental component is:

(8) 
$$f_{1r1}(\omega t) = \frac{4 \sin(W_a/2)}{\pi} \sin(\omega t + \varphi_a)$$

The output pulse of the other H-bridge unit is the same as the first one, so the fundamental component of it is the same as equation (8). Then the composed fundamental component of output pulses is:

(9) 
$$2 \cdot f_{1r1}(\omega t) = \frac{8 \sin(W_a/2)}{\pi} \sin(\omega t + \varphi_a)$$

The active power absorbed by the first and second H-bridge is:

(10) 
$$P_{a1} = P_{a2} = 2 \int_{\pi/2 - \varphi_a - W_a/2}^{\pi/2 - \varphi_a + W_a/2} \sqrt{2} u_{dc} \cdot I \cos \omega t d\omega t$$

$$= 4\sqrt{2} u_{dc} \cdot I \sin \varphi_a \cdot \sin \frac{W_a}{2}$$

(1) Regulate width of output pulses:

First of all, the sketch map of output pulses before and after regulation is shown in Fig.8.

Blue real lines represent output pulses before regulation. Red and blue broken lines represent output pulses after regulation.  $\Delta W_{a1}, \Delta W_{a2}$  are the variation value of width of the first and second H-bridge units after regulation respectively.

Mathematical expression of the blue pulses is shown as follows:

(11)

$$f_2(\omega t) = \begin{cases} 1 & 2m\pi + \pi/2 - \varphi_a - W_a/2 - \Delta W_{a1}/2 \leq \omega t \leq 2m\pi + \pi/2 - \varphi_a + W_a/2 + \Delta W_{a1}/2 \\ -1 & 2m\pi + 3\pi/2 - \varphi_a - W_a/2 - \Delta W_{a1}/2 \leq \omega t \leq 2m\pi + 3\pi/2 - \varphi_a + W_a/2 + \Delta W_{a1}/2 \\ 0 & \text{others} \end{cases}$$

$$m = \dots, -1, 0, 1, \dots$$

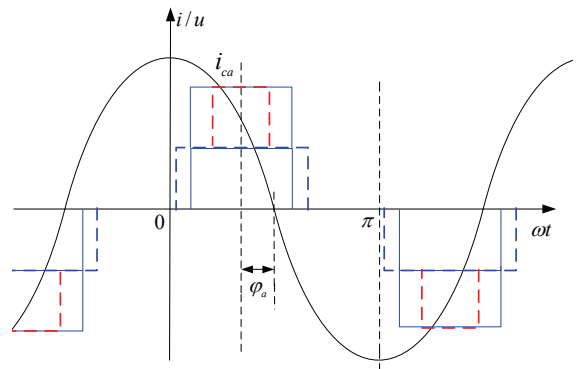


Fig.8 Sketch map of width of output pulses regulated  
Expand it in Fourier series, the fundamental component of it is:

(12) 
$$f_{2r1}(\omega t) = \frac{4 \sin(W_a + \Delta W_{a1})/2}{\pi} \sin(\omega t + \varphi_a)$$

Mathematical expression of the red pulses is shown as follows:

(13)

$$f_3(\omega t) = \begin{cases} 1 & 2m\pi + \pi/2 - \varphi_a - W_a/2 - \Delta W_{a2}/2 \leq \omega t \leq 2m\pi + \pi/2 - \varphi_a + W_a/2 + \Delta W_{a2}/2 \\ -1 & 2m\pi + 3\pi/2 - \varphi_a - W_a/2 - \Delta W_{a2}/2 \leq \omega t \leq 2m\pi + 3\pi/2 - \varphi_a + W_a/2 + \Delta W_{a2}/2 \\ 0 & \text{others} \end{cases}$$

$$m = \dots, -1, 0, 1, \dots$$

Expand it in Fourier series, the fundamental component of it is:

(14) 
$$f_{3r1}(\omega t) = \frac{4 \sin(W_a + \Delta W_{a2})/2}{\pi} \sin(\omega t + \varphi_a)$$

The total composed fundamental component of the two units is:

(15)

$$f_{2r1}(\omega t) + f_{3r1}(\omega t) = \frac{4 \sin(W_a + \Delta W_{a1})/2}{\pi} \sin(\omega t + \varphi_a) + \frac{4 \sin(W_a + \Delta W_{a2})/2}{\pi} \sin(\omega t + \varphi_a)$$

In order to guarantee the output is unchanged after regulation, the following equation is got from the above-mentioned equation:

(16) 
$$\sin \frac{W_a + \Delta W_{a1}}{2} + \sin \frac{W_a + \Delta W_{a2}}{2} = 2 \sin \frac{W_a}{2}$$

We assume that the total active power absorbed from the grid has been controlled based on this equipment's total losses and the parameters of the two H-bridge units are unequal. The active power required by the first and second H-bridge to maintain rated DC voltage is  $P_{a1}^*, P_{a2}^*$ . The variation amount of active power, which the first and second H-bridge units need absorb, is  $\Delta P_{a1}^*, \Delta P_{a2}^*$ .  $\Delta P_{a1}^* = P_{a1}^* - P_{av}^*$ ,  $\Delta P_{a2}^* = P_{a2}^* - P_{av}^*$ .  $P_{av}^*$  is half of the total active power absorbed from the grid. Evidently,  $\Delta P_{a1}^*$  is equal to  $\Delta P_{a2}^*$  in reverse.

(17)

$$\Delta P_{a1}^* = 2 \int_{\pi/2 - \varphi_a - W_a/2 - \Delta W_{a1}/2}^{\pi/2 - \varphi_a + W_a/2 + \Delta W_{a1}/2} \sqrt{2} u_{dc} I \cos \omega t d\omega t$$

$$- 2 \int_{\pi/2 - \varphi_a - W_a/2}^{\pi/2 - \varphi_a + W_a/2} \sqrt{2} u_{dc} I \cos \omega t d\omega t$$

$$= 4\sqrt{2} u_{dc} I \sin \varphi_a \cdot \sin \frac{W_a + \Delta W_{a1}}{2} - 4\sqrt{2} u_{dc} I \sin \varphi_a \cdot \sin \frac{W_a}{2}$$

$$\begin{aligned}
&= -\Delta P_{a2}^* = -2 \int_{\pi/2 - \varphi_a - W_a/2}^{\pi/2 - \varphi_a + W_a/2 + \Delta W_{a2}/2} \sqrt{2} u_{dc} I \cos \omega t d\omega t \\
&- 2 \int_{\pi/2 - \varphi_a - W_a/2}^{\pi/2 - \varphi_a + W_a/2} \sqrt{2} u_{dc} I \cos \omega t d\omega t \\
&= -4\sqrt{2} u_{dc} I \sin \varphi_a \cdot \sin \frac{W_a + \Delta W_{a2}}{2} + 4\sqrt{2} u_{dc} I \sin \varphi_a \cdot \sin \frac{W_a}{2}
\end{aligned}$$

Based on the relation of absorbing active power of two H-bridge units in the above equation, the following equation is got:

$$(18) \quad \sin \frac{W_a + \Delta W_{a1}}{2} + \sin \frac{W_a + \Delta W_{a2}}{2} = 2 \sin \frac{W_a}{2}$$

The equation (18) is the same as the equation (16). Because the output voltage is guaranteed be unchanged after regulation, the total active power absorbed after regulation is the same with before regulation. Moreover, the cascade multilevel inverter only has two H-bridge inverter units. So the variation amount of active power of the first H-bridge unit is certain to be equal to the second H-bridge unit in reverse. So the equation (18) is the same as the equation (16). The relation of absorbing active power of two H-bridge units in the equation (18) is already included in the equation (16). So solving the equation (17),  $\Delta W_{a1}$  and  $\Delta W_{a2}$  will be got. That is to say, so long as the variation value of output pulses width after regulation meets equation (18), the final output of the inverter will be unchanged and DC capacitor voltage of the two H-bridge unit will be equal.

(2) Regulate angles of output pulses:

First of all, sketch map of output pulses before and after regulation is shown in Fig.9.

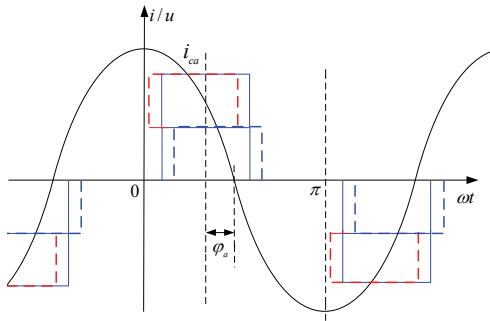


Fig.9 Sketch map of angles of of output pulses regulated

$\Delta\varphi_{a1}, \Delta\varphi_{a2}$  are variation value of output pulse angles of the two H-bridge units respectively. Mathematical expression of the blue pulses is shown as follows:

$$(19) \quad f_s(\omega t) = \begin{cases} 1 & 2m\pi + \pi/2 - \varphi_a + \Delta\varphi_{a1} - W_a/2 \leq \omega t \leq 2m\pi + \pi/2 - \varphi_a + \Delta\varphi_{a1} + W_a/2 \\ -1 & 2m\pi + 3\pi/2 - \varphi_a + \Delta\varphi_{a1} - W_a/2 \leq \omega t \leq 2m\pi + 3\pi/2 - \varphi_a + \Delta\varphi_{a1} + W_a/2 \\ 0 & \text{others} \end{cases}$$

$$m = \dots, -1, 0, 1, \dots$$

Expand it in Fourier series, the fundamental component of it is:

$$(20) \quad f_{4T1}(\omega t) = \frac{4 \sin \frac{W_a}{2}}{\pi} \sin(\omega t + \varphi_a - \Delta\varphi_{a1})$$

Mathematical expression of the red pulses is shown as follows:

$$(21) \quad f_s(\omega t) = \begin{cases} 1 & 2m\pi + \pi/2 - \varphi_a + \Delta\varphi_{a2} - W_a/2 \leq \omega t \leq 2m\pi + \pi/2 - \varphi_a + \Delta\varphi_{a2} + W_a/2 \\ -1 & 2m\pi + 3\pi/2 - \varphi_a + \Delta\varphi_{a2} - W_a/2 \leq \omega t \leq 2m\pi + 3\pi/2 - \varphi_a + \Delta\varphi_{a2} + W_a/2 \\ 0 & \text{others} \end{cases}$$

$$m = \dots, -1, 0, 1, \dots$$

Expand it in Fourier series, the fundamental component of it is:

$$(22) \quad f_{5T1}(\omega t) = \frac{4 \sin \frac{W_a}{2}}{\pi} \sin(\omega t + \varphi_a - \Delta\varphi_{a2})$$

The same method as in situation (1) is used, and then the composed fundamental component of the inverter is:

$$(23) \quad f_{4T1}(\omega t) + f_{5T1}(\omega t) = \frac{4 \sin \frac{W_a}{2}}{\pi} \sin(\omega t + \varphi_a - \Delta\varphi_{a1}) + \frac{4 \sin \frac{W_a}{2}}{\pi} \sin(\omega t + \varphi_a - \Delta\varphi_{a2})$$

In order to guarantee that the output is just the same as before regulation, the following equation is got:

$$(24) \quad \begin{cases} \sin(\varphi_a - \Delta\varphi_{a1}) + \sin(\varphi_a - \Delta\varphi_{a2}) - 2 \sin \varphi_a = 0 \\ \cos(\varphi_a - \Delta\varphi_{a1}) + \cos(\varphi_a - \Delta\varphi_{a2}) - 2 \cos \varphi_a = 0 \end{cases}$$

Solving equation (24), the result is got:

$$(25) \quad \Delta\varphi_{a1} = \Delta\varphi_{a2} = 0$$

The equation (25) means angles of the two H-bridge units are not changed, which is contradicted with the premise. So that is to say, the final output of the H-bridge multilevel inverter will be changed by just regulating the output pulses angle.

(3) Regulate both the width and angles of output pulses

The sketch map of output pulses before and after regulation is shown in Fig.10.

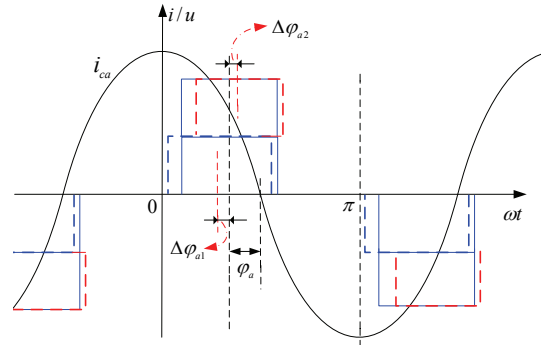


Fig.10 Sketch map of width and angles of output pulses regulated

$\Delta W_{a1}, \Delta W_{a2}$  are the variation value of width of the first and second H-bridge units after regulation respectively.  $\Delta\varphi_{a1}, \Delta\varphi_{a2}$  are the variation value of output pulse angles of the two H-bridge units respectively. Mathematical expression of blue pulses is shown as follows:

$$(26) \quad f_s(\omega t) = \begin{cases} 1 & 2m\pi + \pi/2 - \varphi_a + \Delta\varphi_{a1} - W_a/2 - \Delta W_{a1}/2 \leq \omega t \leq 2m\pi + \pi/2 - \varphi_a + \Delta\varphi_{a1} + W_a/2 + \Delta W_{a1}/2 \\ -1 & 2m\pi + 3\pi/2 - \varphi_a + \Delta\varphi_{a1} - W_a/2 - \Delta W_{a1}/2 \leq \omega t \leq 2m\pi + 3\pi/2 - \varphi_a + \Delta\varphi_{a1} + W_a/2 + \Delta W_{a1}/2 \\ 0 & \text{others} \end{cases}$$

$$m = \dots, -1, 0, 1, \dots$$

Expand it in Fourier series, the fundamental component of it is:

$$(27) \quad f_{6T1}(\omega t) = \frac{4 \sin \frac{(W_a + \Delta W_{a1})}{2}}{\pi} \sin(\omega t + \varphi_a - \Delta\varphi_{a1})$$

Mathematical expression of the blue pulses is shown as follows:

$$(28) \quad f_s(\omega t) = \begin{cases} 1 & 2m\pi + \pi/2 - \varphi_a + \Delta\varphi_{a2} - W_a/2 - \Delta W_{a2}/2 \leq \omega t \leq 2m\pi + \pi/2 - \varphi_a + \Delta\varphi_{a2} + W_a/2 + \Delta W_{a2}/2 \\ -1 & 2m\pi + 3\pi/2 - \varphi_a + \Delta\varphi_{a2} - W_a/2 - \Delta W_{a2}/2 \leq \omega t \leq 2m\pi + 3\pi/2 - \varphi_a + \Delta\varphi_{a2} + W_a/2 + \Delta W_{a2}/2 \\ 0 & \text{others} \end{cases}$$

$$m = \dots, -1, 0, 1, \dots$$

Expand it in Fourier series, the fundamental component of it is:

$$(29) \quad f_{7T1}(\omega t) = \frac{4 \sin(W_a + \Delta W_{a2}) / 2}{\pi} \sin(\omega t + \varphi_a - \Delta \varphi_{a2})$$

The same method as in situation (2) is used, and then the composed fundamental component of the H-bridge inverter is got:

$$(30) \quad \begin{aligned} & f_{6T1}(\omega t) + f_{7T1}(\omega t) \\ &= \frac{4 \sin(W_a + \Delta W_{a1}) / 2}{\pi} \sin(\omega t + \varphi_a - \Delta \varphi_{a1}) + \\ & \frac{4 \sin(W_a + \Delta W_{a2}) / 2}{\pi} \sin(\omega t + \varphi_a - \Delta \varphi_{a2}) \end{aligned}$$

In order to guarantee the output is just the same as before regulation, the following equation is got:

$$(31) \quad \begin{aligned} & \sin \frac{W_a + \Delta W_{a1}}{2} \cdot \cos(\varphi_a - \Delta \varphi_{a1}) + \sin \frac{W_a + \Delta W_{a2}}{2} \cdot \cos(\varphi_a - \Delta \varphi_{a2}) \\ &= 2 \sin \frac{W_a}{2} \cdot \cos \varphi_a \end{aligned}$$

$$(32) \quad \begin{aligned} & \sin \frac{W_a + \Delta W_{a1}}{2} \cdot \sin(\varphi_a - \Delta \varphi_{a1}) + \sin \frac{W_a + \Delta W_{a2}}{2} \cdot \sin(\varphi_a - \Delta \varphi_{a2}) \\ &= 2 \sin \frac{W_a}{2} \cdot \sin \varphi_a \end{aligned}$$

The variation value of the active power which the first and second H-bridge units need absorb to maintain DC capacitor voltage is shown as follows:

$$(33) \quad \begin{aligned} \Delta P_{a1}^* &= 2 \int_{\pi/2 - \varphi_a + \Delta \varphi_{a1} - W_a/2 + \Delta W_{a1}/2}^{\pi/2 - \varphi_a + \Delta \varphi_{a1} + W_a/2 + \Delta W_{a1}/2} \sqrt{2} u_{dc} I \cos \omega t dt \\ & - 2 \int_{\pi/2 - \varphi_a - W_a/2}^{\pi/2 - \varphi_a + W_a/2} \sqrt{2} u_{dc} I \cos \omega t dt \\ &= 4 \sqrt{2} u_{dc} I \sin(\varphi_a - \Delta \varphi_{a1}) \cdot \sin \frac{W_a + \Delta W_{a1}}{2} - 4 \sqrt{2} u_{dc} I \sin \varphi_a \cdot \sin \frac{W_a}{2} \\ &= -\Delta P_{a2}^* = -2 \int_{\pi/2 - \varphi_a + \Delta \varphi_{a2} - W_a/2 + \Delta W_{a2}/2}^{\pi/2 - \varphi_a + \Delta \varphi_{a2} + W_a/2 + \Delta W_{a2}/2} \sqrt{2} u_{dc} I \cos \omega t dt \\ & - 2 \int_{\pi/2 - \varphi_a - W_a/2}^{\pi/2 - \varphi_a + W_a/2} \sqrt{2} u_{dc} I \cos \omega t dt \\ &= -4 \sqrt{2} u_{dc} I \sin(\varphi_a - \Delta \varphi_{a2}) \cdot \sin \frac{W_a + \Delta W_{a2}}{2} + 4 \sqrt{2} u_{dc} I \sin \varphi_a \cdot \sin \frac{W_a}{2} \end{aligned}$$

Based on the relation of absorbing active power of two H-bridge units in the above equation, the following equation is got:

$$(34) \quad \begin{aligned} & \sin(\varphi_a - \Delta \varphi_{a1}) \cdot \sin \frac{W_a + \Delta W_{a1}}{2} + \sin(\varphi_a - \Delta \varphi_{a2}) \cdot \sin \frac{W_a + \Delta W_{a2}}{2} \\ &= 2 \sin \varphi_a \cdot \sin \frac{W_a}{2} \end{aligned}$$

The equation (34) is the same as the equation (32).

Solving the equation (31) and (32), several pairs of  $\Delta W_{a1}$ ,

$\Delta W_{a2}$  and  $\Delta \varphi_{a1}$ ,  $\Delta \varphi_{a2}$  will be got based on  $\Delta P_{a1}^*$  and  $\Delta P_{a2}^*$ .

This method can guarantee that the final output is unchanged and DC capacitor voltage is equal. That is to say, so long as the variation value of output pulses width and angles is regulated properly, the final output of the inverter is unchanged and DC capacitor voltage is equal.

(4) Add DC value to output pulses;

The sketch map of output pulses before and after

regulation is shown in Fig.11. All the output pulses mentioned above are symmetric. The output pulses may become asymmetric if DC value is added to output pulses. As shown in Fig.11, the width of output pulse of the first H-bridge inverter unit changes in the positive cycle, moreover, output pulse angle is changed. The variation value of pulse angle is  $\Delta \varphi_{a1}$  and the variation value of pulse width is  $\Delta W_{a1}$ . The width and angles of output pulses also changes in the negative cycle. The variation value of pulse angle is  $\Delta \varphi_{a2}$  and the variation value of pulse width is  $\Delta W_{a2}$ . If  $\Delta W_{a1}$  isn't the same as  $\Delta W_{a2}$ , the output pulse voltage will have DC value. The similar regulation as in the second H-bridge inverter unit is used. Evidently, the output pulse voltage of the cascade multilevel inverter should have no DC value and phase deflection. So the width and angles of output pulses of the second H-bridge inverter unit should vary by contraries. The variation value of pulse angle is  $\Delta \varphi_{a3}$  and pulse width is  $\Delta W_{a3}$  in the positive cycle. The variation value of pulse angle is  $\Delta \varphi_{a4}$  and pulse width is  $\Delta W_{a4}$  in the negative cycle.

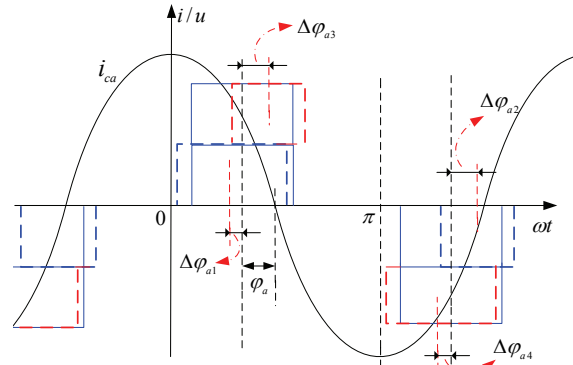


Fig.11 Sketch map of output pulses added DC value

Mathematical expression of the blue pulses is shown as follows:

$$(35) \quad f_s(\omega t) = \begin{cases} 1 & 2m\pi + \pi/2 - \varphi_a + \Delta \varphi_{a1} - W_a/2 - \Delta W_{a1}/2 \leq \omega t \leq 2m\pi + \pi/2 - \varphi_a + \Delta \varphi_{a1} + W_a/2 + \Delta W_{a1}/2 \\ -1 & 2m\pi + 3\pi/2 - \varphi_a + \Delta \varphi_{a2} - W_a/2 - \Delta W_{a2}/2 \leq \omega t \leq 2m\pi + 3\pi/2 - \varphi_a + \Delta \varphi_{a2} + W_a/2 + \Delta W_{a2}/2 \\ 0 & \text{others} \end{cases}$$

$$m = \dots, -1, 0, 1, \dots$$

Expand it in Fourier series, DC value of it is:

$$(36) \quad f_{sT0}(\omega t) = \frac{(\Delta W_{a1} - \Delta W_{a2})}{2\pi}$$

Expand it in Fourier series, the fundamental component of it is:

$$(37) \quad \begin{aligned} f_{sT1}(\omega t) &= \frac{2}{\pi} [\sin(\varphi_a - \Delta \varphi_{a1}) \sin(W_a/2 + \Delta W_{a1}/2) + \sin(\varphi_a - \Delta \varphi_{a2}) \\ & \sin(W_a/2 + \Delta W_{a2}/2)] \cos \omega t + \frac{2}{\pi} [\cos(\varphi_a - \Delta \varphi_{a1}) \sin(W_a/2 + \Delta W_{a1}/2) \\ & + \cos(\varphi_a - \Delta \varphi_{a2}) \sin(W_a/2 + \Delta W_{a2}/2)] \sin \omega t \end{aligned}$$

Mathematical expression of the red pulses is shown as follows:

$$(38) \quad f_s(\omega t) = \begin{cases} 1 & 2m\pi + \pi/2 - \varphi_a + \Delta \varphi_{a3} - W_a/2 - \Delta W_{a3}/2 \leq \omega t \leq 2m\pi + \pi/2 - \varphi_a + \Delta \varphi_{a3} + W_a/2 + \Delta W_{a3}/2 \\ -1 & 2m\pi + 3\pi/2 - \varphi_a + \Delta \varphi_{a4} - W_a/2 - \Delta W_{a4}/2 \leq \omega t \leq 2m\pi + 3\pi/2 - \varphi_a + \Delta \varphi_{a4} + W_a/2 + \Delta W_{a4}/2 \\ 0 & \text{others} \end{cases}$$

$$m = \dots, -1, 0, 1, \dots$$

Expand it in Fourier series, DC value of it is:

$$(39) \quad f_{9T0}(\omega t) = \frac{(\Delta W_{a3} - \Delta W_{a4})}{2\pi}$$

Expand it in Fourier series, the fundamental component of it is:

$$(40) \quad f_{9T1}(\omega t) = \frac{2}{\pi} [\sin(\varphi_a - \Delta\varphi_{a3}) \sin(\frac{W_a}{2} + \frac{\Delta W_{a3}}{2}) + \sin(\varphi_a - \Delta\varphi_{a4}) \sin(\frac{W_a}{2} + \frac{\Delta W_{a4}}{2})] \cos \omega t + \frac{2}{\pi} [\cos(\varphi_a - \Delta\varphi_{a3}) \sin(\frac{W_a}{2} + \frac{\Delta W_{a3}}{2}) + \cos(\varphi_a - \Delta\varphi_{a4}) \sin(\frac{W_a}{2} + \frac{\Delta W_{a4}}{2})] \sin \omega t$$

The total DC value is:

$$(41) \quad f_{8T0}(\omega t) + f_{9T0}(\omega t) = \frac{(\Delta W_{a1} - \Delta W_{a2})}{2\pi} + \frac{(\Delta W_{a3} - \Delta W_{a4})}{2\pi}$$

The total fundamental component is:

$$(42) \quad f_{8T1}(t) + f_{9T1}(t) = \frac{2}{\pi} [\sin(\varphi_a - \Delta\varphi_{a1}) \sin(\frac{W_a}{2} + \frac{\Delta W_{a1}}{2}) + \sin(\varphi_a - \Delta\varphi_{a2}) \sin(\frac{W_a}{2} + \frac{\Delta W_{a2}}{2}) + \sin(\varphi_a - \Delta\varphi_{a3}) \sin(\frac{W_a}{2} + \frac{\Delta W_{a3}}{2}) + \sin(\varphi_a - \Delta\varphi_{a4}) \sin(\frac{W_a}{2} + \frac{\Delta W_{a4}}{2})] \cos \omega t + \frac{2}{\pi} [\cos(\varphi_a - \Delta\varphi_{a1}) \sin(\frac{W_a}{2} + \frac{\Delta W_{a1}}{2}) + \cos(\varphi_a - \Delta\varphi_{a2}) \sin(\frac{W_a}{2} + \frac{\Delta W_{a2}}{2}) + \cos(\varphi_a - \Delta\varphi_{a3}) \sin(\frac{W_a}{2} + \frac{\Delta W_{a3}}{2}) + \cos(\varphi_a - \Delta\varphi_{a4}) \sin(\frac{W_a}{2} + \frac{\Delta W_{a4}}{2})] \sin \omega t$$

In order to guarantee the output is just the same as before regulation, the following equation is got:

$$(43) \quad \Delta W_{a1} - \Delta W_{a2} + \Delta W_{a3} - \Delta W_{a4} = 0$$

$$(44) \quad \sin(\varphi_a - \Delta\varphi_{a1}) \sin(\frac{W_a}{2} + \frac{\Delta W_{a1}}{2}) + \sin(\varphi_a - \Delta\varphi_{a2}) \sin(\frac{W_a}{2} + \frac{\Delta W_{a2}}{2}) + \sin(\varphi_a - \Delta\varphi_{a3}) \sin(\frac{W_a}{2} + \frac{\Delta W_{a3}}{2}) + \sin(\varphi_a - \Delta\varphi_{a4}) \sin(\frac{W_a}{2} + \frac{\Delta W_{a4}}{2}) = 4 \sin(\frac{W_a}{2}) \sin \varphi_a$$

$$(45) \quad \cos(\varphi_a - \Delta\varphi_{a1}) \sin(\frac{W_a}{2} + \frac{\Delta W_{a1}}{2}) + \cos(\varphi_a - \Delta\varphi_{a2}) \sin(\frac{W_a}{2} + \frac{\Delta W_{a2}}{2}) + \cos(\varphi_a - \Delta\varphi_{a3}) \sin(\frac{W_a}{2} + \frac{\Delta W_{a3}}{2}) + \cos(\varphi_a - \Delta\varphi_{a4}) \sin(\frac{W_a}{2} + \frac{\Delta W_{a4}}{2}) = 4 \sin(\frac{W_a}{2}) \cos \varphi_a$$

The variation value of the active power which the first and second H-bridge units need absorb to maintain DC capacitor voltage is shown as follows:

$$(46) \quad \Delta P_{a1}^* = \int_{\frac{\pi}{2}-\varphi_a+\Delta\varphi_{a1}-\frac{W_a}{2}-\frac{\Delta W_{a1}}{2}}^{\frac{\pi}{2}-\varphi_a+\Delta\varphi_{a1}+\frac{W_a}{2}+\frac{\Delta W_{a1}}{2}} \sqrt{2} u_{dc} I \cos \omega t dt + \int_{\frac{3\pi}{2}-\varphi_a+\Delta\varphi_{a2}-\frac{W_a}{2}-\frac{\Delta W_{a2}}{2}}^{\frac{3\pi}{2}-\varphi_a+\Delta\varphi_{a2}+\frac{W_a}{2}+\frac{\Delta W_{a2}}{2}} \sqrt{2} u_{dc} I \cos \omega t dt - 2 \int_{\frac{\pi}{2}-\varphi_a-\frac{W_a}{2}}^{\frac{\pi}{2}-\varphi_a+\frac{W_a}{2}} \sqrt{2} u_{dc} I \cos \omega t dt = 2\sqrt{2} u_{dc} I \sin(\varphi_a - \Delta\varphi_{a1}) \cdot \sin \frac{W_a + \Delta W_{a1}}{2} + 2\sqrt{2} u_{dc} I \sin(\varphi_a - \Delta\varphi_{a2}) \cdot \sin \frac{W_a + \Delta W_{a2}}{2} - 4\sqrt{2} u_{dc} I \sin \varphi_a \cdot \sin \frac{W_a}{2} = -\Delta P_{a2}^* = - \int_{\frac{\pi}{2}-\varphi_a+\Delta\varphi_{a3}-\frac{W_a}{2}-\frac{\Delta W_{a3}}{2}}^{\frac{\pi}{2}-\varphi_a+\Delta\varphi_{a3}+\frac{W_a}{2}+\frac{\Delta W_{a3}}{2}} \sqrt{2} u_{dc} I \cos \omega t dt - \int_{\frac{\pi}{2}-\varphi_a+\Delta\varphi_{a4}-\frac{W_a}{2}-\frac{\Delta W_{a4}}{2}}^{\frac{\pi}{2}-\varphi_a+\Delta\varphi_{a4}+\frac{W_a}{2}+\frac{\Delta W_{a4}}{2}} \sqrt{2} u_{dc} I \cos \omega t dt + 2 \int_{\frac{\pi}{2}-\varphi_a-\frac{W_a}{2}}^{\frac{\pi}{2}-\varphi_a+\frac{W_a}{2}} \sqrt{2} u_{dc} I \cos \omega t dt = -2\sqrt{2} u_{dc} I \sin(\varphi_a - \Delta\varphi_{a3}) \cdot \sin \frac{W_a + \Delta W_{a3}}{2} - 2\sqrt{2} u_{dc} I \sin(\varphi_a - \Delta\varphi_{a4}) \cdot \sin \frac{W_a + \Delta W_{a4}}{2} + 4\sqrt{2} u_{dc} I \sin \varphi_a \cdot \sin \frac{W_a}{2}$$

Based on the relation of absorbing active power of two H-bridge units in the above equation, the following equation is got:

$$(47) \quad \sin(\varphi_a - \Delta\varphi_{a1}) \cdot \sin \frac{W_a + \Delta W_{a1}}{2} + \sin(\varphi_a - \Delta\varphi_{a2}) \cdot \sin \frac{W_a + \Delta W_{a2}}{2} + \sin(\varphi_a - \Delta\varphi_{a3}) \cdot \sin \frac{W_a + \Delta W_{a3}}{2} + \sin(\varphi_a - \Delta\varphi_{a4}) \cdot \sin \frac{W_a + \Delta W_{a4}}{2} = 4 \sin \varphi_a \cdot \sin \frac{W_a}{2}$$

The equation (47) is the same as the equation (44). Solving the equation (43), (44) and (45), several pairs

of  $\Delta W_{a1}$ ,  $\Delta W_{a2}$ ,  $\Delta W_{a3}$ ,  $\Delta W_{a4}$  and  $\Delta\varphi_{a1}$ ,  $\Delta\varphi_{a2}$ ,  $\Delta\varphi_{a3}$ ,

$\Delta\varphi_{a4}$  will be got based on  $\Delta P_{a1}^*$  and  $\Delta P_{a2}^*$ . This method also can guarantee that the final output is unchanged and DC capacitor voltage is equal.

Summarily, we have analyzed the relationship between variation of switching state and final output of the H-bridge inverter qualitatively and quantitatively. Both of them prove: by regulating width, both width and angles of output pulses will not influence the overall output effect of H-bridge inverter. They can be used to reassign the active power absorbed from the grid. Besides, add dc value to modulation waves of H-bridge inverter can also achieve the same result and is an effective method, too. But just regulating output pulse angles will make the final output of H-bridge inverter changed and cannot be used here.

The analysis of the situation (4) not only presents the regulation method of adding DC value to output pulses, but also illuminates that the regulation of the situation (4) is the sum of all regulation means. That is to say, regulating width of output pulses, regulating phases of output pulses, regulating both width and phases of output pulses, and adding DC value to output pulses are the special applications of regulating width and phases of output pulses in the positive and negative cycle respectively in the situation (4).

In the analysis of the above paragraphs mentioned, switching frequency is assumed be power frequency and the system is supposed be sinusoidal modulation. Each H-bridge inverter unit generates a quasi-square wave. Evidently, the above analysis process is irrespective to switching frequency, shape of the command voltage and the output current. That is to say, when switching frequency is higher than power frequency, the similar analysis process can be drawn in a switching period. When the command voltage and output current of the equipment are nonsinusoidal, the same conclusion can be drawn too.

In the situations, we use Fig.12 to analyze the process of the active power absorbed by DC capacitor of the H-bridge in a switching period. Output pulse shown in Fig.12 is output voltage of H-bridge inverter in one switching frequency. In Fig.12,  $W_a$  is width of the output pulse,  $T_s$  is

switching period and  $I_{dc}$  is the sampling current value in the switching period. Based on the theory of the average model, equivalent output voltage of the first H-bridge unit in one switch period in average model is  $\frac{W_a}{\omega T_s} u_{dc}$ . From the

above sentence, we find that width of output pulse is proportional to output voltage. In the switching cycle, the active power absorbed by the H-bridges is:

$$(48) \quad P_a = \int_{\frac{\pi}{2}-\varphi_a-\frac{W_a}{2}}^{\frac{\pi}{2}-\varphi_a+\frac{W_a}{2}} u_{dc} \cdot I_{dc} d\omega t = u_{dc} I_{dc} W_a$$

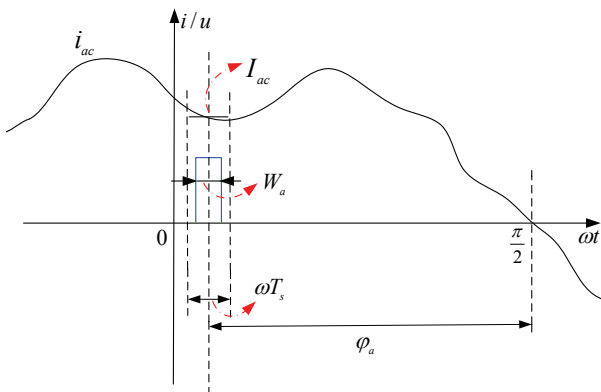


Fig.12 The output current and pulse of inverter in the switching cycle

So from the equation (48), it is clear that, if we want to change the active power absorbed by the H-bridge, we can't do other than to change output pulse width  $W_a$ . Output command voltage of H-bridge for tracking output current is modulated by triangle carrier wave based on pulse width modulation(PWM) method to produce H-bridge output pulse. From the above paragraph, we know that width of output pulse is proportional to output voltage. So width of output pulse is proportional to output command voltage. So we may regulate output command voltage of each H-bridge to change output pulse width  $W_a$  so as to change the active power absorbed by the H-bridge.

There are still three output command voltage regulation means to redistribute active power and guarantee dc capacitor voltage equal to the reference value. (i.e. regulating magnitude of the command voltage of each H-bridge unit, regulating both magnitude and phases of the command voltage of each H-bridge unit, adding DC value to the command voltage of each H-bridge unit). In the same way, output pulses also may be marked by the vectors to analyze the balancing control fundament with Fig.3- Fig.6. In the situations, the command voltage for tracking nonsinusoidal output current is nonsinusoidal and the vector picture represents the instantaneous value of the command voltage in a switching period.

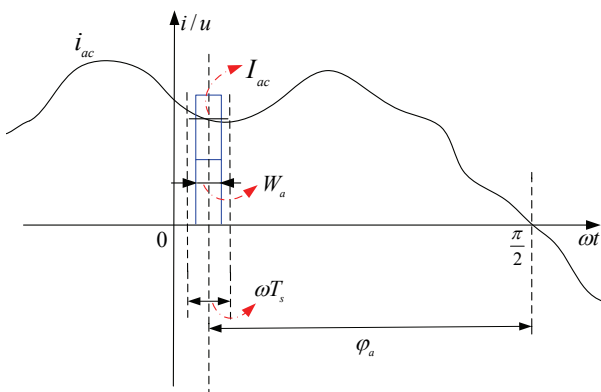


Fig.13 Sketch map of the command voltage without regulation in a switching cycle

In a switching period, all three regulation means may be shown from the point of output pulses with Fig.13 and Fig.14. From Fig.14, we may find that though the modulation waves of output command voltages of the three regulation means are different, the regulation results are all width changes of output pulses in a switching period. Because switching frequency might be very high and variety of output command voltage might be large, power balance

might not be achieved in a switching period. So we still think one fundamental cycle as one fundamental unit to achieve DC voltage balance. In other words, we only think that active power absorbed by H-bridge is controlled based on the H-bridge's total losses in one fundamental cycle. In the situations, we may analyze quantitative relationships of the balancing control fundament with Fig.13 and Fig.14 which is similar to Fig.7- Fig.11.

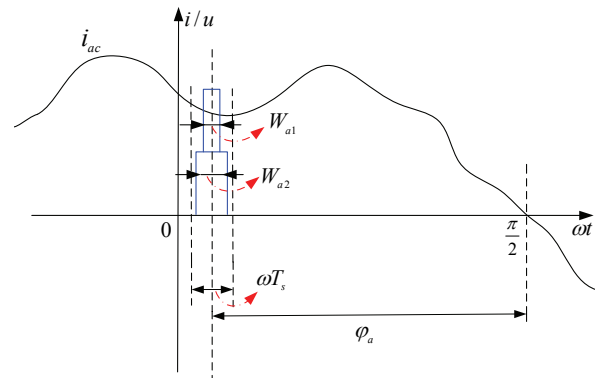


Fig.14 Sketch map of the command voltage regulated in a switching cycle

The active power absorbed by the first and second H-bridge in Fig.13 is:

$$(49) \quad P_{a1} = P_{a2} = \int_{\pi/2 - \varphi_a - W_a/2}^{\pi/2 - \varphi_a + W_a/2} u_{dc} \cdot i_{ac} d\omega t = u_{dc} I_{ac} W_a$$

The total composed output voltage of the two units is:

$$(50) \quad u = u_1 + u_2 = \frac{2W_a}{\omega T_s} u_{dc}$$

In order to guarantee the output is unchanged after regulation, the following equation is got:

$$(51) \quad \frac{2W_a}{\omega T_s} u_{dc} = \frac{W_{a1}}{\omega T_s} u_{dc} + \frac{W_{a2}}{\omega T_s} u_{dc}$$

$$2W_a = W_{a1} + W_{a2}$$

It's assumed that there are  $N$  switching periods in a fundamental cycle. The variation amount of active power, which the first and second H-bridge units need absorb, is:

$$(52) \quad \sum_{i=1}^N \Delta P_{a1}^*(i) = \sum_{i=1}^N (u_{dc} I_{ac}(i) W_{a1}(i) - u_{dc} I_{ac}(i) W_a(i))$$

$$= - \sum_{i=1}^N \Delta P_{a2}^*(i) = \sum_{i=1}^N (u_{dc} I_{ac}(i) W_a(i) - u_{dc} I_{ac}(i) W_{a2}(i))$$

So long as the variation value of output pulses width is regulated properly to meet equation (51) and (52), the final output of the inverter will be unchanged and DC capacitor voltage of the two H-bridge unit will be equal. The control methods in the literature [11], [12], [13], [14], [15] and [16] are means of regulating magnitude of the command voltage of each H-bridge unit. In this literature [17] and [18], the control methods are the means of regulating both magnitude and phases of the command voltage of each H-bridge unit. Through this above research, we find that we may combine different DC voltage control method with different PWM strategy to produce different regulation means of regulating width and phases of output pulses. For different application conditions, we may adopt these different regulation means to achieve DC voltage balance. This above research illustrates the fundament of DC voltage balancing control with two H-bridge units. Evidently, the theory adapts to system composed of multi-H-bridge units.



## Simulation

Theoretic analysis results are necessary to be further verified by the simulation. The parameters of the circuit for the simulation are: switch frequency is 50Hz, DC capacitor is 3300uF, the equivalent resistance,  $R_{dc,i1}$  and  $R_{dc,i2}$  ( $i=a,b,c$ ), of the two units are equal,  $R_{dc,i1} = R_{dc,i2} = 100\Omega$ . Simulation waveforms of switching states and DC flying capacitor voltages of the two H-bridge inverters are given as follows.

When the sum of the active power is controlled, the two dc capacitor voltages are just the same, because for the two H-bridge units, the function of switching state is just the same and the equivalent resistances are equal, as shown in Fig.15.

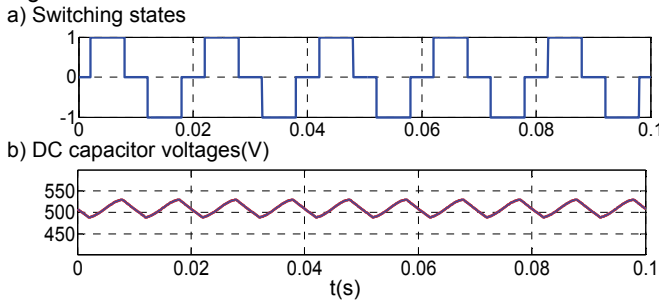


Fig.15 Switching states and DC capacitor voltages of the two H-bridge units

The two dc capacitor voltages are differ from each other when output pulses width of the two H-bridge units are fine-tuned, as shown in Fig.16.

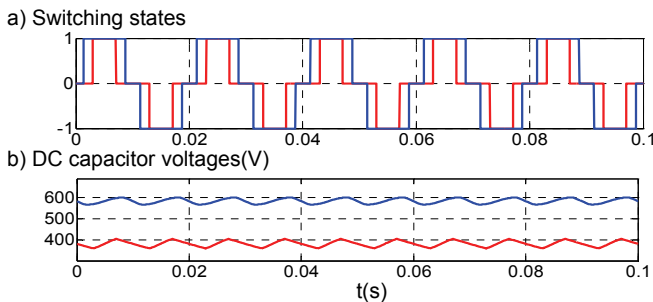


Fig.16 Switching states and DC capacitor voltages of the two H-bridge units when output pulses width are changed

The two dc capacitor voltages are differ from each other when output pulses width and angles of the two H-bridge units are fine-tuned, as shown in fig.19.

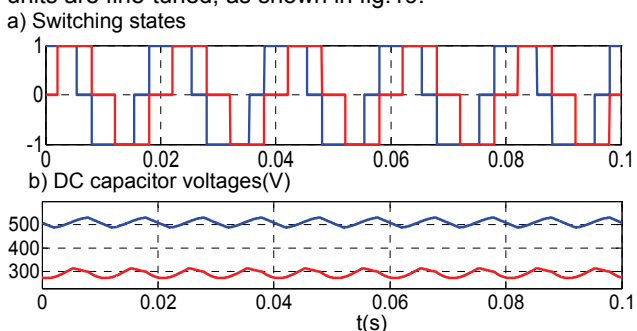


Fig.19 Switching states and DC capacitor voltages of the two H-bridge units when output pulses width and angles are changed

The two dc capacitor voltages are differ from each other when dc value is added to modulation wave of one H-bridge unit is fine-tuned, as shown in fig.20.

a) modulation waves(V)

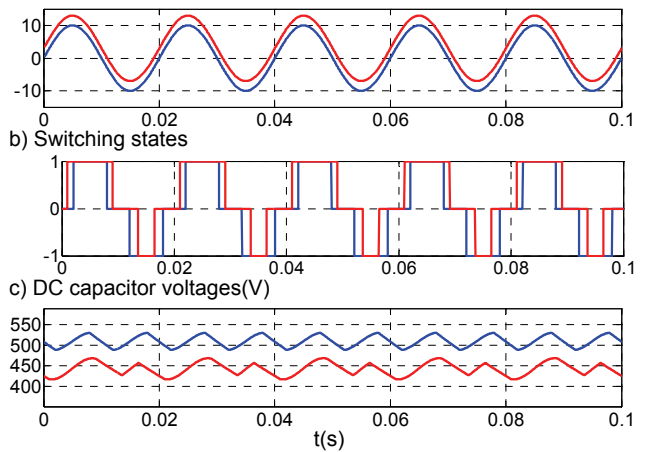


Fig.20 Modulation waves, switching states and DC capacitor voltages of the two H-bridge units when DC value is added to modulation wave of the first one

By simulation, we have proved that: the active power absorbed from the grid can be reassigned during different H-bridge units by changing switching state of them. The switching state reflects on output pulses. That is to say, by regulating width of output pulses, width and angles of output pulses or adding dc value to modulation waves of H-bridge units, we can redistribute the active power absorbed from the grid and then make dc capacitor voltages equal to the reference value.

## Conclusion

In this paper, DC capacitor voltage balancing problem was investigated for the power quality conditioner with cascade multilevel inverter. The issue of voltage balance was discussed from output pulses of H-bridge inverters point of view, and a new and general model was introduced as a way to investigate the charge and discharge process of H-bridge DC capacitor regardless of the type of PWM used. The hypothesis of various voltage balancing control methods is explained clearly. Furthermore, all regulation means of output pulses of H-bridge inverters for redistributing active power is reported and quantified. The simulation results illustrate that the research is satisfactory.

This research of the paper may be not only applicable to STATCOMs and APFs, but also applicable to the rectifier with cascade multilevel inverter [14] and energy storage system with cascade multilevel inverter [12], etc. Moreover, the research by this paper may be applied to asymmetric cascaded multilevel converter with different DC voltages and different switching frequency too. The paper provides a theory basis for idiographic design of DC voltage control methods to achieve DC voltage balance in different application conditions of cascade multilevel inverters.

Based on the investigations reported in this paper and the results reported by other researchers, it can be concluded that DC capacitor voltage balancing problem does not limit the usefulness of this topology for practical applications. This problem can be solved in a satisfactory way using various techniques, depending on idiographic design of regulation means of output pulses.

## REFERENCES

- [1] Akagi H. New trends in active filters for power conditioning. IEEE Trans Industry Applications, 1996, 32(6):1312-1322.
- [2] Lavopa E, Zanchetta P, Sumner M, Cupertino F. Real-Time Estimation of Fundamental Frequency and Harmonics for Active Shunt Power Filters in Aircraft Electrical Systems. IEEE Trans Industry Electronics, 2009, 58(8): 2875 - 2884.
- [3] Grino R, Cardoner R, Costa-Castello R, Fossas E. Digital Repetitive Control of a Three-Phase Four-Wire Shunt Active

- Filter. IEEE Trans Industry Electronics, 2007, 54(3): 1495 - 1503.
- [4] Corasaniti V F, Barbieri M B, Arnera P L, Valla M I. Hybrid Power Filter to Enhance Power Quality in a Medium-Voltage Distribution Network. IEEE Trans Industry Electronics, 2009, 56(8): 2885 - 2893.
- [5] Dixon J W, Venegas G, Moran L A. A series active power filter based on a sinusoidal current-controlled voltage-source inverter. IEEE Trans Industrial Electronics, 1997, 44(5): 612-620.
- [6] Cristian Lascu, Lucian Asiminoaei, Ion Boldea, and Frede Blaabjerg. Frequency Response Analysis of Current Controllers for Selective Harmonic Compensation in Active Power Filters. IEEE Trans Industry Electronics, 2009, 56(2): 337-347.
- [7] Akira N, Takahashi I, Akagi H. A new neutral-point-clamped PWM inverter. IEEE Trans Industry Applications, 1981, 17(3): 518-523.
- [8] Rodriguez J, Lai J S, Peng F Z. Multilevel Inverters: A Survey of Topologies, Controls, and Applications. IEEE Trans Industry Applications, 2002, 49(4): 724-738.
- [9] Rudnick H, Dixon J, Moran L. Delivering clean and pure power. IEEE Power and Energy Magazine, 2003, 1(5): 32-40.
- [10] F Z Peng, J S Lai, J W McKeever, and J VanCoevering. A multilevel voltage-source inverter with separate dc sources for static var generation. IEEE Trans Industry Applications, 1996, 32(5): 1130-1138.
- [11] Hirofumi Akagi, Shigenori Inoue, and Tsurugi Yoshii. Control and performance of a transformerless cascade PWM STATCOM with star configuration. IEEE Trans Industry Applications, 2007 43(4): 1041-1049.
- [12] Laxman Maharjan, Shigenori Inoue, and Hirofumi Akagi. A transformerless energy storage system based on a cascade multilevel PWM converter with star configuration. IEEE Trans Industry Applications, 2008 44(5): 1621-1630.
- [13] Chen Junling, Yin Zhizhu, Wang Ping and Li Yaohua. Capacitor voltage balancing control of cascaded multilevel inverter for high-power active power filters. Proceedings of the 2008 IEEE DRPT, 2008, pp. 1683-1687.
- [14] Alan J Watson, Patrick W Wheeler, and Jon C Clare. A Complete Harmonic Elimination Approach to DC Link Voltage Balancing for a Cascaded Multilevel Rectifier. IEEE Trans Industry Applications, 2007 54(6): 2646-2653.
- [15] Geng Wang, Yongdong Li, Xiaojie You. A novel control algorithm for cascade shunt active power filter. Proceedings of the 2004 IEEE PESC, 2004, pp. 771-775.
- [16] Fujii K, De Doncker R W. A Novel DC-link Voltage Control of PWM-Switched Cascade Cell Multi-Level Inverter Applied to STATCOM. Proceedings of the 2005 IEEE IAS, 2005, pp. 961-967.
- [17] Sirisukprasert S, Huang A Q, Lai J S. Modeling, analysis and control of cascaded-multilevel converter-based STATCOM. Proceedings of the 2003 IEEE PESGM, 2003, pp. 13-17.
- [18] Liu Y, Luo F L. Trinary hybrid multilevel inverter used in STATCOM with unbalanced voltages. IEEE Proceedings Electric Power Applications, 2005, 152(5): 1203-1222.
- [19] Celanovic N, Voroyevich D. A comprehensive study of neutral-point voltage balancing problem in three-level neutral-point-clamped voltage source PWM inverters. IEEE Trans Power Electronics, 2000, 15(2): 242-249.

**Yingjie He:** lecturer. dr. the School of Electrical Engineering, Xi'an Jiaotong University, No.28 Xianning West Road, Xi'an, Shaanxi, 029-82665223, China (e-mail:yjhe@mail.xjtu.edu.cn).

**Jinjun Liu:** prof. dr. the School of Electrical Engineering, Xi'an Jiaotong University, No.28 Xianning West Road, Xi'an, Shaanxi, 029-82665223, China (e-mail:jjliu@mail.xjtu.edu.cn).

**Fang Zhuo:** prof. dr. the School of Electrical Engineering, Xi'an Jiaotong University, No.28 Xianning West Road, Xi'an, Shaanxi, 029-82665223, China (e-mail:zzfz@mail.xjtu.edu.cn).